Single-View Geometry

EECS 442 – David Fouhey
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https://web.eecs.umich.edu/~fouhey/teaching/EECS442_W23/
Application: Single-view modeling

A. Criminisi, I. Reid, and A. Zisserman, Single View Metrology, IJCV 2000
Application: Measuring Height
Application: Measuring Height

- CSI before CSI
- Covered criminal cases talking to random scientists (e.g., footwear experts)
- How do you tell how tall someone is if they’re not kind enough to stand next to a ruler?
Application: Camera Calibration

Calibration a HUGE pain
Application: Camera Calibration

- What if 3D coordinates are unknown?
- Use scene features such as vanishing points
Camera calibration revisited

- What if 3D coordinates are unknown?
- Use scene features such as vanishing points
Recall: Vanishing points

All lines having the same *direction* share the same vanishing point.

Slide credit: S. Lazebnik
Calibration from vanishing points

Consider a scene with 3 orthogonal directions $v_1, v_2$ are finite vps, $v_3$ infinite vp

Want to align world coordinates with directions $v_1, v_2, v_3$.
Calibration from vanishing points

\[
P_{3 \times 4} \equiv \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}
\]

It turns out that

\[
p_1 \equiv P [1,0,0,0]^T \quad \text{VP in X direction}
\]

\[
p_2 \equiv P [0,1,0,0]^T \quad \text{VP in Y direction}
\]

\[
p_3 \equiv P [0,0,1,0]^T \quad \text{VP in Z direction}
\]

\[
p_4 \equiv P [0,0,0,1]^T \quad \text{Projection of origin}
\]

Note the usual \( \equiv \) (i.e., all of this is up to scale) as well as where the 0 is
Calibration from vanishing points

Let’s align the world coordinate system with the three orthogonal vanishing directions:

\[
\begin{align*}
    e_1 &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\
    e_2 &= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\
    e_3 &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\end{align*}
\]

\[
\lambda v_i = K[R, t] \begin{bmatrix} e_i \\ 0 \end{bmatrix}
\]

\[
\lambda v_i = KRe_i
\]

Drop the t

\[
R^{-1}K^{-1}\lambda v_i = e_i
\]

Inverses
Calibration from vanishing points

So \( e_i = R^{-1}K^{-1}\lambda v_i \), but who cares?

What are some properties of axes?

Know \( e_i^T e_j = 0 \) for \( i \neq j \), so \( K, R \) have to satisfy

\[
(R^{-1}K^{-1}\lambda_j v_j)^T(R^{-1}K^{-1}\lambda_i v_i) = 0
\]

\[
(R^T K^{-1}\lambda_j v_j)^T(R^T K^{-1}\lambda_i v_i) = 0
\]

\[
\lambda_i \lambda_j (R^T K^{-1}v_j)^T(R^T K^{-1}v_i) = 0
\]

Move scalars

\[
v_j K^{-T} R R^T K^{-1} v_i = 0
\]

Clean up

\[
v_j K^{-T} K^{-1} v_i = 0
\]

\[
R R^T = I
\]
Calibration from vanishing points

- Intrinsics (focal length $f$, principal point $u_0, v_0$) have to ensure that the rays corresponding to vanishing points for 3 mutually orthogonal directions are orthogonal

$$v_j K^{-T} K^{-1} v_i = 0$$
Calibration from vanishing points

1 finite vanishing point, 2 infinite vanishing points
2 finite vanishing points, 1 infinite vanishing point
3 finite vanishing points

Cannot recover focal length, principal point is the third vanishing point
Can solve for focal length, principal point

Slide credit: S. Lazebnik
Directions and vanishing points
Directions and vanishing points
Directions and vanishing points

If $\mathbf{v}$ vanishing point, and $\mathbf{K}$ the camera intrinsics, $\mathbf{K}^{-1}\mathbf{v}$ is the corresponding direction.

Set $u_0, v_0 = 0, 0$

$$\mathbf{K}^{-1} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{v}_1 = [-f, 0, 1]$

$\mathbf{v}_3 = [0, f\times10^{99}, 1]$

$\mathbf{v}_2 = [f, 0, 1]$
Directions and vanishing points

If I normalize each $K^{-1}v_i$, I get:

\[
  K^{-1}v_1 = [-1,0,1], \quad K^{-1}v_2 = [1,0,1], \quad K^{-1}v_3 = [0, 10^{99}, 1]
\]

\[
  K^{-1} = \begin{bmatrix}
  1/f & 0 & 0 \\
  0 & 1/f & 0 \\
  0 & 0 & 1
\end{bmatrix}
\]

$v_1 \ [\text{-f,0,1}]$

$v_2 \ [\text{f,0,1}]$

$v_3 \ [\text{0,f*10^{99},1}]$
Rotation from vanishing points

Know that $\lambda_i v_i = K R e_i$ and have $K$, but want $R$

So: $\lambda K^{-1} v_i = Re_i$

What does $R e_i$ look like?

$$Re_1 = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = r_1$$

The $i$th column of $R$ is a scaled version of $r_i = \lambda K^{-1} v_i$
Calibration from vanishing points

- Solve for K (focal length, principal point) using 3 orthogonal vanishing points
- Get rotation directly from vanishing points once calibration matrix known
- Pros:
  - Could be totally automatic!
- Cons:
  - Need 3 vanishing points, estimated accurately, AND orthogonal with at least two finite!

Slide credit: S. Lazebnik
Finding Vanishing Points

What might go wrong with the circled points?

Image credit: J.P. Tardif
Finding Vanishing Points

• Find long edges $E = \{e_1, ..., e_n\}$

• All $\binom{n}{2}$ intersections of edges $v_{ij} = e_i \times e_j$ are potential vanishing points

• Try all triplets of popular vanishing points, check if the camera’s focal length, principal point “make sense”

• What are some options for this?
Finding Vanishing Points

Image credit: J.P. Tardif
Measuring height
Measuring height
Measuring height

Camera height

5.3

3.3

2.8
Measuring height without a ruler

Compute $Z$ from image measurements: We’ll need more than vanishing points to do this

Slide credit: S. Lazebnik
Projective invariant

- We need to use a *projective invariant*: a quantity that does not change under projective transformations (including perspective projection)
Projective invariant

• We need to use a projective invariant: a quantity that does not change under projective transformations (including perspective projection)

• The cross-ratio of four points:

\[
\frac{\|P_3 - P_1\| \|P_4 - P_2\|}{\|P_3 - P_2\| \|P_4 - P_1\|}
\]

This is one of the cross-ratios (can reorder arbitrarily)
Measuring height

\[
\frac{\|T - B\|}{\|R - B\|} \frac{\|\infty - R\|}{\|\infty - T\|} = \frac{H}{R}
\]

scene cross ratio

\[
\frac{\|t - b\|}{\|r - b\|} \frac{\|v_z - r\|}{\|v_z - t\|} = \frac{H}{R}
\]

image cross ratio
Measuring height without a ruler
vanishing line (horizon)

image cross ratio

\[
\frac{||t - b|| ||v_Z - r||}{||r - b|| ||v_Z - t||} = \frac{H}{R}
\]
Remember This?

- Line equation: \( ax + by + c = 0 \)
- Vector form: \( l^T \mathbf{p} = 0, \; l = [a, b, c], \; \mathbf{p} = [x, y, 1] \)
- Line through two points?
  - \( l = \mathbf{p}_1 \times \mathbf{p}_2 \)
- Intersection of two lines?
  - \( \mathbf{p} = l_1 \times l_2 \)
- Intersection of two parallel lines is at infinity
vanishing line (horizon)

\[ v \cong (b \times b_0) \times (v_x \times v_y) \]

image cross ratio

\[
\frac{\|t - b\| \|v_z - r\|}{\|r - b\| \|v_z - t\|} = \frac{H}{R}
\]
Example Gone Wrong

Know length of red → can figure out height of blue because they intersect at vanishing point v

Wrong! Any two lines always intersect! Need to point to same 3D direction / VP.
Example Gone Wrong

**Wrong!** Need to connect feet to the horizon (at infinity – thank homogenous coordinates), and then to Jimmy’s head.
Examples


Slide credit: S. Lazebnik

Figure from UPenn CIS580 slides
Another example

- Are the heights of the two groups of people consistent with one another?

Piero della Francesca, *Flagellation*, ca. 1455


Slide credit: S. Lazebnik
Measurements on planes
Measurements on planes

Slide credit: S. Lazebnik
Image rectification: example

Piero della Francesca, *Flagellation*, ca. 1455

Slide credit: S. Lazebnik
Application: 3D modeling from a single image


Slide credit: S. Lazebnik
Application: 3D modeling from a single image

J. Vermeer, *Music Lesson*, 1662

Application: Object Detection

“Reasonable” approximation:

\[ y_{\text{object}} \approx \frac{h y_{\text{camera}}}{v_0 - v} \]
Application: Object detection

(a) input image

Diagram Credit: D. Hoiem
Application: Object detection

Diagram Credit: D. Hoiem
Application: Image Editing

K. Karsch and V. Hedau and D. Forsyth and D. Hoiem, Rendering Synthetic Objects into Legacy Photographs, SIGGRAPH Asia 2011
Application: Estimating Layout

V. Hedau, D. Hoiem, D. Forsyth
Recovering the spatial layout of cluttered rooms ICCV 2009