Who Owns the Zebra?

I erred in writing the October/November issue by not mentioning that "speed" problems are for personal edification only; solutions are not published. So I must apologize to those who have responded.

Let me thank everyone who answered my request for "speed" problems. I could use more, but the supply is no longer critical. As for regular problems, the most descriptive comment is that today, November 15, I am using problems received in April. Please be patient.

Problems

As usual, we begin with a bridge problem—this one from John Rudy:

11 Given the hands shown and the bidding as listed,

North:
♠ 8 6
♥ A K x
♦ Q 9 7
♣ A 10 9 x x

West:
♠ A Q 10 x x
♥ Q J 10 x x
♦ x
♣ J x

East:
♠ J 9
♥ 9 x x x
♦ x x x
♣ K Q x

South:
♠ K x x
♥ x
♦ A K J 10 8 5
♣ x x x

South	West	North	East
1 diamond	1 spade	2 clubs	pass
2 diamonds	2 hearts	3 hearts	pass
3 no-trump	4 hearts	5 diamonds	pass
pass	pass		

and West's lead of the ♥ Q, show that if the declarer wins the first trick with the ♥ A then he must lose two spade tricks and a club when East gets in with the ♣ K.

The following is from Arthur W. Anderson:

12 Show that for every odd positive integer n, sin nx can be expressed in the form $\sin nx = a_1 \sin x + a_3 \sin^3 x + \ldots + a_n \sin^n x$ and derive a general formula for the coefficients a_k.

A geometry problem from George E. Keith, Jr.:

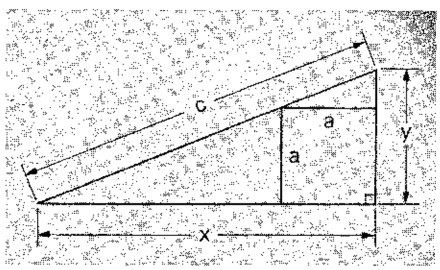
13 Given a convex quadrilateral ABCD with diagonals AC and BD, and given that
AC = BD
Angle BAC = angle CAD
Angle CBD = angle BDA
prove that the quadrilateral is a trapezoid.

Here's a navigation problem from Clark Thompson:

14 Starting from a point 40°N 88°W, a man walked 200 miles due north, then 400 miles due west, 400 miles south, 400 miles east, and finally 200 miles north. To his amazement, he was not at his starting point. How far away was he?

Our final problem is from John L. Sampson:

15 Given the following, find x and y in terms of a and c.



Speed Department

Our first quickie is from John E. Prussing:

SD5 Two trains start at 10 a.m., one going from Boston to Washington and the other from Washington to Boston. The first train takes six hours for the trip and the second train takes nine hours. Each travels at a constant rate with no stops. At what time of day do the trains meet each other?

Finally, Alec Henderson and *Readers Digest* offer the following; Mr. Henderson notes that while it does not involve any mathematics, it requires a good deal of logic:

SD6 The facts essential to solving the problem—which can indeed be solved by combining deduction, analysis, and sheer persistence—are as follows:
1. There are five houses, each of a different color and inhabited by men of different nationalities, with different pets, drinks, and cigarettes.
2. The Englishman lives in the red house.
3. The Spaniard owns the dog.
4. Coffee is drunk in the green house.
5. The Ukrainian drinks tea.
6. The green house is immediately to the right (your right) of the ivory house.
7. The Old Gold smoker owns snails.
8. Kools are smoked in the yellow house.
9. Milk is drunk in the middle house.
10. The Norwegian lives in the first house on the left.
11. The man who smokes Chesterfields lives in the house next to the man with the fox.
12. Kools are smoked in the house next to the house where the horse is kept.
13. The Lucky Strike smoker drinks orange juice.
14. The Japanese smokes Parliaments.
15. The Norwegian lives next to the blue house.
Questions: Who drinks water? And who owns the zebra?

Solutions

41 A canary is hovering inside a submarine when the submarine finds its neutral equilibrium in water. When the canary gently lands on the submarine deck, does the submarine go down?

Robert D. Shooshan solved this one; he writes: "This problem reminds me of a cartoon I once saw, a three-ton truck crossing a bridge with a three-ton load limit. The driver, beating on the body with a stick, explained to his helper 'We have a two-ton load of pigeons and I want to keep them flying until we get across this bridge.'

"The canary hovering inside the submarine is exerting a force on the surrounding air equivalent, in the vertical direction, to its weight. The air is pushing against the deck while the bird is hovering. When the canary gently lands, its weight on the deck is equivalent to, and replaces, the downward air current. Therefore, the submarine remains in equilibrium."

Also solved by Charles H. Blake, Jerry Blum, Dale Epstein, Rockwell Hereford, Mark Leonard, James Marler, Jr., Eric Rosenthal, R. Robinson Rowe, Frank

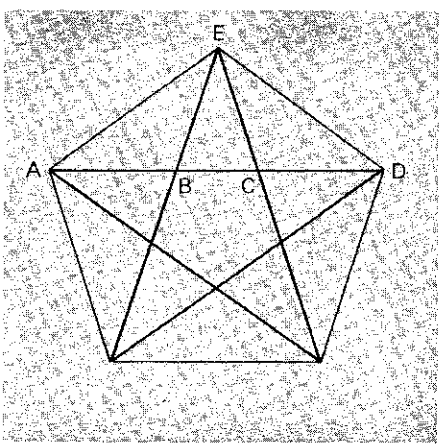
Rubin, John Rudy, Les Servi, and James Sinclair.

42 Find the next term in the following series:
18 46 94 63 52
The Perseverance Award goes to Donald C. Berkey and Dr. Robert E. Hoffman. Mr. Berkey's first solution was mailed August 25. Two days later it was amended. On September 9 Mr. Berkey and Dr. Hoffman reported the following seven equations, with the value $n = 5$ given in parenthesis and italics just after each equation:
$18 + 99 \sin^2 (n\pi/4) - 31.5n + 10n^2$ (*160*)
$18 - 24.75 \sin^2 (n\pi/2) + 67.5 n - 14.75n^2$ (*−38*)
$18 - 64.5n + 150.25n^2 - 66n^3 + 8.25n^4$ (*358*)
$18 + 88 \sin^2 (n\pi/3) - 114 + 92.5n^2 - 16.5n^3$ (*−236*)
$72 \sin^2 [(n + 1)\pi]/6 - 94.5n + 133.75n^2 - 54n^3 + 6.75n^4$ (*340*)
$18 - 10 \sin^2 (n\pi/2) + 34/3 \sin^2 (n\pi/3) + 42 \sin^2 (n\pi/4) + 34 \sin^2 (n\pi/6)$ (*46*)
$18 + 12 \sin^2 (n\pi/2) + 684 \sin^2 (n\pi/6) - 192.5n^2 + 40.5n^3$ (*448*)
Finally Dr. Hoffman determined that (as they expected) any number K will satisfy the fifth term by the following formula:
$18 - 10 \sin^2 (n\pi/2) + [(80 - K)/3] \sin^2 (n\pi/3) + [(19 + K/2)] \sin^2 (n\pi/4) + (80 - K) \sin^2 (n\pi/6) + [(K - 46)/4]n.$
Finally on September 16 they arrived at the same solution as the proposer, namely 61; the series is generated by writing 9^2, 8^2, 7^2, etc., with the digits reversed.

Also solved by Jerry Blum, David Cantor, Walter C. Janney, Mrs. Martin L. Lindenberg, James Marler, Jr., Eric Rosenthal, Robert Rosin, R. Robinson Rowe, Heinrich Ruschat, Jerrold Sabath, George S. Sacerdote, and Les Servi.

43 Outline the geometrical method of drawing the five-star insignia of our top military commanders. If W is the width of any star, can anyone determine the ratio K of the distance between adjacent star points to W?

The following is from Eric Rosenthal:



I assume AD is the "width."
In triangle ABD, AB/(sin AEB) = AE/(sin ABE)
AB = AE[(sin AEB)/(sin ABE)]
= AE[(sin 36°)/(sin 108°)]
Now (sin 36°)/(sin 108°) = (sin 36°)/(sin 72°) = (sin 36°)/(2 sin 36° cos 36°) = 1/(2 cos 36°), so AB = AE/(2 cos 36°).
In triangle BCE, BC/(sin BEC) = BE/(sin BCE)
BC = BE[(sin BEC)/(sin BCE)]
= BE(sin 36°)/(sin 72°).
Since AB = BE and (sin 36°)/(sin 72°) = 1/(2 cos 36°),
BC = AB/(2 cos 36°)
= AE/(2 cos 36°) (2 cos 36°).
Now AD = AB + BC + CD
= 2(AB + BC) = 2[AE/(2 cos 36°)] + AE/(4 cos² 36°)
= AE[1/(cos 36°) + 1/(4 cos² 36°)].
In the solution to problem 30 in the July/August issue, you give
$\sin 36° = \sqrt{10 - 2\sqrt{5}}/4.$
So $(\cos^2 36°) = 1 - (\sin^2 36°) = 1 - (10 - 2\sqrt{5})/16 = (6 - 2\sqrt{5})/16$
and $\cos 36° = (\sqrt{6 + 2\sqrt{5}})/4.$
So K/W = AE{AE[1/(cos 36°) + 1/4(cos² 36°)]}

$$= \frac{1}{(\cos 36°)/(\cos^2 36°) + .25/(\cos^2 36°)}$$

$$= (\cos^2 36°)/[.25 + (\cos 36°)]$$

$$= \frac{(6 + 2\sqrt{5})/16}{.25 + (\sqrt{6 + 2\sqrt{5}})/4}$$

$$= (3 + \sqrt{5})/2(1 + \sqrt{6 + 2\sqrt{5}}).$$

Also $(\cos^2 36°)/[.25 + (\cos 36°)] \approx 0.65451/(0.25 + 0.65451) = 0.65451/1.05902 \approx 0.61803.$

Also solved by John E. Prussing and Frank Rubin.

44 A certain physicist is studying a nuclear reaction with very precise equipment. He can sort out and store separate atoms and observe when a gamma ray is emitted by any of his collection, but he cannot tell which one has changed or tell whether a neutron was released or absorbed also. His equipment has a balance sensitive enough to weigh any or all of the atoms and finally to sort out the single atom and determine the process. After 282 atoms are isolated, a gamma ray is detected; as the weighing process is exceedingly tedious, the physicist seeks to determine the method that will yield the answer in the least number of weighings. While the physicist is checking the data on a computer—with a tremendous amount of identical parallel circuits feeding in—there is a breakdown; the operators have difficulty in isolating defective circuits. The physicist therefore comments, "If you people have as sensitive a nulling ammeter as I have a balance and can parallel all the inputs and outputs as you please, I can determine which component is open or shorted in less than 11 comparisons if the total number of elements is not much more than 68,000." To fulfill his promise, what (1) was the least number of weighings of the atoms made by the physicist; and what (2) is the greatest number of circuits he could check with 11 comparisons?

Only Frank Rubin responded:

A simple induction establishes that in n weighings you can determine the odd-weight item, with direction of error, from $(3^n - 3)/2$ items. In this case $n = 6$ is required. (Note that $x_{n+1} = 3(x_n + 1)$ is the identity used for the induction.) But the second half of the problem is incomprehensible. How many inputs and how many outputs does each circuit have? How many input/output combinations must be tried before concluding that a given circuit is open or shut? The nature of each circuit seems relevant. Suppose they are binary triggers which change level outputs on every second pulse input. What then?

45 What time is it when the spread between a clock's hands (measured the short way) is an integral multiple of 13 minutes? No fractional minutes are involved, and the hands are pointed in different directions.

Many computer programs were submitted: the following, by Mark Leonard, is in FOCAL. 2:48 is the desired answer (the only one before 4:00).

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*WRITE ALL
C-FOCAL , 8/68

01.10 FOR H=1,60; DO 2.0
01.20 TYPE !!!; QUIT

02.11 C H IS POSITION OF HOUR
        HAND, IN MINUTES.
02.12 C MINUTE HAND POSITION IS
        12 TIMES HOUR HAND
02.13 C SO THE DIFFERENCE IS 11
        TIMES H.
02.20 SET D=11*H
02.30 IF (D-60) 2.4 2.31,2.31
02.31 SET D=D-60; GOTO 2.3
02.40 C DIFFERENCE IS NOW LESS
        THAN 60.  SEE IF LESS
        THAN 30.
02.41 IF (D-30) 2.51,2.99,2.42
02.42 SET D=60-D
02.50 C D IS NOW SHORT WAY,
        LESS THAN 30.
02.51 IF (D) 2.99,2.61,2.52
02.52 SET D=D-13; GOTO 2.51
02.60 C IF WE GET HERE D WAS AN
        INTEGRAL MULTIPLE OF 13.
02.61 SET HOUR=FITR(H/5)
02.62 SET MINUTE=12*(H-<5*HOUR>)
02.70 TYPE !,%2.0,"TIME",HOUR,
        ":",MINUTE
02.99 RETURN
*GO
TIME=  2:= 48
TIME=  4:= 36
TIME=  7:= 24
TIME=  9:= 12
TIME= 12:=  0
*C EXECUTION TIME WAS ABOUT 15
   SECONDS, INCLUDING TYPING TIME.
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Also solved by F. Steele Blakall III, Jerry Blum, Charles Bures, Rockwell Hereford, John Hornstein, Richard Lipes, Lyall D. Morrill, Jr., John E. Prussing, R. Robinson Rowe, Frank Rubin, John Rudy, James Sinclair, David B. Smith, and one anonymous correspondent.

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