Puzzle Corner

Allan J. Gottlieb

Congratulations!

I have another dedication to make, this time on a pleasant note. As I am getting married, this issue of "Puzzle Corner" is hereby dedicated to my fiancee, Alice; no longer can she be referred to as "the girlfriend." I'm not exactly sure what this will do to "Puzzle Corner"—or to other aspects of my life; and any suggestions will be appreciated.

Some free advice: when any readers are ever within 500 miles of San Diego, go see the zoo—it's fantastic. Finally, if anyone has suggestions to make concerning problem selection, don't be bashful; I shall be grateful. Write to me—and send problems and solutions—at the Department of Mathematics, University of California, Santa Cruz, Calif., 95060.

Problems

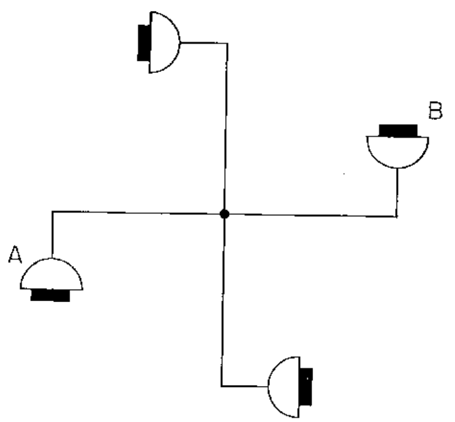
First is a chess problem from Philip D. Bell:

61 Set up chess pieces as though to start a game. White *must*, in proper sequence, make the following four moves: 1. P—KB3; 2. K—B2; 3. K—N3; and 4. K—R4. What are the four legal moves to be made by Black after which White is checkmated?

P. Markstein sends the following geometry problem:

62 Given line segments of length h_c, t_c, and m_c construct a triangle such that the altitude has length h_c, the angle bisector has length t_c, and the median has length m_c when these three lines emanate from the same angle.

Here is a new perpetual motion machine to work on—from Morris Markovitz:



63 This "ferris wheel" is constructed under atmospheric pressure. Metal cups are attached to each arm, and a pliable membrane seals the top of each cup. Glued to the center of each membrane is a weight. The "ferris wheel" is now submerged in water. The weight at cup A stretches the membrane, increasing the cup's volume. The weight at cup B compresses the cup's volume. Thus, cup A is more buoyant and the "wheel" rotates in a clockwise direction forever.

A number-theory teaser from Frank Rubin:

64 Consider the infinitely nested square root

$$\sqrt{a_1 + \sqrt{a_2 + \sqrt{a_3 + \ldots}}}$$

Prove that the nest converges when $a_n = n$. Does it converge when $a_n = n^2$? $n!$? How about when $a_n = (n^2!)^{n^2}$?

Please recall problem 29 from last year (April, 1971): "In how many different ways can eight numbers be rearranged such that no number occupies its original position? Write out all the possibilities. Find the answer for n numbers in general." John Bobbitt asks the following (apparently hard) question about it:

65 What fraction of the arrangements possible meet the criteria of having every digit change its position? In other words, does the sequence $a_n = k_n/n!$ converge?

Speed Department

William Glassmire sends in the following, which he says has "some historical significance":

1 It is well known that an 8 × 8 checkerboard with diagonally opposite corner squares removed cannot be covered with 31 dominoes (each of which covers two squares of the board). Suppose that two squares are removed arbitrarily, subject only to the restriction that one is black and the other is white. Is it always possible to then cover the board with 31 dominoes?

A. Porter wants you to:

2 Find the relationship between the sides of a rectangle which guarantees that doubling the short side gives a new rectangle having the same relationship.

Solutions

41 Given these hands, against South's contract of six diamonds, West leads ♥Q:

North:
♠ A K 5
♥ 10 7 4 2
♦ Q J 7 6
♣ J 10

West:
♠ 10 4
♥ Q J 6 5
♦ 10 9 4 2
♣ 9 7 6

East:
♠ J 9 8 2
♥ 9 8 3
♦ 8
♣ K Q 5 3 2

South:
♠ Q 7 6 3
♥ A K
♦ A K 5 3
♣ A 8 4

How does South manage to bring home a small slam?

The following is from Jeffrey A. Miller: Declarer must get two heart ruffs in his hand and establish a fifth black trick by squeezing East, in addition to taking his two good hearts and three trump tricks (he must yield one trump trick to West in order to get his two ruffs). He must also use the diamond suit to enter the dummy once. The order of play: 1. Declarer wins ♥A. 2. Declarer wins ♦A. 3. Declarer wins ♥K. 4. Declarer leads low trump to Dummy, and if West ducks Dummy inserts ♦7 (this would allow declarer to take all 13 tricks); assuming West goes with ♦10, Dummy will win with ♦J; East pitches low club. 5. Dummy leads small heart. 6. Declarer enters Dummy by leading small spade to ♠K. 7. Declarer ruffs last heart high in hand (East discards low club). 8. Since Declarer is now out of trumps, he must enter the Dummy with a low spade to Dummy's ♠A. 9. Dummy leads high ♦Q; East pitches a low club, Declarer likewise discards a low club, and West follows with ♦4. 10. Now Dummy plays its last trump; East is squeezed in the black suits; if he unguards is ♠J, Declarer will discard a low club; if he unguards his ♣K Declarer will discard a low spade; West will win the trick with his high trump and must return a club. 11. Declarer wins ♣A. 12 and 13. Declarer wins his two good black tricks (either ♠Q and ♠7 or a club to Dummy's ♣10 and ♣Q).

Also solved by Richard Bator, Andrew Fillat, Winslow H. Hartford, George Heyman, Stanley A. Horowitz, Elmer C. Ingraham, Michael A. Kay, Mrs. Martin S. Lindenberg, John W. Meader, Joseph Orenstein, R. Robinson Rowe, Patrick J. Sullivan, Smith D. Turner, Dr. Stephen S. Washburne, and George J. Wynne.

42 What is the smallest number (N) of n digits (n > 1) which, if you remove the digit (d) from the units place and relocate it in front of the n's place, exactly multiplies the number N by that digit d?

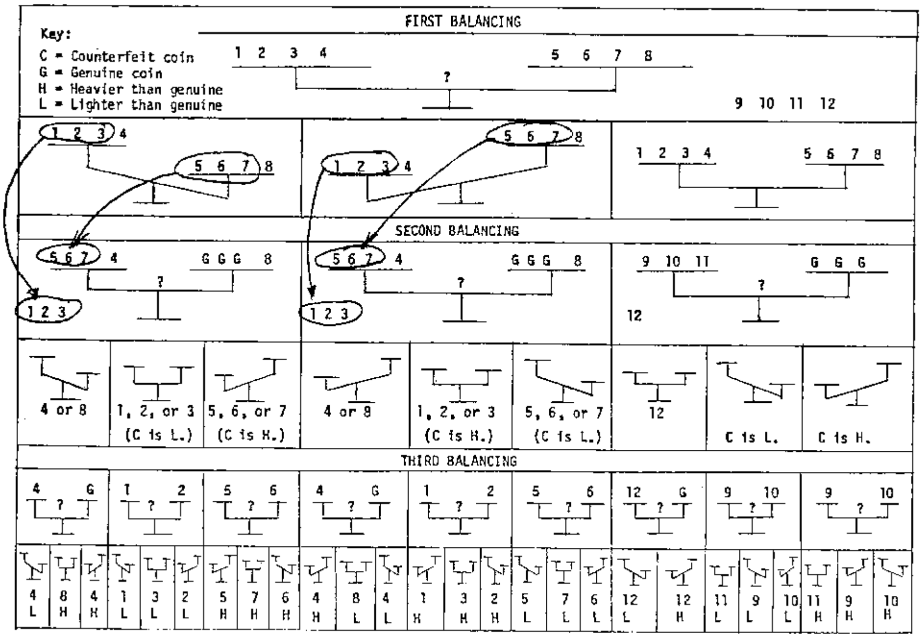
R. Robinson Rowe notes that a "trivial" answer, unintended by the proposer, is N = 11. If instead of specifying n > 1 he required d > 1, then the answer is N = 102564. The complete set for all d is:

d	*n*	*N*
1	1	1
2	18	105, 263, 157, 894, 736, 842
3	28	1, 034, 482, 758, 620, 689, 655, 172, 413, 793
4	6	102, 564
5	42	102, 040, 816, 326, 530, 612, 244, 897, 959, 183, 673, 469, 387, 755
6	58	1, 016, 949, 152, 542, 372, 881, 355, 932, 203, 389, 830, 508, 474, 576, 271, 186, 440, 677, 966
7	22	1, 014, 492, 753, 623, 188, 405, 797
8	13	1, 012, 658, 227, 848
9	44	10, 112, 359, 550, 561, 797, 752, 808, 988, 764, 044, 943, 820, 224, 719

43 A "most interesting" number is 012345679 (8 is missing). In the range 0 to 81, multiplying by any multiple of 9 gives an answer with all digits the same; multiplying by any other multiple of 3 gives an answer containing three different digits; multiplying by any other number gives an answer containing nine different digits (none repeated); and the missing digit is cyclical with increasing multiplier except that 0, 3, 6, and 9 are never missing. Why?

Raymond Gaillard submitted this solution:

$x \equiv 16 \pmod{39}$; $x \equiv 27 \pmod{56}$;
$27 + 56t \equiv 16 \pmod{39}$;
$56t \equiv -11 \pmod{39} \equiv 28 \pmod{39}$;
$2t \equiv 1 \pmod{39}$ or $2t \equiv 1 + 39u$.
$x \equiv 27 + 2(28)t \pmod{39}$;
$x \equiv 27 + 28(1 + 39u)$;
$x \equiv 55 + 28 \cdot 39u \equiv 55 + 1092u$.



For u = 1 this becomes
$x \equiv 1147$.
So $x \equiv 1147 \pmod{39 \cdot 56}$ satisfies the condition; i.e., $1147 \equiv 27 \pmod{56}$ and $1147 \equiv 16 \pmod{39}$.

Also solved by R. Robinson Rowe.

44 Find the set of angles x and y for which $\sin(x + y) = \sin x + \sin y$; and prove that your set is exhaustive.

I like John E. Prussing's solution; do you? He writes: Use the identity $\sin x + \sin y = 2 \sin \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y)$, and define
$w = \tfrac{1}{2}(x + y)$ and $z = \tfrac{1}{2}(x - y)$.
Then the equation to be solved is simply
$2 \sin w \cos z = \sin 2w$.
Subtracting from this the familiar identity,
$2 \sin w \cos w = \sin 2w$,
one obtains the equation
$\sin w(\cos z - \cos w) = 0$.
The solutions to this are
(i) $\sin w = 0$ and (ii) $\cos z = \cos w$.
The general solution to (i) is
$w = k\pi$, implying $x + y = 2k\pi$.
The general solution to (ii) is obtained by using the identity for the cosine of a sum to yield
$\cos \tfrac{1}{2}x \cos \tfrac{1}{2}y + \sin \tfrac{1}{2}x \sin \tfrac{1}{2}y =$
$\cos \tfrac{1}{2}x \cos \tfrac{1}{2}y - \sin \tfrac{1}{2}x \sin \tfrac{1}{2}y$,
which yields
$2 \sin \tfrac{1}{2}x \sin \tfrac{1}{2}y = 0$.
The general solutions to this are
$x = 2k\pi$, y arbitrary, and
$y = 2k\pi$, x arbitrary.
Also solved by Harold Donnelly, R. Robinson Rowe, and Victor W. Sauer.

45 You are given a stack of 12 coins, which appear identical to one another, and are told that one is counterfeit and can be distinguished only by its weight, which is not the same as the genuine coins. Unfortunately, you do not know whether the counterfeit coin weighs more or less than the genuine ones. Using only a balance, how do you find the counterfeit in a minimum number of balancing operations?

Apparently there was some confusion about this. A balancing operation involves simply one balancing—not comparing one fixed group to each of several other groups. Benjamin Whang sent me the above "pictorial" solution, noting that the problem as stated does not require to determine whether the counterfeit is heavier or lighter; it only requires to *find* the counterfeit. He notes that the middle section of three in the third balancing is not really necessary, since it can be considered a mirror image of the left section.

This problem was popular; solutions also came from Captain P. O. Chapman, Carl L. Estes, II, Bruce Fauman, Raymond Gaillard, Carl J. Greever, Maurice A. Hoffman, Stanley A. Horowitz, Elmer C. Ingraham, W. J. Hart, Lowell Kolb, Hubert duB. Lewis, Mrs. Martin S. Lindenberg, R. Robinson Rowe, Christopher Scholz, John R. Selin, W. H. Stephenson, Jr., Dr. Stephen Washburne, George J. Wynne, and the "team" of Ronald G. McKeown, Raul F. Pupo, William P. Quinn, and Thomas W. Schwegel.

Better Late Than Never

Raymond Gaillard has submitted a solution to problem 39.

Books

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During the 1970's and 1980's, long-range planning—particularly that of the larger corporations—will be increasingly concerned with two kinds of change: changes in the business environment due to social and political causes, and the development of the multinational firm. These are the general themes of two books upon which this reviewer has been asked to comment.

The first is a synthesis from a General Electric study concerned with the developing social and political trends, and aims at the integration of these trends into business plans. The second book stems from work done at Salford University (England) and provides a good overview of the history, current state and possible future of multinational companies, and discusses their political and social impacts. The authors of both books have supplemented the findings of their studies with information gained through interviews with educators, businessmen and government representatives, as well as information from the available literature.

Developing trends affecting the business environment stemming from social changes were interpreted in the General Electric study as the interaction of eight significant forces:

- ☐ Increasing affluence
- ☐ Economic stabilization
- ☐ The rising tide of education
- ☐ Changing attitudes toward work and leisure
- ☐ The growing interdependence of institutions
- ☐ The emergence of the "post-industrial" society
- ☐ Increasing pluralism and individualism
- ☐ The urban/minority problem

Institutional changes—in government, the labor force, business, unions, and educational institutions—are segregated off into a chapter of their own. Also considered separately are the impacts of changing value systems—changes in attitudes towards work and leisure, emphasis on "quality of life," rejection of authoritarianism and dogmatism, emphasis on pluralism and individualism. Consequences of student revolts are discussed, with an anticipation of probable youth-related changes in values (utilizing Maslow's "hierarchy of needs" as a framework). An interesting profile of significant value-system changes from 1969 to 1980 is presented, in relation to such pairs as war/peace (in the sense of military might versus economic development), nationalism/internationalism, federal government/state and local government, public enterprise/private enterprise, materialism/"quality of life", work/leisure.

All of the above may suggest a picture of the world in which tastes and habits change in an autonomous fashion, and business can only keep informed and take its chances. But the authors conclude by suggesting a more positive role—active, rather than reactive. Business should not regard itself merely as self-contained or self-regulating, but be willing to work in national and community coalitions.

The Business Environment of the Seventies
Earl B. Dunckel, William K. Reed, Ian H. Wilson
McGraw-Hill Book Company, 1970, 129 pp., $15.

Invisible Empires
Louis Turner
Harcourt Brace Jovanovich, Inc., 1971, 228 pp., $6.95