Geometry, Milk, and a Norwegian

Puzzle Corner:
Allan J. Gottlieb

My graduate school tenure has nearly ended. Things are getting pretty hectic. Right now Alice and I are typing my thesis and I am frantically trying to learn German. Things are so tight that today, January 1 (Happy New Year!), I am watching only one football game. What a sacrifice!

Harry Nelson puzzle creations are available from the creator at Box 643, Livermore, Calif.; write him for details.

Problems

The following novel bridge problem is by Walter F. Penny:

FEB1 A scrambled sequence of letters is formed by choosing a key word (containing no repeated letters) and writing the remainder of the alphabet in order after it. The letters of this sequence are written as capitals on the 26 cards ♠A through ♥2. The letters of another sequence (based on a different key word) are written as lower case on the 26 cards ♦A through ♣2. On this basis a bridge game, with a contract of four spades held by south, might be written as shown in the box at the bottom of the page. What are the key words?

A geometry problem from John T. Rule:

FEB2 Given a parallelogram ABCD. Choose two random points M and N on the parallel sides AB and CD. Draw MC, MD, NA, and NB and intersecting at E and F. Prove that the line EF divides the parallelogram into two equal areas.

This long division puzzle was submitted by Fred Heutink, who writes that the first reaction of Puzzle Corner fans "is likely to be, 'This is impossible; there must be a misprint. And what is the meaning of that X?' I have been very diligent about not making misprints (and so have the editors). And the meaning of the X will become clear as the problem progresses. No cheap tricks like leading zeros are used. But you do have to climb out of your comfortable rut in order to get around the apparent impossibility."

FEB3 Fill in the dashes with digits to make it a correct long division:

```
                7-4-
          __________
  -5--3-)9---------
         ---1--
         ------
          ----X-
          -1---7
          ------
            --2---
            -----0
            ------
              4-----
              ------
              ------
                   0
```

Here's one for animal fans from Bruce (no relation to Bobby) Orr:

FEB4 Who drinks water? And who owns the zebra?

1. There are five houses, each of a different color and each inhabited by men of different nationalities, with different pets, drinks, and cigarettes.
2. The Englishman lives in the red house.
3. The Spaniard owns the dog.
4. Coffee is drunk in the green house.
5. The Ukranian drinks tea.
6. The green house is immediately to the right (*your* right) of the ivory house.
7. The Old Gold smoker owns snails.
8. Kools are smoked in the yellow house.
9. Milk is drunk in the middle house.
10. The Norwegian lives in the first house on the left.
11. The man who smokes Chesterfields lives in the house next to the man with the fox.
12. Kools are smoked in the house next to the house where the horse is kept.
13. The Lucky Strike smoker drinks orange juice.
14. The Japanese smokes Parliaments.
15. The Norwegian lives next to the blue house.

This one, from Judith Q. Longyear, looks kinda tough to me:

FEB5 In each square of an infinite checker board, put the smallest (no negatives) integer not already occurring to the left in that row or above in that column. What number is in the 19th row and the 38th column?

```
0 1 2 3 4 5 6 . .
1 0 3 2 5 4 . .
2 3 0 1 6 . .
3 2 1 0 7 . .
4 . .
. .
.
```

Speed Department

In accordance with the plan announced last month, we publish at the end of this month's column the answers to the following two "speed" problems:

An easy one, for a change, from R. Robinson Rowe:

SD1 How many *Review* readers have noticed that "Institute" is divisible into three three-letter words (see right)? Now that they've noticed, let them equate each letter to a different digit so that the indicated summation will make MIT the greatest. Then identify 42657 as one of its lyrical symbols.

```
 I N S
 T I T
 U T E
------
 M I T
```

Joseph Horton wants you to:

SD2 Find English words containing the following letters consecutively in the given order:

a. WSST
b. YCAM
c. PERMI
d. BPOE
e. EIPT
f. NDICT
g. SIMMO

Solutions

The following are solutions to the problems published in the October/November (1972) issue of *Technology Review*:

O/N1 Suppose your arrogant chess opponent, instead of just giving you the first move, lets you set up your pieces in any positions you want, as long as you keep them on your half of the board. He then reserves to himself the privilege of first move (his pieces begin in the normal position). What is the best arrangement in order for you to force mate as quickly as possible?

I believe the proposer, Douglas Goodman, has the best solution:

The following particular position of pieces is "best" among positions which mate in two moves in the sense that it can be reached in the fewest legal moves from the normal starting position (the underlined pieces are the crucial ones).

R	N	B	Q	K	B	N	R
P	P	P	P	P	P	P	P
P		N̲	P	N̲	Q̲		P
				R̲	R̲	P	B̲
	P	P		P	P		
		B		K			

The moves to reach it are:
1. -- N-KB3
2. KN-Q6 (check), BPxN
3. NxP (mate)

or

1. -- P-K-3
2. QxKBP (mate)

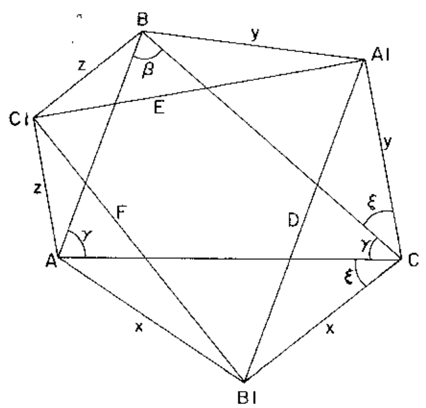
Since this is an original problem, it is possible that there is a better solution—but I doubt it very much.

Also solved by 24 other Puzzle Corner readers: John B. Allen, Ted Altman, Albert Bingaman, Gerald Blum, Ray Brinker, Joseph Carr, Andrew Fink, Brian Forst, Peter Groot, Walter Hausz, Doug

W	N	E	S	W	N	E	S	W	N	E	S	W	N	E	S	W	N	E	S	W	N	E
H	Z	Y	R																			
G	W	P	D																			
			V	O	B	F																
			E	T	S	C																
			p	a	e	t																
			d	r	c	h																
					o	i	b	A														
								Q	M	J	U											
											x	z	f	u								
													y	L	s	X						
																N	g	K	l			
																			j	w	v	n
																			l	q	k	m

Hoylman, Jon Kelly, Julius Leonhard, Brian MacDowell, Roy McDonald, Harry Nelson, Robert Potash, Roy Schweiker, Steve Shalom, Lee Sheridan, Michael Speciner, Michael Sutherland, Herve Thiriez, and Luis Villolobus.

O/N2 On the sides of a triangle ABC are erected three isoceles triangles with base angles of 15° and vertices A′, B′, and C′ external to ABC. Prove that triangle A′B′C′ is equilateral.



As many noticed, the angle should have been 30°. The following solution is from Bogdan Marcovici, who calls it a "simple-minded trigonometry" solution:

Let angle BAC $= \alpha$, angle ACB $= \gamma$, and angle CBA $= \beta$. Let CB′ = AB′ = x, CA′ = A′B = y, and AC′ = C′B = z. Let angle BCA′ = angle A′BC = angle B′CA = angle B′AC = = angle C′AB = angle C′BA $= \xi$. Let A′B′ = D, A′C′ = E, and B′C′ = F. Then, by the law of sines,

$AC/\sin\beta = BC/\sin\alpha = AB/\sin\gamma$; and

$AC = 2x\cos\xi$, $BC = 2y\cos\xi$, and $AB = 2y\cos\xi$.

Since AB′C is isosceles, etc., we have

$x/\sin\beta = y/\sin\alpha = z/\sin\gamma \overset{\Delta}{=} K$.

Since the triangle is arbitrary, let K = 1; thus $x = \sin\beta$, $y = \sin\alpha$, and $z = \sin\gamma$. Since the angle B′CA′ $= \alpha + 2\xi$, etc., and using the law of cosinees, the lengths D, E, and F are given by:

$D^2 = \sin^2\beta + \sin^2\alpha - 2\sin\beta\sin\alpha\cos(\gamma + 2\xi)$

$E^2 = \sin^2\gamma + \sin^2\alpha - 2\sin\gamma\sin\alpha\cos(\beta + 2\xi)$

$F^2 = \sin^2\gamma + \sin^2\beta - 2\sin\gamma\sin\beta\cos(\alpha + 2\xi)$

Assuming $\exists\,\xi$ such that D = E = F, the condition for, say, D = F is $\sin^2\alpha - 2\sin\beta\sin\alpha\cos(\gamma + 2\xi) = \sin^2\gamma - 2\sin\gamma\sin\beta\cos(\alpha + 2\xi)$, after cancelling $\sin^2\beta$.

Equivalently,

$\sin^2\alpha - \sin^2\gamma = 2\sin\beta\,[-\sin\gamma\cos(\alpha + 2\xi) + \sin\alpha\cos(\gamma + 2\xi)]$. (1)

Since $2 + \beta + \gamma = 180°$, $\sin\beta = \sin(\alpha + \gamma)$.

Also, since

$\cos(\alpha + 2\xi) = \cos\alpha\cos 2\xi - \sin\alpha\sin 2\xi$, and

$\cos(\gamma + 2\xi) = \cos\gamma\cos 2\xi - \sin\gamma\sin 2\xi$,

the parenthesis in (1) equals

$-[\sin\gamma\cos\alpha\cos 2\xi - \sin\gamma\sin\alpha\sin 2\xi - \sin\alpha\cos\gamma\cos 2\xi + \sin\alpha\sin\gamma\sin 2\xi]$

$= -\cos 2\xi\,[\sin\gamma\cos\alpha - \sin\alpha\cos\gamma]$

$= -\cos 2\xi\sin(\gamma - \alpha)$

$= \cos 2\xi\sin(\alpha - \gamma)$.

Thus (1) implies

$(\sin^2\alpha - \sin^2\gamma) = 2\cos 2\xi\sin(\alpha + \gamma)\sin(\gamma - \alpha)$

$= 2\cos 2\xi\sin(\alpha + \gamma)\sin(\alpha - \gamma)$.

But $\sin^2\alpha - \sin^2\gamma = ½[1 - 2\sin^2\gamma - 1 + 2\sin^2\alpha]$

$= ½[\cos 2\gamma - \cos 2\alpha] = ½[-2\sin(\alpha + \gamma)\sin(\gamma - \alpha)]$

$= \sin(\alpha + \gamma)\sin(\alpha - \gamma)$.

Hence the condition (1) implies

$2\cos 2\xi = 1$,

$\cos 2\xi = ½$, $2\xi = 60°$, and $\xi = 30°$.

The same argument obviously can be used to show that D = E, which implies D = F = E → A′B′C′ is equilateral.

Also solved by 27 other readers: Allen Andersson, Adam Apt, Edward Barry, Gerald Blum, K. J. ("Charlie") Bossart, Jorge D'Almeida, Zachary Gilstein, Kyoichi Haruta, Walter Hausz, K. Heindlhofer, I. L. Hopkins, Winthrop Leeds, Edward Mowka, Mark Novak, Harold Phinney, Robert Potash, Robert Rogoff, R. Robinson Rowe, John T. Rule, Donald Savage, Gilbert Shen, K. Schoenherr, Jay Sinnett, David B. Smith, Norman Wickstrand, J. Woolston, and Harry Zaremba.

O/N3 We cover the globe with a set of geodetic points in such a way that the distances from any point to three of its closest neighbors are the same. If we further stipulate that one of the points lies in Cambridge, Mass., and that another one lies due north of the first one, (1) what is the location of the second point? and (2) how many points fall in the U.S.? Assume the earth to be a perfect sphere.

The following is from R. Robinson Rowe:

For a geodetic net as described, its points must be the vertices of an inscribed regular polyhedron. With equal distances from each point to its three nearest neighbors, there must be three edges converging at each vertex, such as in a tetrahedron, cube, or dodecahedron. To have one point in Cambridge at latitude N 42° 22′ and another due north, the edge length must be less than the polar distance of 47° 38′, which limits the choice to the dodecahedron. With this edge fixed, the geographical coordinates of the 20 dodecahedral vertices have been computed and are shown in the table at the bottom of the page in longitudinal order with a rough landmark location for each. The computation was simpler than it may appear. Two points (the fourth and fifth) were specified. Four more were computed by solving four spherical triangles. Since the initial edge was on a meridian, these four could be reflected to four more on the other side of the meridian. This made ten, and their antipodes were the second ten to complete the set. The required answers are: (1) the second point is in the Arctic Ocean on the Cambridge meridian 400 miles from the North Pole, and (2) only one point (that in Cambridge) falls in the U.S.

Also solved by Paul Burstein, John Crawford, Brian Forst, Peter Groot, Doug Hoylman, Bogdam Marcovici, Bruce Parker, Gilbert Shen, and the proposer, Karel Jan Bossart

O/N4 Given a set of N elements arranged in a particular lineal order, rearrange the elements in a new lineal order to satisfy the following two conditions: (1) no element to be in its original position; and (2) no two elements which were originally consecutive (they may still be adjacent as long as their order is reversed).

Michael Sutherland says he "keeps thinking that there's something I'm missing in this problem," but here is his solution:

It would seem that, for N odd and greater than 3, the following procedure will satisfy the conditions:

a. Remove the middle element.

b. Reverse the order of the remaining elements.

c. Place the removed element either first or last in the order.

The set {1,2,3,4,5,6,7} arranged thus: 1 2 3 4 5 6 7 can be rearranged thus: 7 6 5 3 2 1 4 and satisfy the conditions.

Also solved by 25 other readers: Ted

Longitude				*Latitude*				*Location*
W	4°	59′	43.75″	S 50°	50′	05.36″		In Atlantic, 400 mi. NW. of Bouvet Øya
W	34	24	36.11	N 14	09	56.65		In Atlantic, 700 mi. W. of Cape Verde Is.
W	47	55	42.01	S 24	50	41.27		Near Iguape, Brazil
W	71	05		N 42	22			In Cambridge, Mass.
W	71	05		N 84	10	37.15		In Arctic, 400 mi. S. of North Pole
W	94	14	17.99	S 24	50	41.27		In Pacific, 1,400 mi. SW. of Peru
W	107	45	23.89	N 14	09	56.65		In Pacific, 500 mi. SW. of Mexico
W	137	10	16.25	S 50	50	05.36		In middle of South Pacific
W	151	20	07.89	N 18	35	04.54		In Pacific, 200 mi. SW. of Hawaii
W	170	49	52.11	S 18	35	04.54		In Pacific, 50 mi. NW. of Niue Is.
E	175	00	16.25	N 50	50	05.36		In Pacific, 100 mi. S. of Buldir Is.
E	145	35	23.89	S 14	09	56.65		In Coral Sea near Cooktown, Australia
E	132	04	17.99	N 24	50	41.27		In Philippine Sea near Dalto Is.
E	108	55		S 42	22			In Indian Ocean 500 mi. SW. of Australia
E	108	55		S 84	10	37.15		In Antarctica, 400 mi. N. of South Pole
E	85	45	42.01	N 24	50	41.27		In India, near Gaya
E	72	14	36.11	S 14	09	56.65		In Indian Ocean 500 mi. S. of Diego Garcia Is.
E	42	49	43.75	N 50	50	05.36		In U.S.S.R. near Borisoglebsk
E	28	39	52.11	S 18	35	04.54		In Africa near Victoria Falls
E	9	10	07.89	N 18	35	04.54		In Africa near Agades

Altman, Peter Anderson, Allan Andersson, Gerald Blum, Richard Bumby, Edward Gershuny, Peter Groot, Walter Hausz, Dennis Hegler, Doug Hoylman, N. Judell, M. Kunstenaar, Judith Longyear, Roy McDonald, W. J. Mitchell, Bruce Parker, Harold Phinney, Robert Potash, R. Robinson Rowe, Donald Savage, G. S. Sacerdote, Steve Shalom, Gilbert Shen, J. Woolston, and Harry Zaremba.

O/N5 In four tosses of a pair of dice, what are the odds against making a seven on the first throw and the point six on the second and fourth tosses without losing one's turn to roll?

There were a variety of answers, but Captain J. Woolston's looks right to me: If you can only lose your turn in making a point with 7, the odds against this particular sequence are:

$1 - (1/6 \cdot 5/36 \cdot 25/36 \cdot 5/36) = 1 - 625/279{,}936 = 279{,}311/279{,}936.$

Also solved by Ted Altman, Allen Andersson, Gerald Blum, Brian Frost, Peter Groot, Steve Krimbill, M. Kunstenaar, Tom Murphy, R. Robinson Rowe, Michael Sutherland, and the proposer, Harry Zaremba.

Speed Department Answers

SD1 The column totals lead to the equations:

$S + E = 10$

$N + T = 9$

$I + T + U + 1 = M$

With different digits for I, T, and U, M must be at least 7. To make MIT the greatest, try M = 9. This will make I + T + U = 8, with a choice between 1 + 2 + 5 and 1 + 3 + 4. Again, to make MIT the greatest, choose the first, making I = 5 and T = 2. Then from the second equation, N = 7. With 1, 2, 5, 7, and 9 assigned, the only digits left which will satisfy the first equation are 4 and 6. To decide which is S and which is E, the clue is in the lyrical symbol. If E = 4 and S = 6, 42657 is deciphered as ETSIN, but the other choice, E = 6 and S = 4, deciphers 42657 as STEIN of the Stein Song. Hence the summation is

```
5 7 4
2 5 2
1 2 6
-----
9 5 2
```

SD2 Here is one word (of many) in each case:

a. Newsstand
b. Sycamore
c. Permit
d. Subpoena
e. Receipt
f. Indict
g. Persimmon

Alan J. Gottlieb teaches mathematics at North Adams State College; send solutions and new problems to him at the Department of Mathematics, North Adams State College, North Adams, Mass. 01247.

Book Reviews

Continued from page 10

most offensive industries out of Moscow and impose some control on the remainder. There has been improvement. In 1962, the sulfur oxide readings were down to the Philadelphia level, though still worse than Cincinnati, and the particulate concentration had fallen by over half since 1956, though it was still higher than New York's. Tom Lehrer would be right at home. ("We'll all go together when we go.")

Look at the Incentives

Mind you, there are plenty of laws. The Conservation Law of 1960 runs 10 closely-printed pages and the Water Law of 1970 is twice as long. One would even imagine that a centralized socialist state would have an advantage over your average bourgeois democracy when it comes to getting laws passed. They run about neck and neck when it comes to enforcing them: not very well. The reason for this failure is the same in both places. No complicated modern socialist economy can be completely centralized, in the sense that not every decision can be made or policed by the supreme authority. Cabinet ministers, deputy associate assistant ministers, bureaucrats, and factory managers will all have something to do with what actually happens. So what actually happens will depend on their incentives.

If the cellulose factory manager has a plan to fulfil, with money or medals for over-fulfilling it, he or she will regard grinding out the cellulose as an objective of personal and social importance and will regard the health of the fish in Lake Baikal as the frivolous concern of environment freaks and the commercial fishermen who probably tend to be local yokels. Moreover, to a graduate of a course in cellulose engineering, cellulose will seem like one of the loveliest, most interesting materials on earth, much more interesting than fish. The boss will see it the same way; he will get his kicks from production, too. Moreover, he will have to compete for promotion with somebody who is running a bunch of cellulose plants in a different part of the country, on a different body of water, where there is no uproar about ecology.

In the nature of the case, the distribution of knowledge about the production of cellulose and attention to it is such that, in any bureaucratic infighting over enforcement of environmental directives, the advantage lies all with the factory manager and his immediate superiors. It's no wonder that the regulations face an uphill battle. If you think we in the United States are any better off, read the interesting article "Clean Rhetoric, Dirty Water" by A. Myrick Freeman and Robert H. Haveman in the Summer, 1972, issue of *The Public Interest.*

We may be better off in one respect, as Professor Goldman points out. One thing that does discourage the wasteful use of valuable natural resources is a high price for them. In a socialist society, land and natural resources are *par excellence* the property of "everybody." How can one charge the people for what is rightfully theirs? So land tends to be regarded as a free good, and builders haul away the sand and pebbles from the Black Sea beaches by the millions of cubic meters, and the removal of this buffer opens the way to extensive erosion by wave action.

Taking the Tax Route?

There is a moral here for everyone, especially for us environment freaks, in both countries. Detailed regulation cannot cover every case unambiguously, so there will have to be negotiation. In the negotiation, the polluter will usually win a lot, if not everything. (Remember, the polluter feels virtuous, too.) It is likely to be far more effective to use the law to change incentives, and the price system offers a way to do that, through the imposition of user charges, effluent taxes, and fees on those who use the environment to dispose of waste.

It won't be easy, because taxes can be eroded just as regulations can. But the tax route is easier to administer and puts the administering authority at less of an informational disadvantage compared with the operator on the ground. Think of the hassle over whether the automobile companies can or can't meet a particular set of emission standards by 1975. Suppose instead that the Congress had legislated a graduated tax on exhaust emissions to be paid by each car produced in 1975 according to its measured characteristics. The industry would fight like tigers, of course, and threaten higher prices (yes indeed) and unemployment (not necessarily). But the industry could hardly say that it can't possibly, because it can. And once the tax was on the books, the payoff to every automotive engineer and manager would shift to favor cleaner exhaust. Think about it.

On Self-Defeating Prognostications

Book Review:
Dennis L. Meadows
Dartmouth College

The Doomsday Syndrome
John Maddox
McGraw Hill Book Co., New York,
1972, x + 293 pp., $6.95

How comforting it must be to live in the world of John Maddox. The problems which occupy the attention of many institutions and people in our world either do not exist in his world or need only marginal improvements in laws and technology for their solution. Population growth rates are falling in Maddox's world, and his less industrialised countries are about to emerge into a new age of abundance; the spectre of famine has finally been put to rout, and man's activities on earth are so inconsequential that there is no potential for serious disruption of the environment. The only serious problem in Maddox's world seems to be that an increasing number of individuals manifest the "doomsday syndrome."

Because there are no other serious problems in his world, Maddox has written a little tract describing the syndrome, listing its serious implications, identifying several who suffer from it, and indicating why it is inappropriate in his world.

Those suffering from the syndrome can be identified by several symptoms: no appreciation for the innate wisdom and flexibility of political institutions; a feeling that the course of modern technology does not serve the best interests of the globe's citizens; a tendency to emphasize the unity of the living world; a preoccupation with the analogy of the spaceship earth; an attitude that one should always be prepared for the worst; a certain disrespect for the science of economics;