Puzzle Corner

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A "Most Interesting" Number

A great deal happened this summer. The excitement of planning for a year in Santa Cruz was severely tempered by a family loss: my mother suffered a heart attack on July 13 and died early the following day. Since she enjoyed reading this column, I dedicate this installment of Puzzle Corner to her memory.

Since this issue is the first in a new volume, here are the "rules" for Puzzle Corner: every month we publish five problems and several "speed problems," selected from those suggested by readers. The first selection each month will be either a bridge or a chess problem. Three months later we select for publication one of the answers—if any—to each problem received by then from readers, and we publish a list of other readers submitting correct answers. Answers received too late or additional comments of special interest are published as space permits under "Better Late Than Never." Except under unusual circumstances, no answers or discussions are published concerning "speed problems." As you see, readers' participation is not only welcome; it's essential to the success of "Puzzle Corner." Address problems and answers to me at the Department of Mathematics, University of California, Santa Cruz, Calif. 95060.

Problems

Our bridge problem for this month is from John W. Meader, who calls the following "an easy little bridge problem":

41 Given these hands, against South's contract of six diamonds, West leads ♥Q:

North:
♠ A K 5
♥ 10 7 4 2
♦ Q J 7 6
♣ J 10

West:
♠ 10 4
♥ Q J 6 5
♦ 10 9 4 2
♣ 9 7 6

East:
♠ J 9 8 2
♥ 9 8 3
♦ 8
♣ K Q 5 3 2

South:
♠ Q 7 6 3
♥ A K
♦ A K 5 3
♣ A 8 4

How does South manage to bring home a small slam?

The following problem from E. A. Nordstrom is an offshoot of last year's number 37:

42 What is the smallest number (N) of n digits which, if you remove the digit (d) from the units place and relocate it in front of the n's place, exactly multiplies the number N by that digit d? The answer: N = 1. Since that is too easy, replace ". . . n digits . . ." in the problem as originally stated with ". . . n digits ($n > 1$). . ."

Art Delagrange has discovered "a most interesting number" and offers everyone a chance to play with it:

43 The number is 012345679 (8 is missing): in the range 0 to 81, multiplying by any multiple of 9 gives an answer with all digits the same; multiplying by any other multiple of 3 gives an answer containing three different digits; multiplying by any other number gives an answer containing nine different digits (none repeated); and the missing digit is cyclical with increasing multiplier except that 0, 3, 6, and 9 are never missing. Why?

A trigonometry problem has been supplied by Frank Rubin; it was published in *Electronic News* "some time ago," Mr. Rubin writes, "but the contributor did not provide any proof of his answer." Can you?

44 Find the set of angles x and y for which $\sin(x + y) = \sin x + \sin y$; and prove that your set is exhaustive.

Here's a problem for all the G-men in the crowd. It was submitted by Robert Baird, but I have also heard it over dinner from a former M.I.T. roommate, Martin Aldridge:

45 You are given a stack of 12 coins, which appear identical to one another, and are told that one is counterfeit and can be distinguished only by its weight, which is not the same as the genuine coins. Unfortunately, you do not know whether the counterfeit coin weighs more or less than the genuine ones. Using only a balance, how do you find the counterfeit in a minimum number of balancing operations? (As a hint—if you need one—the minimum number of weighings is three.)

Speed Problems

Here's one from Ely Shelleen:

What day of the week (if any) can never be: (1) February 29; (2) The first day of a new century?

Greg Schaffer wants you to prove:

For all real x and positive integers n, $2 \leq (1 + x)^n + (1 - x)^n$

Solutions

31 South has rashly bid six spades:

North:
♠ 2
♥ A Q 10 9 8 7
♦ A K Q
♣ J 3 2

West:
♠ 6 5 4 3
♥ K J
♦ 8 7
♣ K Q 10 9 4

East:
♠ 10 9 8 7
♥ 6 5 4 3 2
♦ 6 5 4 3
♣ —

South:
♠ A K Q J
♥ —
♦ J 10 9 2
♣ A 8 7 6 5

How can South make the contract against a spade lead?

The following solution is from Leon Kaaty, who found the fact that East had only one conceivable entry a "dead giveaway": The declarer wins the opening spade lead and cashes his four top spades, discarding the ♦A, ♦K, and ♦Q in dummy. Next the ♦J, ♦10, and ♦9 are cashed and dummy's three clubs are discarded. Declarer now leads ♦2, throwing off a low heart from dummy. At this point East is the only player with a diamond remaining, and he is forced to win the trick; but he has only hearts left thereafter. The forced heart return gives the declarer a free finesse which he takes. Upon cashing the two high hearts in the dummy, West's ♥K and ♥J fall, establishing North's heart suit to win the rest. The problem was also solved by 33 other bridge fans—the list is too long to print—including the proposer, Edwin G. Davis.

32 Prove that a regular icosahedron having the same volume as a regular dodecahedron has the same perpendicular distance from the center to a face.

Norman L. Apollonio is hereby unanimously declared guilty of submitting an incorrect problem. The following proof was supplied by the foreman of the jury, Kard Jan Bossart: According to the problem, icosahedrons and dodecahedrons of equal volume would have equal inscribed spheres. This goes contrary to intuition; the 20-faced solid should be a closer approximation to the sphere than the 12-faced one. This led me to compute the table below (to slide rule accuracy) for the ratio of the volumes of polyhedron to inscribed sphere:

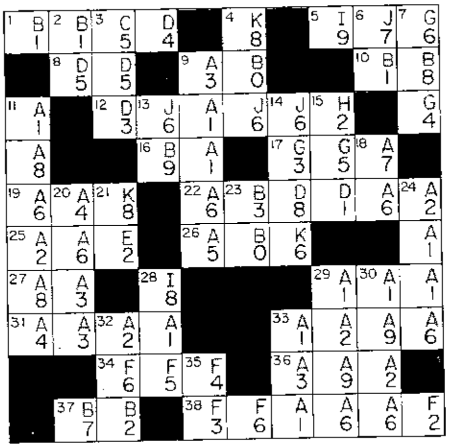
Sphere	Icosah.	Dodecah.
1	1.20	1.33
Octah.	Cube	Tetrah.
1.56	1.91	3.31

It appears that—for once—my intuition was right and that consequently Mr. Apollonio was wrong. (Mr. Bossart follows this with an analysis developing the volume of the two polyhedrons in terms of the radius of an inscribed sphere: in the icosahedron, $V = 5.02r^3$; in the dodecahedron, $V = 5.58r^3$. Space does not permit publication of his development.) Three other jurors concurred: Ted Leahy, William Ackerman, and R. Robinson Rowe.

33 Two players play a game in which each player alternately selects dates of the year subject to the restrictions: (1) The first date must be in January, and each subsequent selection must (2) agree with the immediately preceding date in either month or number and (3) be later in the calendar year. The winner is the player who is able to select December 31. Which player has the advantage and what is the winning strategy?

John G. Miller writes: The game favors the first opponent to pick January 20 or the next applicable date in the following sequence: February 21, March 22, April 23, May 24, June 25, July 26, August 27, September 28, October 29, November 30. The strategy is: once on the sequence, stay there. If your opponent starts by picking January 1 through January 19, you pick January 20. If he picks January 21 through January 31, you pick the later month which corresponds to his day of the month. Any time he picks a lower day of the month than the sequence shows, choose his month but raise the day of the month to the sequence. Any time he raises the day of the month beyond that of the sequence for that month, choose the later month in the sequence that corresponds to his numbered day. If he starts with January 20, stay loose and see if he knows the sequence. There were 22 other successful strategists—once more, too many to list—including the proposer, John E. Prussing.

34 In the crossword printed below, the problem is to determine the total quantity of vehicles (both types A and B) purchased (horizontal 5). It is known that the center of gravity (CG) of type A vehicles is identical to that of type B vehicles. The customer bought some A vehicles and some B vehicles. All values are positive whole numbers. There were 45 charter members of the Society of Aeronautical Weight Engineers (S.A.W.E.). Use the speed of light as 186,284 mi./sec., and 4,633 lb./in.sq./slug ft. sq.



Horizontal:

1. Weight of vehicle A.
5. Total quantity of vehicles A and B purchased.
8. Sum of digits of 22 horiz. and 11 vert.
9. Robert's age.
10. Age of the second Fred in Robert's family.
12. Total cost of vehicles A and B purchased.
16. Sum of the ages of the three Freds in Robert's family.
17. CG of forward section of vehicle B.
19. Weight of aft section of vehicle A.
22. Moment from station O of vehicle A.
25. Unit price of vehicle A.
26. Weight of forward section of vehicle A.
27. Weight-reduction ideas considered, but not implemented, for vehicle B.
29. Non-prime number.
31. Next in series 7, 6, 5 . . .
33. Product of the ages of the three Freds.
34. Weight of vehicle B.
36. Unit price of vehicle B.
37. Age of eldest Fred.
38. Moment from station O of vehicle B.

Vertical:

2. Cost-increase weight-reduction ideas implemented in vehicle B.
3. CG of vehicles A and B.
4. CG of aft section of vehicle A.
6. Cost-decrease weight-reduction ideas implemented in vehicle B.
7. CG of aft section of vehicle B.
9. Robert discovered that when his name was arranged as a division problem, there were four numerical solutions:

   ENNIS
   J) ROBERT

   One quotient is 311 - - .
11. Speed of light (*).
13. Greater than 60.
14. Greater than the square of the age of the eldest Fred.
15. Total weight-reduction ideas considered for vehicle B.
18. LXXVI.
20. Lb. in. sq. per slug ft. sq.
21. No-cost-change weight-reduction ideas implemented in vehicle B.
23. Robert's age. He is 12 years older than the middle Fred.
24. Square of the number of charter members of S.A.W.E.
28. Quantity of vehicles B purchased.
29. Square of atomic number of krypton.
30. Year of election of the 70th U. S. Congress.
32. Weight of forward section of vehicle B.
33. Decimal equivalent of the binary 10,000,011.
35. Prime number.

M. A. Clark, writing all the way from Rome, has submitted a most detailed analysis of his method of solution, which is far too long to print here; readers wishing copies should write to the Editor of the *Review,* Room E19-430, M.I.T., Cambridge, Mass. 02139. The crossword in the left column shows the solution in black numbers and the sequence of stages in which Mr. Clark completed it in grey letters. Mr. Clark found clue 30V the most difficult—he "had to be assisted by an American!" There were also solutions from the proposer, John Mandl, and from William Ackerman, John C. Ebert, Robert C. Fleetham, Robert H. Griffin, R. Robinson Rowe, Sammy Loebl and Loren Bonderson jointly, and one anonymous reader. This is exactly five more solutions than received by the S.A.W.E. when the problem was published in that Society's *Journal.*

35 Given the following sequence of numbers:
1,2,1,3,2,1,2,3,4,1,2,1,3,2,1,2,3,5,2,3,4,1,4,3, 1,4,5,2,3,1,5,2,1,2,4,6,1,2,1,4,2,1,2,6, . . .
find the method of formation.

As expected only the proposer, Preston Bush, solved this problem: Elements of the sequence are formed consecutively and are as small (integers > 0) as possible without violating two rules: (1) No two consecutive blocks of numbers can be identical, e.g. 1,2,1,2, is not allowed and 1,1 is not allowed; and (2) No three equally spaced numbers can all have the same value, e.g. *1*,2,*1*,3,*1* is not allowed. According to the second rule, for example, the 10th and 15th elements are 1 so the 20th cannot be, and elements 21 and 26 are 4 so element 31 cannot be.

Better Late Than Never

Late responses have been received as follows:

12 William Ackerman.
21 R. H. Gaunt and John E. Burchard.
23 Smith D. Turner.
24 Ann Giffels.
26 W. C. Backus.
27 Harold Donnelly.
29 Harold Donnelly.

In addition, A. R. Latven submits the following discussion of problem 24 which will interest many readers: Despite your evident uncertainty, the solution to the problem given by D. R. Wheeler in *Technology Review* for June is unquestionably correct. One of my favorite fun-flights as a private pilot confirms that geo-algebraic calculation. I release a constant-altitude balloon into a wind of unknown velocity and bee-line my 3-km./min.$^{-1}$ aluminum bird in *any* compass direction chosen by my companions for a precise leg of 30 minutes, then turn 180° and exactly 30 minutes later my prop bursts that balloon every time. It's quite a thrill to see the balloon suddenly appear head-on within seconds of the projected time, sometimes as far as 30 kilometers from the point of release! The fun part comes from not knowing just where the collision will occur geographically.

The collision point, of course, is point 2 in Wheeler's diagram; it represents the point to which a balloon is carried by the prevailing wind in the period of time under consideration. It also represents the direction and distance of drift of all aircraft regardless of compass heading or airspeed. A 747 will drift just as far, no more and no less, as a Piper Cub in the same period of time. Thus if, simultaneously, a 747 started out northward, a DC-3 eastward, a Lear Jet southward, and a Piper Cub westward—and each proceeded as described all four craft would converge upon point 2 exactly two hours later. What a mess!

Finally, let me point out that the problem as given is specious at best. Upon reaching point 2 after two hours the pilot is said to turn straight to the point of departure. If he doesn't know where he is, he cannot perform this maneuver. If he does know where he is, he must know the distance to the point of departure and thus can calculate windspeed simply by dividing that value by two. You might say that the final turn can be accomplished simply by turning directly into the wind. True enough. However, he has no means of determining wind direction accurately. Tracking the shadow of a cumulus cloud yields a crude approximation at best; even if he looks down upon the long plume from a tall smokestack he knows that it points the direction of the surface wind and that all surface winds shift counterclockwise because of the lessened Cariolis force, often 20° or more. Thus the final turn can only be accomplished effectively above an industrial complex situated on the equator! But this was not included in the problem.

The latter has provided some excellent mental exercise, to say the least!