Allan J. Gottlieb

Bridge Becomes a Monthly Feature

It finally happened. After two years of silence, my stereo system is again working. As I listen to J. S. Bach's violin concerto in A minor, all the effort seems almost worthwhile. The hero of this episode is John Forster (M.I.T. '67); his patience and expertise enabled us to trace the failure sequence to a factory-miswired printed circuit board (two diodes were misplaced). Not only can I now enjoy music—but sleeping will be easier, knowing that my wiring, while "not the neatest ever seen," to use John's sympathetic terminology, was not in error. Now we can hope that the case is closed forever.

Many people have complained that their solutions have appeared in the "Better Late Than Never" department when they though that the solutions were submitted promptly. In order to explain why this happens, let me say that today's date is October 1.

I should like to ask that we have no more contributions concerning last year's problems—meaning those published in or before the July/August, 1970, issue of *Technology Review.* This will help my filing system, and I feel that those problems have by now lost their interest.

Currently, problems are appearing about seven months after I receive them. Thus I have a backlog which will last until the end of the current volume.

Problems

I will try to include exactly one bridge problem in each issue. This month's selection is from C. C. Crystal:

6 Given the following hands, and West's lead of the ♣5, how can South make seven hearts?

North:
♠ —
♥ 10 9 8 7 6
♦ A K
♣ A 10 9 8 7 6

West:
♠ 5 4 3 2
♥ —
♦ 6 5 4 3 2
♣ 5 4 3 2

East:
♠ K J 9 8 7 6
♥ 5 4 3 2
♦ —
♣ K Q J

South:
♠ A Q 10
♥ A K Q J
♦ Q J 10 9 8 7
♣ —

This next problem, of the number-theoretic variety, is from Donald Morrison:

7 Fill in the array below with 36 digits such that the six-digit number "a," read left to right, equals the vertical number "A," read top to bottom; b = B, etc.; and also so that b = 3a, c = 2a, d = 6a, e = 4a, and f = 5a.

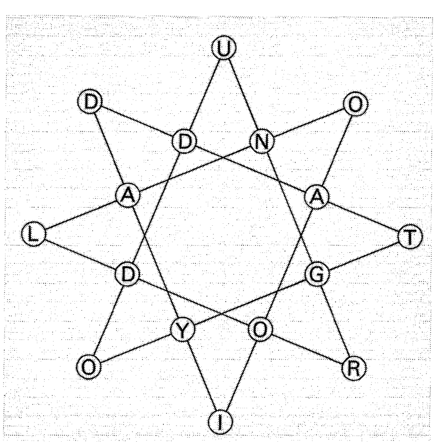
	A	B	C	D	E	F
a						
b						
c						
d						
e						
f						

A geometric problem comes from Joseph T. Patterson:

8 Given the lengths of two sides and the included angle bisector, construct the given triangle using compass and straight-edge.

This puzzle, from Walter Penny, requires both geometry and number theory:

9 The numbers from 1 to 16 were written in the circles of the diagram below in such a way that the sum of any four numbers in a straight line was the same. Then the number 1 was replaced by the first letter of a saying, number 2 by the second letter, etc. The final configuration is shown. What was the saying?



We end this installment with a problem from tenth-grader Leslie Servi:

10 Under what additional conditions is it true that $6x + 1$ *or* $6x - 1$ is a prime number when x is a counting number?

Speed Department

The first speed problem this month is from R. Robinson Rowe:

SD3 Three fishermen each caught a fish. By a coincidence, each man plus his fish weighed 170 lbs. Pairing the men in turn, two men plus the third man's fish weighed 320, 322, and 324 lbs., respectively. What did the three men weigh?

SD4 Finally, John E. Prussing wants you to show that the principal value of the ith root of i is a real number slightly less than 5.

Solutions

36 Show that the equations
$a^2 + b^2 = c^{16}$
and
$a^{16} + b^2 = c^2$
each have an infinite number of solutions with a, b, and c nonzero integers.

The following solution is from Frank Rubin:
First consider the equation
$a^2 + b^2 = c^2$.
It is well known that all solutions to this equation take the form
$a = a_1^2 - b_1^2$,
$b = 2a_1b_1$, and
$c = a_1^2 - b_1^2$.
To achieve
$a^2 + b^2 = c^4$
we require that
$a_1^2 + b_1^2 = c_1^2$; $c^2 = c_1^4$.
So take
$a_1 = a_2^2 - b_2^2$,
$b_1 = 2a_2b_2$, and
$c_1 = a_2^2 + b_2^2$.
Repeating, we would get
$a_2 = a_3^2 - b_3^2$,
$b_2 = 2a_3b_3$,
$c_2 = a_3^2 + b_3^2$,
$a_3 = a_4^2 - b_4^2$,
$b_3 = 2a_4b_4$, and
$c_3 = a_4^2 - b_4^2$.
The minimum solution, derived from $a_4 = 2$, $b_4 = 1$, is $164{,}733^2 + 354{,}144^2 = 5^{16}$. The second half is much easier. We simply take $a_1 = 2^7 \cdot a_2^8$ and $b_1 = b_2^8$, reversing the roles of a and b.

The smallest solution in this case is $2^{16} + 16{,}383^2 = 16{,}385^2$, derived from $a_2 = b_2 = 1$.

Also solved by R. E. Crandall, Edward Friedman, Peter Groot, Donald R. Oestreicher, John E. Prussing, R. Robinson Rowe, and John Rudy.

37 What is the smallest number (N) of n digits which, when removing the digit from the units place and relocating it in front of the n's place, exactly doubles the number?

James L. Funk submitted two solutions; here is his second—revised—answer: The general equation would be $-$ 10N $(10^x - 2/19) + N$ $(N < 9)$. Divide $10^x - 2$ by 19 backwards. The last digit must be 2 $(38/19 = 2)$. The next digit must be 4 $(79/19 = 4 + 3/19)$. The next digit must be 8 $(159/19 = 8 + 7/19)$. Continue to add 9's until you have no carry-over to the left. You will get: 5263157894736842. Substitute 2 for N to get: 105263157894736842; this is your answer.

Also solved by James R. Bledsoe, R. E. Crandall, William J. Deane, Edward Friedman, Peter Groot, Fred Heutink, Thomas Krause, D. C. Matiatos, Donald Oestreicher, S. Patten, Robert Pogoff, John E. Prussing, George Ropes, R. Robinson Rowe, Frank Rubin, John Rudy, Donald E. Savage, W. A. Smith, and S. D. Turner

38 With bidding and lead as shown, how do you play the following hand to maximize the probability of making the contract?

North:	**South:**
♠ A Q J 8 5	♠ K 10 9 6 3
♥ A K 7	♥ 5 4 3
♦ A 7 5	♦ 9 6 3
♣ K 3	♣ A 2

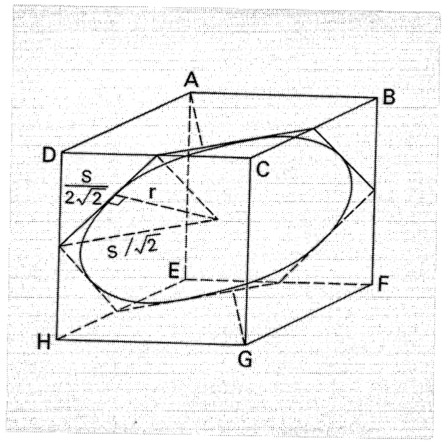
The bidding: West, four clubs; North, double; East, pass; South, four spades; West, North, and East, pass. West opens with the ♣ Q.

Elmer C. Ingraham submits the following: East will, of course, ruff the first trick, else there would be no problem. West will not, of course, ruff the second trick, else there would be no solution. So, you can maximize the probability of making your contract by planning the play to throw West into a forced club return upon which you can drop a loser from one hand while ruffing in the other for your tenth trick. So you win the second trick and count West's distribution; he never had but four cards beside his nine clubs and is pretty well marked with high diamonds to justify his bid. Plan your play to draw trumps and to observe West's play to those and to dummy's red aces; if West still has three red cards or has not thrown a high diamond it may be safe, and wise, to take dummy's ♥ K, thus assuring that he does not have a heart for safe exit. When West has only high diamonds left you can take your club trick and throw him in with the diamond, not caring whether he can win one or two tricks. With luck you might even catch West with a single high diamond, ruff his club return in one hand and your small heart in the other to score an overtrick.

Also solved by Paul D. Berger, Edwin Davis, James W. Dotson, Edward Friedman, Peter Groot, Stanley Horowitz, Fred Price, R. Robinson Rowe, John Rudy, David Silberstein, and S. D. Turner.

39 I want to send a record to a friend but don't want her to guess what's inside from the size and shape of the package. What's the smallest size *cubic* box that will hold a 12-in. record (without its jacket, of course)?

Robert Pogoff of Stewart-Warner Corp. offers the following:



The perpendicular bisecting plane of a diagonal of a cube intersects the face planes in lines which each bisect two edges, and which form a regular hexagon with side length $s/\sqrt{2}$, where s is the length of the edge of the cube. A circle inscribed in this hexagon will thus touch each of the faces of the cube. If r is the radius, then

$r = s/2\sqrt{2} \cdot \sqrt{3}$

$s = (2\sqrt{2}/\sqrt{3})r = (4/\sqrt{6})r$

For the 12-in. disc, $s = (4 \cdot 6)/\sqrt{6} = 4\sqrt{6} = 9.796$ in.

Also solved by Norman L. Apollonio, Major F. H. Cleveland, R. E. Crandall, Peter Groot, R. M. Neudecker, Donald R. Oestreicher, A. Oltmann, R. Robinson Rowe, Eric Schaffer, C. Scholz, Robert D. Shooshan, W. A. Smith, S. D. Turner, S. Thomas Terwilliger, Benjamin Whang and Harry Gray, and "the Green Phantom."

40 Three prisoners stand in a row, each facing the back of the one in front of him, the front one facing a wall perpendicular to the prisoners' line. The prisoners can neither turn around nor see their own heads. But they know that there are five hats, three red ones and two black ones; each prisoner is wearing a hat, and the remaining hats are hidden from view. If any prisoner can state the color of his own hat and provide sufficiently good reasons for his choice (law of averages excluded), he will be set free. After a suitable time, the prisoner nearest the wall announces his answer, is correct, and is set free. How?

Nearly everyone solved this problem. Here is Heinrich Ruschat's answer: The prisoner nearest the wall waits a suitable period of time and when neither of the prisoners second and third from the wall states the color of his hat, the first prisoner knows that he has on a red hat. If the prisoner second from the wall looks forward and sees a black hat, he will know that he does not have on a black hat, for if he did, the prisoner third from the wall would have realized at once his hat must be red (since there are only two black hats) and would have called out at once, stating the color of his hat. Therefore, if the second prisoner doesn't hear a statement from the third prisoner, he will at once state the color of his hat as being red. If the prisoner second from the wall does not quickly state the color of his hat as being red, it is because he does not see a black hat on the prisoner in front of him. A suitable time passes, and in the silence of its passing the prisoner nearest the wall realizes his hat must be red.

Also solved by James R. Bledsoe, Sterling G. Brisbin, Jr., Thomas A. Casey, Jr., Craig F. Cheng, R. E. Crandall, Edwin Davis, James W. Dotson, Robert Epstein, James Flechtner, Edward Friedman, David Glazer, George Goodstein, Peter Groot, William Grosky, Peter Hall, Stanley Horowitz, Elmer C. Ingraham, Leon M. Kaatz, Thomas Krause, J. B. Linn, R. Loffredo, Dale Madden, William P. Mitchell, Donald F. Morrison, R. Neff, Donald R. Oestreicher, Stephen Perrenod, A. Porter, Fred Price, John E. Prussing, J. D. Ring, Eliot Roberts, R. Robinson Rowe, Frank Rubin, Greg Schaffer, C. Scholz, A. Roger Seymour, Robert D. Shooshan, David Silberstein, W. C. Sussky, S. D. Turner, Ralph Wanger, Benjamin Whang, Ronald E. Wilson, "the Green Phantom," and the proposer, John Rudy.

Better Late Than Never

Solutions to two of last year's problems have arrived:

16 Peter Lobban

23 Donald Morrison and S. Thomas Terwilliger.

Allan J. Gottlieb received his S.B. degree in mathematics at M.I.T. in 1967 and has since been at Brandeis University, where he is now a teaching assistant in the Department of Mathematics. Send correspondence to him at the Department of Mathematics, Brandeis University, Waltham, Mass. 02154.

October/November Crostic Solution

Spectrum lines serve not only to identify the atom by which they are produced, but also to tell a good deal about the circumstances of the gas in which they originated.
—P. W. Merrill, *Space Chemistry.*