Perpetual Motion Rides Again

A few requests: *Please* refer to problems by number. If you use some code name (e.g., "the monkey problem last spring") I will probably ignore your letter; contrary to (apparently) popular opinion, I do not have a photographic memory for problems and numbers. On a more pleasant note, bridge problems are still (January 19) in short supply, so please send along any you have found interesting. I am far from a bridge expert, so it is not advisable for me to have to make the selections.

Problems

First the usual bridge problem—this one from John P. Rudy (and Alan Truscott of the *New York Times*):

21 Given the following, show how South can complete the contract:

North:
♠ 5
♥ Q 2
♦ A K J 9 8 4 3
♣ J 9 4

West:
♠ 8 3
♥ K J 10 9 8 7 6 3
♦ 7 2
♣ 7

East:
♠ J 7 4
♥ 5
♦ Q 10 5
♣ K Q 10 8 6 3

South:
♠ A K Q 10 9 6 2
♥ A 4
♦ 6
♣ A 5 2

The bidding, North and South being vulnerable:

South	West	North	East
1 spade	4 hearts	5 diamonds	pass
6 spades	pass	pass	pass

West's lead is ♣ 7.

22 Dr. John Prussing wants you to show that the series
$1! + 2! + 3! + \ldots + k!$
is asymptotic as $k \to \infty$ to the sum of the last two terms.

Perpetual motion rides again. Smith D. Turner submits the following problem, noting that it may belong in the "Speed Department" because what is desired is an answer without advanced mathematics suitable for, say, a bright high school senior:

23 From each pound of water passing through a hydraulic turbine we can get more and more energy as we increase the pressure head on the water. It is proposed to place a turbine at the bottom of a tower so high that the energy obtained from each pound of water, when converted to electricity by a generator run by the turbine, will be sufficient to electrolyze that pound of water. The resulting gas mixture, being lighter than air, may rise through an adjoining shaft (wrapped in balloons of infinitesimal weight and infinite stretch, if this idea will help) to the top of the tower, where they may be ignited to reform water, condensed, and returned down the tower. The fact that units in the system are not 100 per cent efficient will not prevent operation, as the tower may be made higher than the theoretical height, producing enough additional power to offset losses. But no perpetual-motion system is economic unless power can be drawn from it. This can be done by making the tower still higher than necessary to electrolyze the water and offset losses; from the lifting effect of the rising gases; from the heat generated by the burning gases; and by use of the superheated steam formed by the combustion to power a turbine. Aside from possible *practical* difficulties (such as the height of the tower):
1. Will the system run as described?
2. If so, does it constitute perpetual motion; or, if not, from what source does the energy come?
3. If it would not run, point out any fallacy in the reasoning above.

Here is a navigation problem from J. J. Shipman and V. J. Knight:

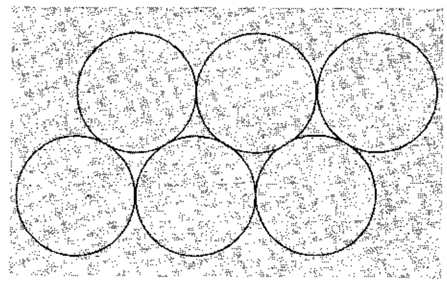
24 An airplane pilot flies a triangular course, flying first due north for a time t_o, then due south for the same time t_o, and finally returning by a straight line to his starting point. The course is triangular because of a wind of unknown direction and velocity V_w. Assuming the pilot has a stop watch and an air speed indicator which shows his speed relative to the air, and he maintains this air speed constant, how can he determine the direction and velocity of the wind?

From Douglas J. Hoylman:

25 Suppose that a football team scores only touchdowns and points-after-touchdown—i.e., that all scores are either 6 or 7 points. Then 12, 13, and 14, for example, are possible final scores, but 11 and 15 are not. What is the highest unattainable score? In "generalized" football, the only possible scores are a and b, both integers being greater than 1. Under what conditions is there a maximum attainable total score, and what is it?

Speed Department

James R. Bledsoe submits the following:



SD7 Given six coins touching each other as shown. Form them into a hexagon (circle) in three moves, moving only one coin at a time. The coins must be moved by sliding (not picked up), and each time a coin's position is altered it must be moved to a spot where it touches two other coins.

Finally, from Frank Rubin:

SD8 In an idle moment I stretched a string around the earth's equator (about 250,000,000 in.). Unfortunately, the string was 1 in. too long, so I built a tower to hold the string taut from one point. How tall was the tower?

Solutions

6 Given the following hands, and West's lead of the ♣ 5, how can South make seven hearts?

North:
♠ —
♥ 10 9 8 7 6
♦ A K
♣ A 10 9 8 7 6

West:
♠ 5 4 3 2
♥ —
♦ 6 5 4 3 2
♣ 5 4 3 2

East:
♠ K J 9 8 7 6
♥ 5 4 3 2
♦ —
♣ K Q J

South:
♠ A Q 10
♥ A K Q J
♦ Q J 10 9 8 7
♣ —

My girl friend says that it does her heart good to see a husband and wife work together. Hence the following solution, from Liz and Neil Doppelt, was a unanimous choice, even if Alice and I don't agree on the definition of working together. The letter is from Liz Doppelt: "Neil and I are really pleased about the monthly bridge problem, as we compete to see who can solve it first. This month I won. South wins the club lead with the ♣ A in dummy and pitches a diamond from his hand. He then leads a heart from dummy, winning in his hand. The ♠ A is led and dummy pitches the ♦ K. The ♠ 10 is led and ruffed, and another trump is led. South again leads a spade and ruffs in dummy. A trump is led and, after winning, South plays his last trump and pitches the ♦ A. All trumps are out and the declarer's diamonds are all good."

Also solved by the longest list of readers —43—ever known to respond to a single probem in Puzzle Corner. This list is simply too long to print, which says something about the popularity of bridge problems with readers of *Technology Review* —or perhaps something about the problem, which was originally submitted by C. C. Crystal.

7 Fill in the array below with 36 digits such that the six-digit number "a," read left to right, equals the vertical number "A," read top to bottom; b = B, etc.; and also so that b = 3a, c = 2a, d = 6a, e = 4a, and f = 5a.

	A	B	C	D	E	F
a						
b						
c						
d						
e						
f						

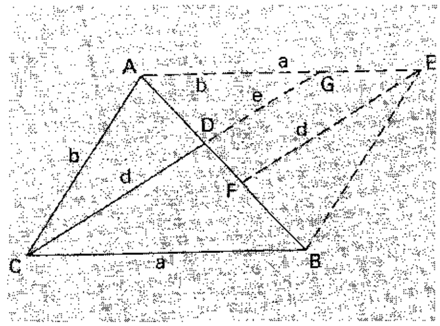
The following is from Harold Donnelly: By testing the last digit of A to be a_0, where $a_0 \epsilon \{1, 2, \ldots, 9\}$, then digits of f must be a_0, $3a_0$ mod 10, $2a_0$ mod 10, $6a_0$ mod 10, $4a_0$ mod 10, and $5a_0$ mod 10. Divide f by 5 and see if we get an A ending in the proper last digit. The only possibility is $a_0 = 7$. The solution:

	A	B	C	D	E	F
a	1	4	2	8	5	7
b	4	2	8	5	7	1
c	2	8	5	7	1	4
d	8	5	7	1	4	2
e	5	7	1	4	2	8
f	7	1	4	2	8	5

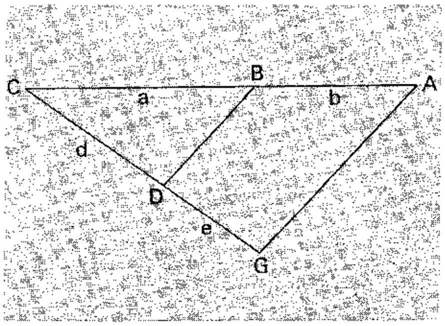
Also solved by Bob Baird, John D. Chisholm, Daniel S. Diamond, Leon M. Kaatz, Deena Koniver, Hubert duB. Lewis, Henry Lett, Victor J. Newton, R. Robinson Rowe, Greg Schaeffer, Frank G. Smith, and Mary J. Youngquist.

8 Given the lengths of two sides and the included angle bisector, construct the given triangle using compass and straight-edge.

This answer came from Robert Pogoff: In ΔABC, CD bisects angle ACB; CB = a, CA = b, and a > b.

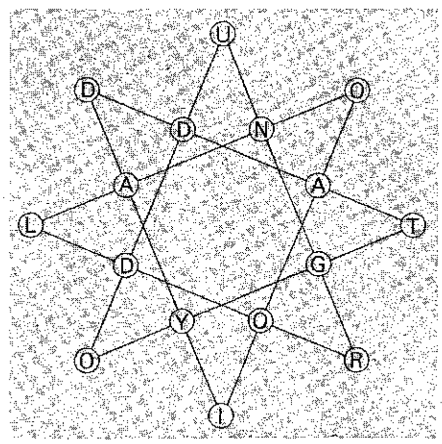


Draw AE parallel to CB; BE parallel to CA; and EF parallel to CD. Extend CD to G. Let the length of DG = e. Then angle AGC = angle GCB = angle ACD; therefore ΔAGC is isosceles: AG = AC = b. ΔBEF ≅ ΔACD; therefore EF = CD = d ΔAEF ≅ ΔBCD; therefore AE = CB = a ΔAGD ~ ΔAEF; therefore e/d = b/a.



The problem, then, is given a, b, and d, determine by means of parallel lines, the length of e of DG, based on the proportion above. Construct a triangle AGC with sides of lengths b, b, and (d + e) as in the upper figure. Construct angle GCB = angle GCA. Lay off CB = a. Join A to B.

Also solved by Harold Donnelly, S. Lindenberg, Mrs. Martin, R. Robinson Rowe, and Greg Schaeffer.



9 The numbers from 1 to 16 were written in the circles of the diagram below in such a way that the sum of any four numbers in a straight line was the same. Then the number 1 was replaced by the first letter of a saying, number 2 by the second letter, etc. The final configuration is shown. What was the saying?

No one gave his reasoning but three people, in addition to the proposer, Walter Penny, agree that the saying is "Do a good turn daily." The solutions came from Bob Baird, Hubert duB. Lewis, and R. Robinson Rowe.

10 Under what additional conditions is it true that 6x + 1 *or* 6x − 1 is a prime number when x is a counting number?

R. Robinson Rowe has a partial solution: In constructing a Sieve of Eratosthenes, the first two steps delete (as composite) all integers divisible by 2 and/or 3, leaving, in particular, all numbers of the forms 6n − 1 and 6n + 1. Beyond the primitive 2 and 3, all primes are of one or the other of these forms. It so happens that for n = 1, 2, 3, . . . one *or* the other generates a prime up to n = 20, 24, 31, 34, 36, . . . Hence for this particular problem, the proposition is true for *n less than 20.*

Better Late Than Never

Additional solutions have come to several of last year's problems and one of this year's, as indicated:

14 Homer D. Schaaf
20 Robert J. F. Roughley and D. Wehn
24 Norman Apollonio
27 Norman Apollonio, J. R. Bledsoe, and (with a generalized version) George L. Uman
28 Anonymous
37 Raymond Mancha
43 W. C. Backus
1 Michael Kay and W. C. Backus

The following, concerning last year's problem 41 (the "canary problem"), comes from Joel Pitlor, who asks what is wrong with his reasoning:
Consider the canary and submarine as a system with the sum of all the external forces equal to zero. The center of gravity of this system is at some point between the canary and the submarine. Since the sum of all the external forces is zero, the center of gravity of the system cannot move. Therefore, as the canary moves down, the submarine must move up to maintain the position of the center of gravity.

Allan J. Gottlieb, who teaches mathematics at Brandeis University, graduated from M.I.T. (S.B.) in 1967. Send new problems, solutions, and comments to him at the Department of Mathematics, Brandeis University, Waltham, Mass. 02154.