Dimes, Triangles, and Radii

Season's greetings. Today is Christmas. It appears that the powers that be have decided that my dress should be "more contemporary." I now own a "semi-Edwardian" jacket and a bandana (which my 12-year-old cousin had to show me how to wear) à la "Midnight Cowboy." If my courage holds out I shall wear this outfit to class. When I ask my relatives why my teachers a M.I.T. were not required to be mod dressers, they explain that we are in a new era now (apparently the mid-1960's—pre-Armstrong—are bygone days).

More questions have come in concerning my amp. Currently it is unused, as a roommate has supplied a slightly inferior *but working* stereo set. Ron Kadomiya, '67, managed to isolate the trouble somewhat, but his mechanical engineering test equipment (a precision set of graduated sledge hammers) did not allow him to solve the mystery completely. Things are looking up now: John Forster, '67, has offered his assistance, and he has electrical engineering test equipment (a precision set of graduated pots [and pans?]) left over from his thesis. Last year Mark Yu offered a one-word appraisal of my problem: change brands. He may be right, but I'm hopeful that someone will get my set running properly.

Enough gab; on to the problems.

Problems

The first problem is from Philip W. Robinson:

16 Can you find a curve having non-constant radius of curvature such that all the centers of curvature lie on the x axis?

The second problem comes to us from David P. Dewan, who calls it "the problem of the six cups:"

17 I placed 15 dimes and 15 nickels in six cups such that each cup contained the same number of coins but a different amount of money. I made six labels showing correctly how much money each cut contained, but attached to each cup an incorrect label. I explained the situation to six logicians and gave a cup to each. I asked each man in turn to feel the size of as many coins as he wanted in his own cup and announce something interesting. The only evidence each man had was the size of the coins he felt, the incorrect label on his own cup, and the statements made by those who preceded him. The first man said, "I feel four coins which are not all the same size; I know that my fifth coin must be a dime." The second man said, "I feel four coins which are all the same size; Iknow that my fifth coin must be a nickel." The third man said, "I feel two coins, but I shall tell you nothing about their size; I know what my other three coins must be." The fourth man said, "I feel one coin; I know what my other four must be." Then everyone knew how the remaining two cups were labelled and what the total value of money in those two cups was. Do you?

Here is a problem from Thomas B. Jabine, '48, which he says is "guaranteed to keep the solver occupied for several hours unless he happens to be very lucky. There is a solution, and no tricks (such as zeros to the left) are involved. There is no quick way to find the answer, but large classes of possible solutions can be readily eliminated in various ways."

18 This is a multiplication problem:

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      x x x
      x x x
      -----
      x x x
    x x x
  x x x
  ---------
  x x x x x
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Each of the 10 digits (0, 1, 2, . . . 9) is to be used exactly twice.

19 Given a triangle ABC and a point P, Smith D. Turner wants a method for constructing a line through P bisecting the area of the triangle.

John F. Mandl has submitted the following, which he says he's had in his file "for years." He says that C "is a remarkable mental gymnast to arrive at the answer without pencil and paper; the estimated time for solution for the average mortal is six hours."

20 A said to the farmer, "I know you own a rectangular plot in that 20-by-20 section, and I know the area of your plot. Is the length greater than twice the width?" B said to the farmer, "Before you answer let me state that I knew the width, and I now know the length." C said, "I did not know the length, width, or area, but now I know the dimensions." What are they?

Speed Department

Only one speed problem has been submitted; here it is (from John J. Sytek):

SD7 Where is the fallacy in the following:

For all θ,

$$e^{i\theta} = e^{i\theta 2\pi/2\pi} = (e^{2\pi i})^{\theta/2\pi} = 1^{\theta/2\pi} = 1$$

Solutions

1 Obtain rational factors of
$x^8 - 4x^4y^4 + 16y^8$

Judith Q. Longyear and Francis T. Leahy, Jr., have shown that no linear factors exist. The solution expected by the proposer, Smith D. Turner, is:
In general, $A^4 - B^2A^2 + B^4$ cannot be factored unless it can be put in the form $M^2 - N^2$, which requires that 2AB be a perfect square. In this case that requirement is fulfilled, since 2AB is $4x^2y^2$. So
$x^8 - 4x^4y^4 + 16y^2 =$
$(x^4 + 2x^2y^2 + 4y^4)^2 - (2xy)^2(x^2 + 2y^2)^2$,
which is in the form $M^2 - N^2$ and can be arranged to yield
$(x^4 + 2x^3y + 2x^2y^2 + 4xy^3 + 4y^4)$
$(x^4 - 2x^3y + 2x^2y^2 - 4xy^3 + 4y^4)$.
Mr. Turner notes that "the solution may be difficult without this gimmick."

2 For the real symmetric matrix Q, let $\Delta = \det(Q)$ and $\Delta_o = 1$. Let Δ_{n-t} be the determinant of Q with its last t rows and columns deleted. It is well known that Q is positive semi-definite if all principal minors of Q are $\geqq 0$ and $\Delta = 0$. Is this last condition equivalent to some (seemingly weaker) condition? (For example, one might conjecture a condition such as "$\Delta_o, \Delta_1, \ldots, \Delta_n \geqq 0$ and $\Delta = 0$," which involves only the leading principal minors. This, though, is clearly insufficient.) Such a weaker condition could be of use in determining whether a matrix is PSD, especially if it uses little more than $\Delta_0, \Delta_1, \ldots, \Delta_n$.

No takers, so far. Keep at it.

3 The contract is six spades, and the opening lead is ♦3. Play to make it.

North
♥ A K 8
♥ —
♦ A K 8 6
♣ A K J 10 8 2

West
♠ Q 9
♥ K J 10 6 2
♦ Q 7 4 3
♣ 7 5

East
♠ J 6 3
♥ 8
♦ J 10 9 4 2
♣ Q 9 6 4

South
♠ 10 7 5 4 2
♥ A Q 9 7 5 4 3
♦ —
♣ 3

The following is from Winslow H. Hartford, who writes that he is "still enjoying Puzzle Corner and solving a few when time permits or the 'new math' isn't too much involved. However, bridge problems and Diophantine equations remain my forte." As to this problem, he notes that it "would be a tough hand to play first crack out of the box, but its secret is timing. Dummy's clubs must be established by immediate ruffs; so the discard on the first trick is a club rather than the more obvious heart." The play as he proposes it:

Trick	W	N	E	S
1	♦3	♦A	♦2	♣3
2	♣5	♣2	♣4	♠2
3	♠9	♠K	♠3	♠4
4	♣7	♣8	♣9	♠5
5	♥2	♦6	♥8	♥A*
6	♠Q	♠A	♠6	♠7
7	♥6	♣K	♣6	♥3
8	♥10	♣A	♣Q	♥4
9	♦4	♣J	♠J	♥5†
10	♦7	♦K	♦J	♥7
11	♥J	♣10	♦4	♥9
12	♦Q	♦8	♦9	♠10
13	♥K	♠8	♦10	♥Q

*South must cash ♥A before the hand is dead.
†East can take his ♠J any time.

Also solved by Stanlay A. Horowitz, Elmer C. Ingraham, Leon M. Kaatz, Atma P. Lalchandaani, Francis T. Leahy, Jr., Michael Mann, John P. Rudy, Ruth Turner, and Eric Weitz.

4 A monkey and his uncle are suspended at equal distances from the floor at opposite ends of a rope which passes through a pulley. The rope weighs four ounces per foot. The weight of the monkey in pounds equals the age of the monkey's uncle in years. The age of the uncle plus that of the monkey equals four years. The unce is twice as old as the monkey was when the uncle was half as old as the monkey will be when the monkey is three times as old as the uncle was when the uncle was three times as old as the monkey. The weight of the rope plus the weight of the monkey's uncle is one-half again as much as the difference between the weight of the monkey and that of the uncle plus the weight of the monkey. How long is the rope? How old is the monkey?

The following solution, neatly typed, is from Leon M. Kaatz:

Let
W_u = uncle's weight (lbs.)
W_m = monkey's weight (lbs.)
W_r = ropes weight (lbs.)
A_u = uncles age (yrs.)
A_m = monkeys age (yrs.)
L = rope length (ft.)
K = number of years ago that the uncle was 3 times as old as the monkey

Since the monkey and the uncle are equi-distant from the ground, $W_u = W_m$. We are given that $W_m = A_u$, therefore $W_u = W_m = A_u$. We are also given that $A_u + A_m = 4$.
Now, $A_u - K = 3 \cdot (A_m - K)$ by the definition of K. Therefore, $3 \cdot (A_u - K)$ = age of the monkey when the monkey is three times as old as the uncle was when the uncle was three times as old as the monkey was.
Therefore, $(3/2) \cdot (A_u - K)$ = age of the uncle when the uncle was ½ as old as the monkey will be when the monkey is 3 times as old as the uncle was when the uncle was 3 times as old as the monkey was.
Now we are given that the unce is twice the age that the monkey was when the uncle's age was $(3/2) \cdot (A_u - K)$. But, $A_u - (3/2) \cdot (A_u - K)$ is how long ago the uncle's age was $(3/3) \cdot (A_u - K)$. Therefore, when the uncle's age was $(3/2) \cdot (A_u - K)$, the monkey's age was $A_m - [A_u - (3/2) \cdot (A_u - K)]$, or $A_m - \frac{1}{2} \cdot (3K - A_u)$. Since the uncle is twice this age, we have $A_u = 2A_m - 3K + A_u$, or $A_m = 3K/2$. Substituting this into the equation $A_u - K = 3 \cdot (A_m - K)$, yields $A_u = 5K/2$. Therefore $A_u = (5/3) \cdot A_m$, and since $A_u + A_m = 4$, we get
$A_u = 2\frac{1}{2}$
$A_m = 1\frac{1}{2}$.

Next, we are given that the weight of the rope plus the weight of the uncle is half again as much as the difference between the monkey's weight and the weight of the monkey plus the uncle. Mathematically, this is:
$W_r + W_{mu} = (3/2) \cdot [(W_m + W_u) - W_m]$.
Since $W_m = W_u = A_u = 2\frac{1}{2}$, this becomes $W_r = 1\frac{1}{4}$ lbs. = 20 oz. Since the rope weighs 4 oz. per ft., the rope must be 5 feet long.
Well I'll be a monkey's uncle, I actually solved that one.

Also solved by J. Douglas Doxsey, Sharon Hubbard, Edgar Keats, James W. Royle, Jr., G. W. Stratfort, and Benjamin Whang.

5 Three thespians came on a cache of bright, shining obols and decided to share it. Silimon took some coins, and Stupidas also helped himself. What is the probability that at most one-third of the coins were left for Preposteros?

I confess to not totally understanding this problem. What seems to me to be the most reasonable submitted solution is the following from Mr. Leahy:
This problem has no mathematical answer without additional assumptions. If all three grab together, each has an equal chance of getting over one-third, of course. If the first selects at random any percentage that he'd like, and the second selects at random any percentage that he'd like of what's left, the last man can expect one-fourth of the pile; and his chance of getting more than one-third is given by:

$$\int_{0.33}^{1.00} \frac{y - 0.33}{y}\, dy =$$

$2/3(2/3 - 1/3\ [\ln 3]) = 0.20$
Hence, 80 per cent of the time the third man gets less than a third of the pile.

Elmer C. Ingraham and Smith D. Turner also responded, and Jerome I. Glaser has the following alternate solution, which he says results in the answer $1/3\ (1 + \log_e 3)$, or about 0.7:
Assume that the large cache contains N coins. The probability that at most N/3 coins are left for Preposteros is equal to one minus the probability that the combined take of Stupidas and Silimon is less than 2N/3. Let n_1 be Slimon's take and n_2 be Stupidas's take. The joint density of n_1 and n_2 is $(1/N)(1/N - n_1)$ and the probability is

$$p = 1 - \int_0^{2N/3} (dn_1/N) \int_0^{2N/3 - n_1} (dn_2/N - n_1) = 1 - (2/3 - 1/3 \log_e 3).$$

Allan J. Gottlieb, who studied at M.I.T. with the Class of 1967, is a teaching assistant at Brandeis University. Send solutions, new puzzles, and other correspondence to him at the Department of Mathematics, Brandeis University, Waltham, Mass., 02154.

January Tech-Crostic Solution

Bilateral networks with several inputs and several outputs must be transformed into the equivalent unilateral networks. Nonlinear operators either may be a function of the input signal only or may be a function of a second input as with automatic gain-controlled amplifiers.

(O. J. M.) Smith, *Feedback Control Systems*