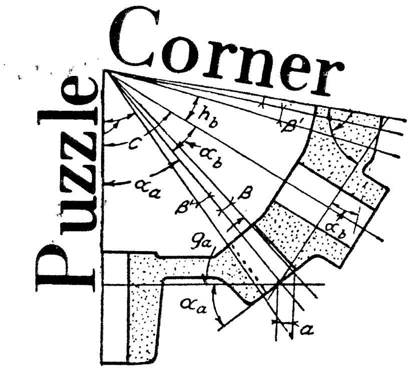


Puzzle Corner

Allan Gottlieb, '67

Address all correspondence to:

Kid's Place
Baker 438
362 Memorial Drive
Cambridge, Massachusetts 02139

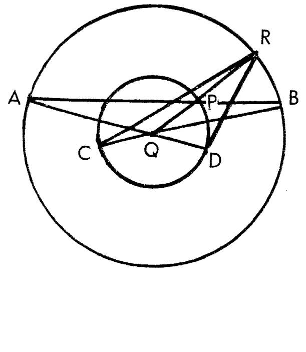
Thank you, one and all. Your many letters containing problems and solutions, when coupled with random threatening letters from my creditors, have caused my mailbox to be continually full. This has done more for my persecution complex than could possibly be accomplished by a twenty-dollar visit to the shrink.

problems

43 — Place 7 unbent cigarettes such that each one is touching the other 6.

◆44 — Prove that none of the Platonic solids may be dissected into another (Two page proof). *Submitted by Mark Yu, '70.*

◆45 — Prove that $\overline{RD}^2 - \overline{RC}^2 = \overline{PB}^2 - \overline{PA}^2$



The circles have center Q, and CB and AD intersect at Q. *Submitted by Mark Yu, '70.*

◆46 — I have received the following letter:

Dear Mr. Gottlieb:

Last year at Wilson High School in Washington, D.C., one of my classmates, Michael Reedy (now at U. of Chicago) developed the Storey Intelligence Integral (named after Thomas Storey, now at the USNA). The following is an example:

Evaluate in five seconds or less:

$$\int_{-e^{-e^2}}^{\sin^3 4}\int_{\int_{-2}^{2}(y^3+3)\,dy}^{25\cosh(\ln 3)}\int_{-e^{-\ln(1/2)}}^{\int_{\pi}^{2\pi}\sin X\,dX}\frac{(\cosh^2(x+2)\sin x^3)^{x^2-x}\,dx}{\ln(e^{2x}+\ln 2x e^{e^x}-1)^{x+2\sin x}}$$

Mark Pelcovits, '70
Hayden 107
East Campus

I will accept any evaluation performed in 5 *weeks* or less.

SPEED department

47(a) — I have received the following letter:

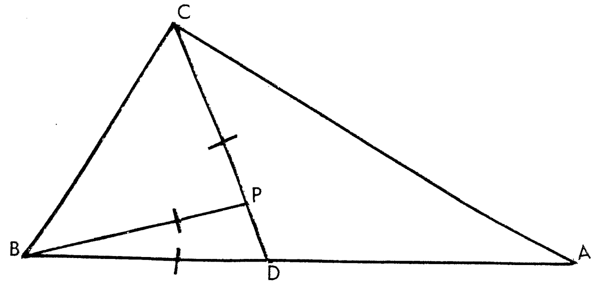
Suggestion for speed problem

$$\lim_{2\to\infty}\left(\frac{1}{2}\right) = \underline{\qquad\qquad}$$

A. Ratner
Ass Editor

(b) Fill in the blank: Ass Editors write__________ problems.

48 — Prove that it is impossible to construct a point P in the general triangle ABC such that CP = BP = BD.



Submitted by Mark Yu, '70.

solutions

29 —

```
  WASMARCH
+ THEBEST
 CANDIDATE
```

The product $H \cdot E \cdot R \cdot B \cdot I \cdot D$ equals 0.

I have been informed that there are several trivial solutions to this problem, which was given in May 1966 as problem number 24. For example, all the characters may be zeros. To force the solution to be unique I am adding the condition that M, which does not equal zero, equals $C \cdot W$.

The following was contributed by Don B. Zagier, '70:

We consider only the case where W, T, and C are non-zero (i.e., the numbers are real 8-, 7-, and 9- digit numbers). This is not a partial solution: since I produce two solutions, though the problem was supposed to have a unique solution, it is the problem rather than the solution that fails to be exhaustive. The solutions are

```
  90790912              90790813
+  9219179     and    +  9320279
 100010091             100111092
```

Method: Plainly, W is 9, C is 1, A is 0. Hence $M = CW = 9$. The third column then gives T is 9, N is 0. $E = 0$ leads to a contradiction, so the seventh column gives $R + E = 10$, $D \equiv B + 1 \bmod 10$. $H = 0$ gives a contradiction, so from the last column $H = E + 1$, so S is 7 from the 8th column. From $H \cdot E \cdot R \cdot B \cdot I \cdot D = 0$ we get $H = E + 1 = D + 2$. There are now two possibilities from the 6th column: $D = 0$, $B = 9$ (which gives first solution) or $B \neq 9$, $D = B + 1$, $D = I$, $R + D = 9$, and R or $B = 0$ (from $H \cdot E \cdot R \cdot B \cdot I \cdot D = 0$) so (since the 4th column gives $D \leqq 7$) $D \neq 9$, $R \neq 0$, so $B = 0$, $D = I = 1$, $R = 8$, and we have the second solution.

Bob Parker, '70, found the second solution, and Mark Yu, '70, found the first plus a completely different solution, "No".

♦ 30 —I have received the following letter:

Dear Mr. Gottlieb:

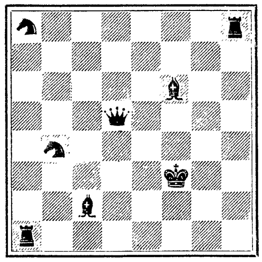
We have discovered (we think) an interesting chess problem which you and your readers may find amusing. It is moderately difficult, but there is a solution. Given the eight rear-rank pieces, place them on a board in such a way that they cover every square. (i.e., any piece of the opposing color placed anywhere on the board may be taken in one move.) The two bishops may not be of the same color.

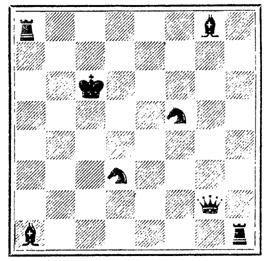
Lawrence Ribbecke
Mitchell Wand
FroshHacKomm HQ
553 Lounge
Burton House

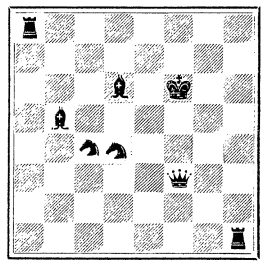
I am not sure whether one must protect his own pieces. The reader may attempt to solve the problem either way.

Zagier again:

I have found several solutions. The problem as stated does not require one to cover one's own pieces (since they are not enemy pieces). Three solutions are:







(red squares represent unprotected pieces); the third is the "best" in that all your own pieces but one are covered.

If anyone can find a solution in which all the pieces are covered, I will welcome it.

31 —Simplify the following:

$$\int_{1}^{\lim_{z \to \infty}\left(1 + \frac{1}{z}\right)^{z}} \left(\frac{\pi}{2} - \tan^{-1}x + \frac{\sum_{k=1}^{\infty} (-1)^{k+1}}{(2k+1)x^{2k+1}} \right) dx$$

$$- \frac{1}{2}(e^{i\alpha} - e^{-i\alpha})^2 + \cos 2\alpha$$

$$- \sum_{n=0}^{\infty} \frac{\cosh y \sqrt{1 - \tanh^2 y}}{\left(\sum_{j=0}^{\infty} \frac{\cosh \gamma \sqrt{1 - \tanh^2 \gamma}}{2^j} \right)^n}$$

CRC's are permitted. *SHOW ALL WORK.*

The following solution was contributed by Mark Pelcovits, '70:

The upper limit of the integral is $r = \lim_{Z\to\infty}\left(1+\frac{1}{Z}\right)^z$.

$\ln r = \lim z \ln(1 + 1/z) = \lim\left(-1/(z+1) + \ln(1+1/z)\right)$ $=1$ by l'Hôpital's rule. $\therefore\ r=e$.

(That's pretty cool but e is often defined *to be* $\lim_{z\to\infty}\left(1+\frac{1}{Z}\right)^z$ *— ed.)*

$$\int_1^e\left(\frac{\pi}{2} - \tan^{-1}x + \frac{\sum_{k=1}^{\infty}(-1)^{k+1}}{(2k+1)x^{2k+1}}\right)dx$$

$$= \int_1^e\left(\frac{\pi}{2} - \frac{\pi}{2} + \frac{1}{x} - \frac{\sum_{k=1}^{\infty}(-1)^{k+1}}{(2k+1)x^{2k+1}} + \frac{\sum_{k=1}^{\infty}(-1)^{k+1}}{(2k+1)x^{2k+1}}\right)dx$$

$$= \int_1^e\frac{dx}{x} = \ln x\Big]_1^e = 1$$

$$-\frac{1}{2}(e^{i\alpha} - e^{-i\alpha})^2 = 2\left(\frac{1}{4i^2}(e^{i\alpha} - e^{-i\alpha})^2\right) = 2\sin^2\alpha = 1 - \cos 2\alpha$$

$$-\sum_{n=o}^{\infty}\frac{\cosh y\sqrt{1-\tanh^2 y}}{\left(\frac{\sum_{j=o}^{\infty}\cosh\gamma\sqrt{1-\tanh^2\gamma}}{2^j}\right)^n} = -\sum_{n=o}^{\infty}\frac{1}{\left(\sum_{j=o}^{\infty}\frac{1}{2^j}\right)^n}$$

(from the identity $\cosh y = 1/\sqrt{1-\tanh^2 y}$)

$= -\sum 1/2^n = -2$ (from the sum of a geometric series)

$\therefore$ the expression equals $1 + 1 - \cos 2\alpha + \cos 2\alpha - 2 =$ 0

Mr. Zagier solved this problem as well.

32 — What non-zero five digit number has its digits reversed when multiplied by 4 ?

The following was received:

ans 21978

```
 21978
     4
 87912
```

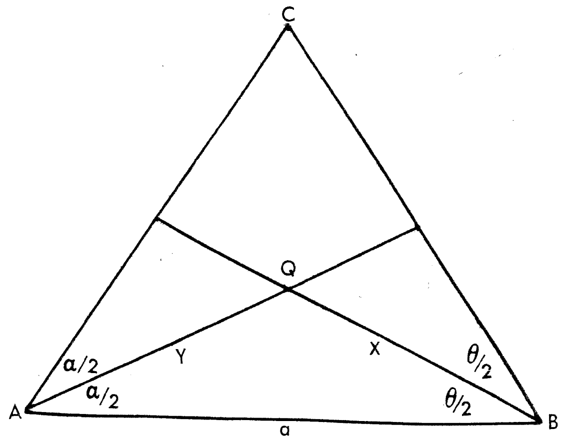
The answer was arrived at by Sheer Randomness!!

Lawrence Smith, '68
Hayden 405
East Campus

This problem was also answered by Messrs. Yu and Parker, and by Zagier again, who even gave a uniqueness proof!

33 — Prove the well-known theorem in geometry that if two angle bisectors of a triangle are equal, then the triangle is isosceles.

As usual Zagier solved the problem. My proof has the advantage of being purely geometrical but his is so much hairier I cannot resist printing it instead.



We show that if one of the bisected angles is α, the other is too. Let AB be of length a, $\angle CAB = \alpha$. Let angle QBA be θ, so that AQ bisects A, BP bisects B, AQ = Y, PB = X. We have to show that $X = Y \Longrightarrow \theta = \alpha$. The sine law in ABP gives

$$\frac{x}{\sin\alpha} = \frac{\alpha}{\sin(\pi - \alpha - \theta/2)} \quad\text{or } x = \frac{a\sin\alpha}{\sin(\alpha + \theta/2)}$$

and similarly from ABQ, $Y = \dfrac{a\sin\theta}{\sin(\theta + \alpha/2)}$.

$$\text{Hence, } \frac{d}{d\theta}(x - y) = -\frac{a\sin\alpha\cos(\alpha+\theta/2)}{2\sin^2(\alpha+\theta/2)}$$

$$-\frac{a\cos\theta\sin(\theta+\alpha/2) - a\sin\theta\cos(\theta+\alpha/2)}{\sin^2(\theta+\alpha/2)}$$

$$= -\frac{a\sin\alpha\cos(\alpha+\theta/2)}{2\sin^2(\alpha+\theta/2)} - a\frac{\sin\alpha/2}{\sin^2(\theta+\alpha/2)}$$

< 0 for all $\theta, \alpha, a > 0$.

That is, x-y is a monotone (decreasing) function of θ. Since plainly $x\text{-}y = 0$ when $\theta = a$, this shows that $\theta = a$ is the only root of $x\text{-}y = 0$, so that the equality of the angle bisectors x and y implies that of the angles A and B, so ABC is isosceles.

Thank you, Don. Let me know if you ever need a recommendation to grad school.

In his letter, Yu states "Concerning #33, I know some guys who'll pay you plenty for its correct proof." Mark, you can tell them to make the check payable to Allan (Kid) Gottlieb.

Due to various deadlines the bulk of this column went to the printer only a few weeks after the October issue appeared. I have subsequently received several additional solutions. Richard Haberman, '67, submitted an elegant proof to Problem 32, and adds that 2178, 219978, 2199978, etc., also have the required property. Bob Parker, '70, solved Problem 30 and came within standard engineering tolerances on Problem 31. ■