Puzzle Corner
Allan J. Gottlieb

California Here I Come!

No snow next year! My thesis adviser has received an appointment at the University of California at Santa Cruz (75 miles south of Berkeley) and was able to get an assistantship for me as well. And with some other help, my girl friend has been accepted as an extension student, and Brandeis was kind enough to give her a leave of absence with credit. The upshot is that starting in September I will be residing in the middle of a redwood forest about one mile from the Pacific Ocean in sunny Central California—enjoy the slush.

All correspondence should now be sent to me at the Mathematics Department, University of California, Santa Cruz, Calif., 95060.

Problems

41 The bridge problem this month, taken from the *New York Times* of January 4, reached me from some guy named Mattill from *Technology Review.* It is the hand with which two M.I.T. entrants—Mike Gurwitz, a recent graduate, and Mark Feldman, a senior—won the first trans-Atlantic bridge competition late last year. How did the winners do it?

North:
♠ K 9 4
♥ 8 2
♦ A Q 10 6 5
♣ 8 6 2

West:
♠ Q 3
♥ Q 10
♦ J 8 7 2
♣ K J 9 5 4

East:
♠ J 10 8 6
♥ A 9 4
♦ 9 4 3
♣ Q 10 3

South:
♠ A 7 5 2
♥ K J 7 6 5 3
♦ K
♣ A 7

Neither side was vulnerable. The bidding:

North	*East*	*South*	*West*
Pass	Pass	1 club	Pass
2 diamonds	Pass	2 hearts	Pass
3 diamonds	Pass	3 N.T.	Pass
Pass	Pass		

West led the ♣ 5.

Here is a fairly easy number theory problem from John E. Prussing:

42 Show that for any positive integer n, $3^{2n+1} + 2^{n+2}$ is divisible by 7.

Here is a somewhat harder one from Warren Himmelberger:

43 Two brothers owned an orange grove. After their oranges were picked, one brother divided them into 39 baskets and found that there were 16 oranges left over. The second brother then took all of the oranges and put them into 56 smaller baskets, finding that there were now 27 oranges left over. What is the smallest number of oranges possible, and what are the next four larger numbers of oranges which fulfill the conditions?

Walter Penney sent in the following combination game and mathematical problem with a message included:

44 The letters of a saying are written in order on squares 1 to 32 of a checkerboard with the usual numbering system. A certain game of checkers might then be written:

1.	R—Y	E—H
2.	E—R	H—B
3.	F—O	N—B
4.	I—H	H—B
5.	O—E	I—N
6.	T—I	W—H
7.	I—H	H—B
8.	B—T	R—W
9.	T—F	E—I
10.	F—O	N—B
11.	S—W	E—T
12.	E—T	W—H
13.	T—F	I—N
14.	F—O	N—B
15.	W—M	I—I
16.	Y—O	B—I
17.	H—B	S—N
18.	O—S	D—H
19.	D—H	Drawn

Finally, a magic square—which he describes as "a square matrix of numbers such that rows, columns, and diagonals all sum to the same total"—from David DeWan:

45 Determine an 8 × 8 upside-down magic square from the 64 three-digit numbers made of the digits 1, 6, 8, and 9.

Speed Department

Our first quickie is from Arthur W. Porter:

SD13 Consider the case of a bookshelf containing a set of nine volumes, with volume I at the left and volume IX at the right. The thickness of the pages (from page 1 to the last page, inclusive) in each book is 1". (The books are paperbacks, so the cover thickness is negligible.) If an enterprising bookworm starts eating at page 1 of volume I and proceeds right, eating his way through to the last page of volume IX, how far does he travel?

Our final selection for Volume 73 is a classic probability problem from Russell A. Nahigian:

SD14 Suppose that one is present in a room in which there are n people. What must n be to make the probability greater than 50 per cent that two persons in the room have the same birthday? (Disregard leap years.)

Solutions

26 Given the following hands, South the successful bidder at 4 spades, and the opponents' lead of ♣ 7:

North:
♠ K
♥ 9 5 4 2
♦ K 7
♣ K Q J 8 6 5

West:
♠ 8 6
♥ K J 10 6 3
♦ A J 9 6 3
♣ 7

East:
♠ 7 4
♥ A Q 8
♦ Q 4 3
♣ A 10 4 3 2

South:
♠ A Q J 10 9 5 3 2
♥ 7
♦ 10 8 2
♣ 9

Do you choose offense or defense?

The following solution is from Charles L. Eater, III:

If the defense plays his hand properly, there is no way the declarer can make four spades. East takes the first trick with the ♣A, then follows with the ♦Q. At this point if declarer refuses to cover with his ♦K, the defense can take its two aces—♥A and ♦A—and declarer will be down. If the declarer covers with the ♦K, he is on the board with no safe lead. If he leads a heart or a diamond he will lose to the defense which can then lead a trump to get rid of dummy's ♠K. At this point a diamond or a heart lead will either force declarer to his hand with a trump—if he played a heart earlier—where he will eventualy be forced to give the defense two diamond tricks; or it will lose to the defense who can then take the necessary tricks. A club lead will force declarer to trump in his hand where he will again have to lead a diamond or a heart eventually. If, at the third trick, he led a spade or club, the same problem would arise—that is, he would eventually be in his hand and would have to lead a diamond or a heart to the defense.

Also solved by John Babbitt, Philip D. Bell, Shirley V. Bugg, Ed Gershuny, Winslow H. Hartford, Elmer C. Ingraham, Michael A. Kay, Mrs. Martin S. Lindenberg, John W. Meader, R. Robinson Rowe, John P. Rudy, Gene Schacht, Les Servi, Richard Simon, Hugh D. Sims, and the proposer, Paul D. Berger.

27 Three hoboes spent the day gathering aluminum cans to sell back to a can company for ½¢ each. That night they left the cans in a pile to be divided equally in the morning. During the night one hobo decided he wanted his share then. He divided the cans into three equal piles and had one odd can left over. He took his pile, pushed the other two piles back together, threw the odd can on the big pile, and stole away into the night with his cans. Later another of the hoboes did the same thing: three equal piles, one odd can left over, took his share, threw the odd can on the big pile, ran away. Still later the third hobo did the same thing and had the same experience with one can. After the hoboes had left, the number of cans in the big pile was exactly divisible by three. In the morning a fourth hobo found the pile left behind by the others and sold them to the company. How much money did he receive? (Hint: it was less than 25¢.)

This was submitted by Winslow H. Hartford:

If 3x is the number of cans left, by calculating back we find the original number was $[27(3x - 1)]/8 + 1 = (81x - 19)/8$. This becomes integral when $x = 3$; $3x = 9$ and the fourth hobo got 4½ cents. There were originally 28 cans:

Pile	Hobo takes
28	9
19	6
13	4
9	

Not much of a day's work! The algorithm is also integral if $x = 11$, $3x = 33$, and the fourth hobo gets 16½ cents. Now:

Pile	Hobo takes
109	36
73	24
49	16
33	

Also $x = 19$ gives $3x = 57$, and the return is 28½ cents.

Also solved by John Babbitt, Clark Baker, Raymond Gaillard, D. P. Gaillard, Ed Gershuny, Elmer C. Ingraham, Michael A. Kay, Mrs. Martin S. Lindenberg, Ross Rapaport, R. Robinson Rowe, James W. Royle, Jr., John P. Rudy, John Schwarz, Les Servi, Ermanno Signorelli, Hugh D. Sims, Smith D. Turner, and Harry Zaremba.

28 Solve the following learning curve formulation for B:

$A + 1 = N^{1-B} + AB$,

knowing that there is only one solution in the range $0 < B < 1$.

Charles Wells writes that he does not believe the problem has an "exact" solution, that it must be solved by approximation. (But, he asks, "what is an 'exact' solution, anyway? The solution to this problem is surely irrational, and irrational numbers can only be approximated.") Mr. Wells continues:

However, one can say when the problem has no solution at all. Note that one must have $N \geqq 0$. Setting $x = (1 - B)$, the problem is to find x with $0 < x < 1$ such that $N^x = Ax + 1$; or, in other words, when does the curve $Y = N^x$ intersect the straight line $Y = Ax + 1$? They intersect at 0. If they intersect again between 0 and 1, then since $Y = N^x$ is concave upward (first derivative is $N^x \log N$, second is $N^x \log^2 N$), the slope of the line $Ax + 1$ must be larger than the slope of the line tangent to the curve at 0 and smaller than the slope of the secant line cutting the curve and 0 and 1. It follows that the problem can be solved if and only if $\log N < A < N - 1$. Note that there are such A for all non-negative N except $N = 1$. The solution, when it exists, can be approximated by bisection (for example).

Also solved by John Prussing, R. Robinson Rowe, John P. Rudy, and Smith D. Turner.

29 In how many different ways can eight numbers be rearranged such that no number occupies its original position? Write out all the possibilities. Find the answer for n numbers in general.

R. Robinson Rowe writes:

This was an interesting problem until I came to the assignment of "check your answer by writing out all the possibilities." Using 1, 2, 3, 4, 5, 6, 7, and 8 for the eight numbers, I have limited my check list to the first two and last two, viz.:

Ordinal	Arrangement
1	21436587
2	21436785
gap	
14832	87654312
14833	87654321

For n numbers, in general, if we let A_n be the number of arrangements:

$A_n = (n - 1)(A_{n-1} + A_{n-2})$, with $A_1 = 0$ and $A_2 = 1$, whence

n		A	.37n!
1		0	0
2		1	1
3	2(0 + 1) =	2	2
4	3(1 + 2) =	9	9
5	4(2 + 9) =	44	44
6	5(9 + 44) =	265	266
7	6(44 + 265) =	1854	1865
8	7(265 + 1854) =	14833	14918

I have shown in the third column that A is approximately proportional to n!; instead of 0.37, the proportionality factor is nearer 0.3769—or 0.367894

In a second letter, Mr. Rowe writes that the above "began to look familiar," and so he "looked it at some more" after mailing the first letter:

Consider the series

$1/2 - 1/6 + 1/24 - 1/120 + 1/720 - \ldots$

with alternating signs and successive factorials as denominators. Its convergents, including an initial null, are

0, 1/2, 1/3, 3/8, 11/30, 53/144, . . .

which are equal to the successive ratios $A_n/n!$

0/1, 1/2, 2/6, 9/24, 44/120, 265/720, . . .

and the limit of the series is

0.367 879 441 172 . . . = 1/e, leading to the approximation $A_n = n!/e$. I have tested this approximation for all n up to 8, for which $A_8 = 40320/e = 14832.89907\ldots$ The result rounded to the nearest integer is correct. Hence for a supplementary answer to the quest for a general formula for the number or arrangements of n numbers with none in the initial position, I submit $A_n = [n!/e]$, where the brackets mean "nearest integer to."

Also solved by John Babbitt, Ed Gershuny, R. Robinson Rowe, Smith D. Turner, Charles Wells, and Harris Zaremba.

30 Given the notations

$f: R \rightarrow R$,

$\Delta_t f(x) = f(x + t) - f(x)$,

show that there exists f continuous such that

$\lim_{x \to +\infty} \Delta_t f(x) = 0$ for all rational t; and

$\lim_{x \to +\infty} \Delta_t f(x)$ does not exist for almost all (measure-theoretic) t.

This was solved only by the proposer, Charles Heiberg:

Define intervals I_n ($n = 1, 2, \ldots$) of length $2n + 1$ so that $I_1 = [0, 3]$, $I_2 = [3, 8]$, $I_3 = [8, 15]$, etc. Define

$g : R \rightarrow R$ by

$g(x) = 0$ if $x \neq (0, 1)$

$g(x) = 1$ if $x = y_2$

$g(x)$ is linear on [0, ½] and [½, 1].

Write I_n as the union of $2n + 1$ closed intervals of unit length

$J^n_{-n}, J^n_{-(n-1)}, \ldots J^n_{-1}, J^n_0, J^n_1, \ldots J^n_n$

such that $i < j \geqq -J_i^n$ is to the left of $-J_j^n$ of the real line.

$$\text{Let } f(x) = \sum_{n=1}^{\infty} \left\{ \sum_{j=-n}^{n} (1 - |j|/n) \right. g(nx + x_j^n)$$

where x_j^n is the left end point of J_j^n.

A

Then $|\Delta_{1/n} f(x)| \leqq 1/n \quad x \geqq x_j^n$, ($n = 1, 2, 3, \ldots$)

So $\Delta_{1/n} f(x) \rightarrow 0$ as $x \rightarrow +\infty$.

But $\{t : \Delta_t f(x) \rightarrow 0 \text{ as } x \rightarrow +\infty\}$ is a group, since

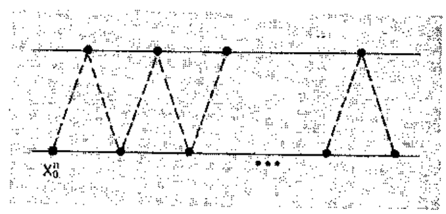
$\Delta_{-t} f(x) = -\Delta_t f(x - t)$

$\Delta_{t+s} f(x) = \Delta_t f(x + s) + \Delta_s f(x)$. So

A

$\Delta_t f(x) \rightarrow 0 \quad t \in Q$.

Now on J_0^n. f(x) looks like this:



(Continued on p. 71, col. 3)

I met him. Will you please tell me how tall he is and how much he weighs now?

Cyrus Francisco
Lexington, Ky.

That Technology Review's *article appeared without attribution was a mechanical error; the author was Deborah Shapley, then Associate Editor of the* Review, *who is now a member of the staff of* Science. *We are informed that Dr. Teller's present weight is 180 lbs., height 5'9"; obviously his "enormous frame" and "domination of the session" were less literal than figurative in Miss Shapley's eyes.*—Ed.

Misguided Amusement

To the Editor:
I find the advertisement placed by Vappi and Co., Inc. in your April, 1971 issue (*p. 75*) in very poor taste. I'm sure I'm not the only woman for whom this picture brings back disgusting memories of dirty construction workers yelling "Hey, Baby, come to poppa," of cabbies wolf-whistling, and of truck-drivers honking madly as they pass. (And those aren't regarded as compliments, especially by women with enough brains and ambition to be at M.I.T.) I hope that you will suggest to your advertisers that this sort of ad might be replaced by something that is not so offensive to some of your subscribers.

Janet Louise Mangold
Pittsburgh, Pa.

Miles and Years Apart

To the Editor:
Fantastic! I never dreamed that another galaxy was so close to us (closer, that is, than Pluto) (*see photo caption, p. 15, May*). Perhaps you mean 340 million light years away?

Joseph J. Di Certo
New York, N. Y.

Mr. Di Certo is correct, of course. It is the Editors' error, but author Michael P. Charette (See *Technology Review* for May, pp. 34-41) *has the last word: "Although a light-year is approximately six trillion miles, thus causing an error of several orders of magnitude, you can assure Mr. Di Certo that if he is planning a trip he will not notice the difference immediately."*—Ed.

Greenhouse Sterility

To the Editor:
The innocent suggestion of David G. Prosser (*"Correspondence Review" for May, 1971, p. 82*) of the advantages of using waste heat for greenhouses (*originally suggested in "Saving Waste Heat" in "Trend of Affairs" for February, p. 55*)—particularly that "it should not be necessary to apply insecticides, pesticides, etc., in the enclosed environment . . ."—was, for me, the last straw. I have read (mostly in publications other than *Technology Review*) the promotion of high-efficiency artificial lighting and many other proposed technological aids to greenhouse agriculture on the same basis—usually with much more pomp and portent—closed system, no bugs, no disease.

Nonsense!

Greenhouses are beloved by bugs and disease, because they are warm and humid and contain few natural hazards and predators. Some destructive species (such as whiteflies) flourish in the greenhouse and are controllable there only by the most persistent or toxic insecticides, whereas outside the greenhouse they are almost unknown as garden pests because they are naturally controlled. And they always find their way in—"completely enclosed" is a dream.

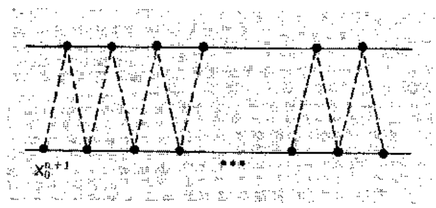
I can only wonder how much more foolishness of this sort exists in "environmental" circles, and how many bandwagoneers are thus now contributing mightily to the surplus fertilizer runoff problem.

Robert M. Rose
Cambridge, Mass.

The writer is Associate Professor of Metallurgy and Materials Science at M.I.T.—Ed.

(Continued from p. 69)

where the distance between small vertical bars is 1/2n.
On J_0^{n+1}, f(x) looks like this:



where the distance between small vertical bars is as above.

So $m\,\{t \in [0, 1] \cdot \ni \cdot \Delta_t f(x_0^n) \leq 1/2,$
$\Delta_t f(x_0^{n+1}) \geq 3/4\} \geq 1/8$

So $\exists\; t_0 \in [0, 1] \cdot \ni \cdot \Delta_{t0} f(x_0^n) \leq 1/2,$

$\Delta_t f(x_0^{n+1}) \geq 3/4$
for infinitely many n.
So $\lim_{x \to \infty} \Delta_{t0} f(x)$ D.N.E. But
$S = \{t : \lim_{x \to \infty} \Delta_t f(x) \text{ exists}\}$ is a group.
$T_0 \notin S$, so S is a proper subgroup of R, so $mS = 0$.

Address problems and solutions to Allan J. Gottlieb at the Mathematics Department, University of California, Santa Cruz, Calif., 95060.