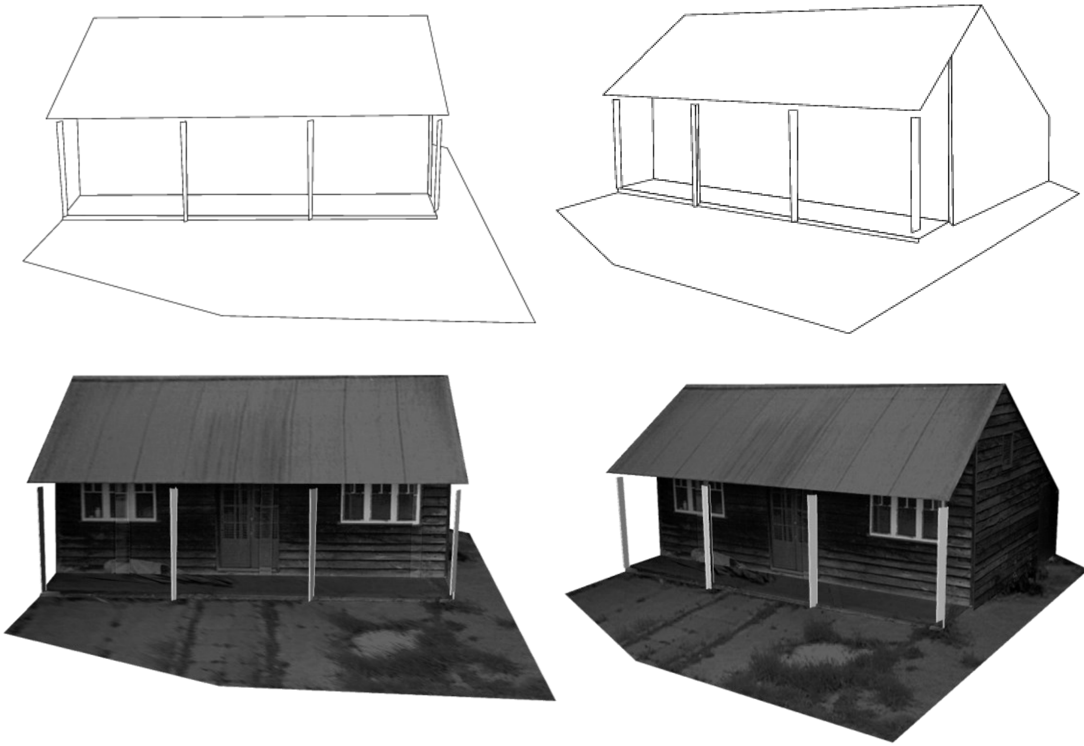
Single-View Geometry

EECS 442 – David Fouhey

Winter 2023, University of Michigan

https://web.eecs.umich.edu/~fouhey/teaching/EECS442_W23/

Application: Single-view modeling



A. Criminisi, I. Reid, and A. Zisserman,
Single View Metrology, IJCV 2000

Application: Measuring Height

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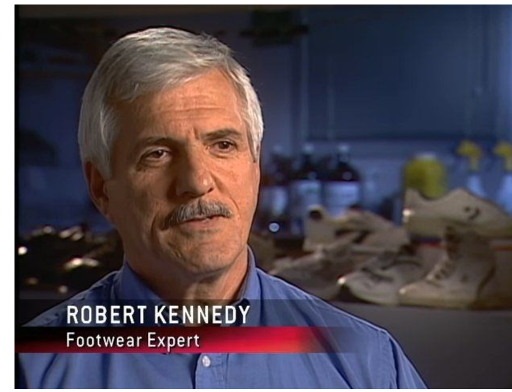
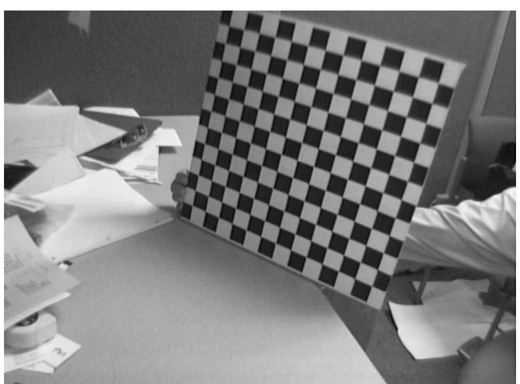


- CSI before CSI
- Covered criminal cases talking to random scientists (e.g., footwear experts)
- How do you tell how tall someone is if they’re not kind enough to stand next to a ruler?

Application: Camera Calibration

Calibration a HUGE pain



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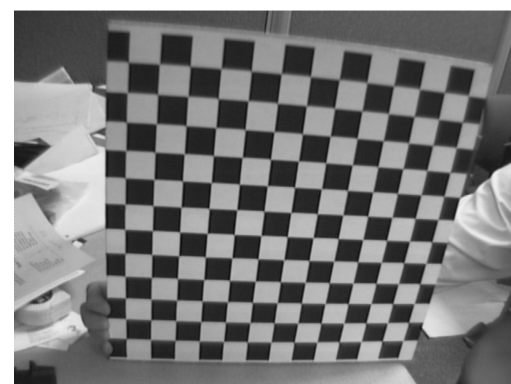


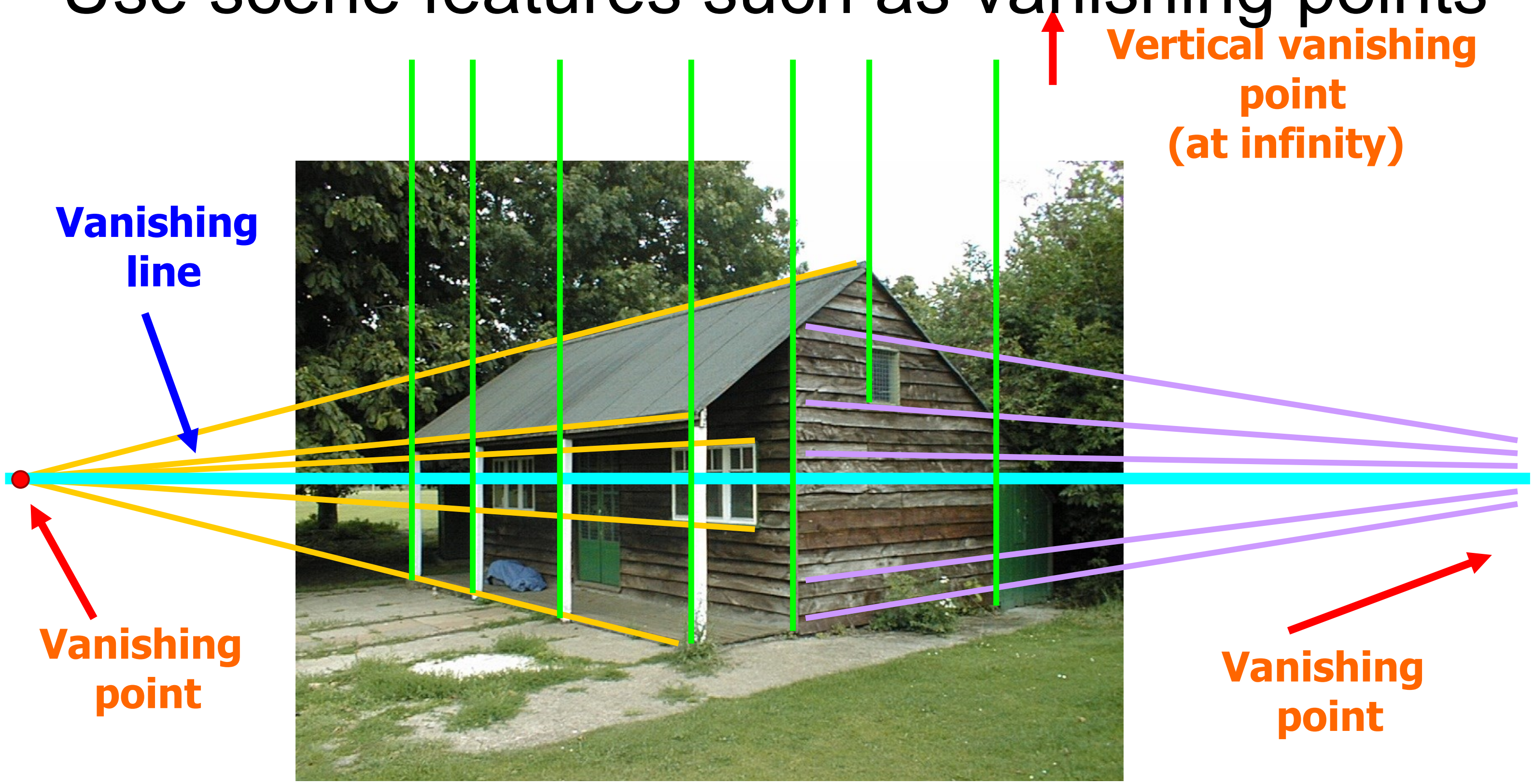
Application: Camera Calibration

- What if 3D coordinates are unknown?
- Use scene features such as vanishing points

Slide from Efros, Photo from Criminisi

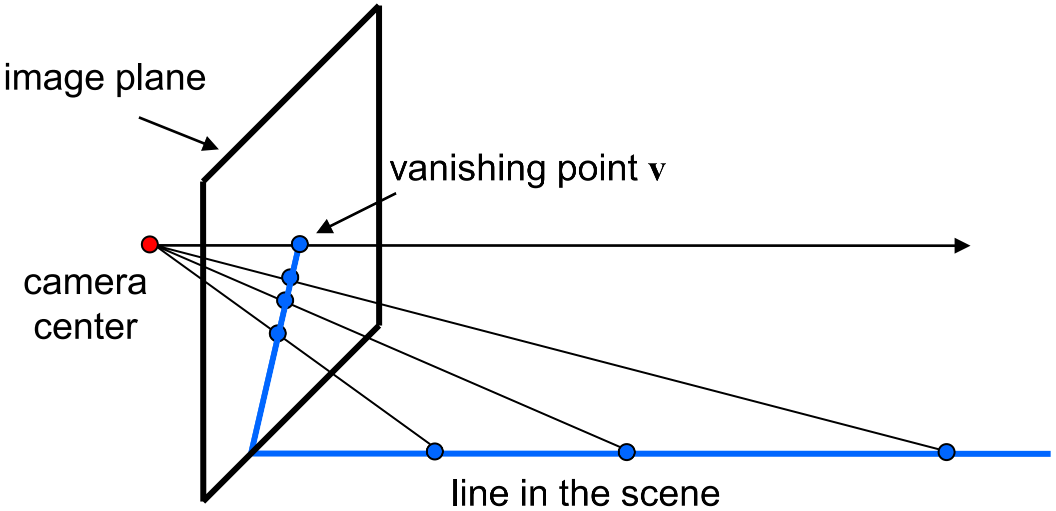
Camera calibration revisited

- What if 3D coordinates are unknown?
- Use scene features such as vanishing points



Slide from Efros, Photo from Criminisi

Recall: Vanishing points



All lines having the same *direction* share the same vanishing point

Slide credit: S. Lazebnik

Calibration from vanishing points

Consider a scene with 3 orthogonal directions

$\mathbf{v}_1$, $\mathbf{v}_2$ are *finite* vps, $\mathbf{v}_3$ *infinite* vp

Want to align world coordinates with directions

Slide credit: S. Lazebnik

Calibration from vanishing points

$$\boldsymbol{P}_{3x4} \equiv [\,\boldsymbol{p}_1 \;\; \boldsymbol{p}_2 \;\; \boldsymbol{p}_3 \;\; \boldsymbol{p}_4\,]$$

It turns out that

$\boldsymbol{p_1} \equiv \boldsymbol{P}\,[1,0,0,0]^T$ VP in X direction

$\boldsymbol{p_2} \equiv \boldsymbol{P}\,[0,1,0,0]^T$ VP in Y direction

$\boldsymbol{p_3} \equiv \boldsymbol{P}\,[0,0,1,0]^T$ VP in Z direction

$\boldsymbol{p_4} \equiv \boldsymbol{P}\,[0,0,0,1]^T$ Projection of origin

Note the usual $\equiv$ (i.e., all of this is up to scale) as well as where the 0 is

Calibration from vanishing points

Let's align the world coordinate system with the three orthogonal vanishing directions:

$$\boldsymbol{e_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{e_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \boldsymbol{e_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda \boldsymbol{v_i} = \boldsymbol{K}[\boldsymbol{R}, \boldsymbol{t}] \begin{bmatrix} \boldsymbol{e_i} \\ 0 \end{bmatrix}$$

$$\lambda \boldsymbol{v_i} = \boldsymbol{K}\boldsymbol{R}\boldsymbol{e}_i \qquad \text{Drop the t}$$

$$\boldsymbol{R}^{-1}\boldsymbol{K}^{-1}\lambda \boldsymbol{v}_i = \boldsymbol{e}_i \qquad \text{Inverses}$$

Calibration from vanishing points

So $\boldsymbol{e_i} = \boldsymbol{R^{-1}K^{-1}}\lambda\boldsymbol{v}_i$, but who cares?

What are some properties of axes?

Know $\boldsymbol{e_i^T e_j} = 0$ for $i \neq j$, so K, R have to satisfy

$$\left(\boldsymbol{R^{-1}K^{-1}}\lambda_j\boldsymbol{v_j}\right)^{\boldsymbol{T}}\left(\boldsymbol{R^{-1}K^{-1}}\lambda_i\boldsymbol{v}_i\right) = \boldsymbol{0}$$

$$\left(\boldsymbol{R^T K^{-1}}\lambda_j\boldsymbol{v_j}\right)^{\boldsymbol{T}}\left(\boldsymbol{R^T K^{-1}}\lambda_i\boldsymbol{v}_i\right) = \boldsymbol{0} \qquad R^{-1} = R^T$$

$$\lambda_i\lambda_j\left(\boldsymbol{R^T K^{-1}}\boldsymbol{v_j}\right)^{\boldsymbol{T}}\left(\boldsymbol{R^T K^{-1}}\boldsymbol{v}_i\right) = \boldsymbol{0} \quad \text{Move scalars}$$

$$\boldsymbol{v_j K^{-T} R R^T K^{-1} v_i} = \boldsymbol{0} \qquad \text{Clean up}$$

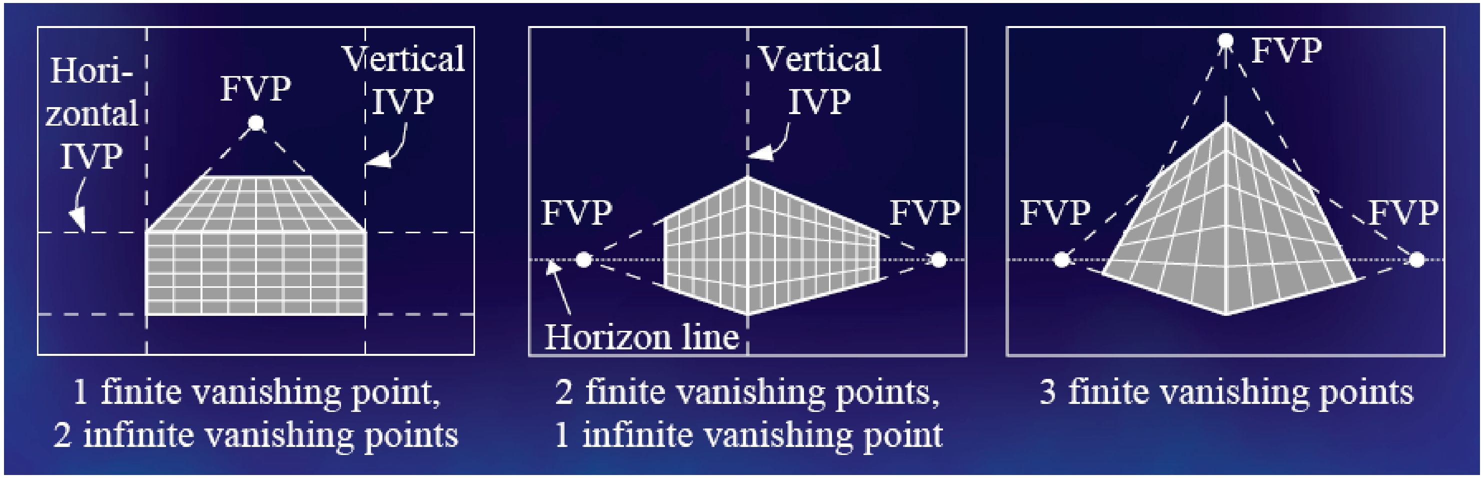
$$\boldsymbol{v_j K^{-T} K^{-1} v_i} = \boldsymbol{0} \qquad RR^T = I$$

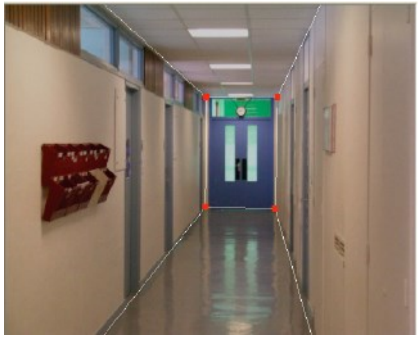
Calibration from vanishing points

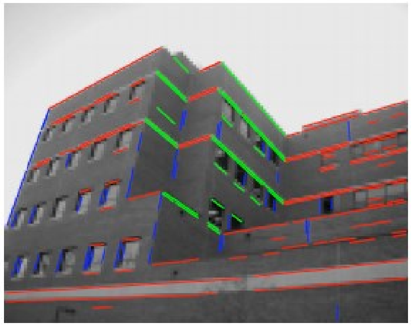
- Intrinsics (focal length f, principal point u_0,v_0) have to ensure that the rays corresponding to vanishing points for 3 mutually orthogonal directions are orthogonal

$$\boldsymbol{v_j K^{-T} K^{-1} v_i = 0}$$

Calibration from vanishing points





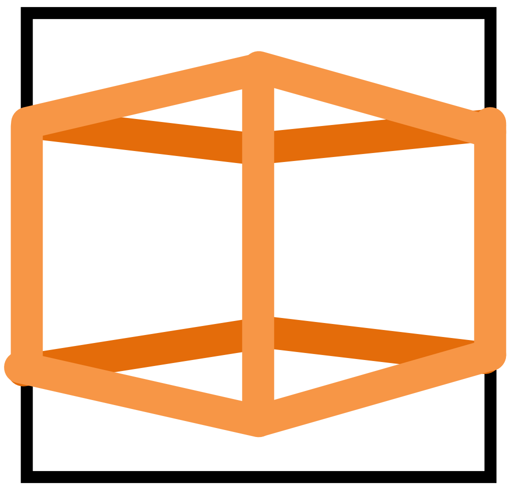


Cannot recover focal length, principal point is the third vanishing point

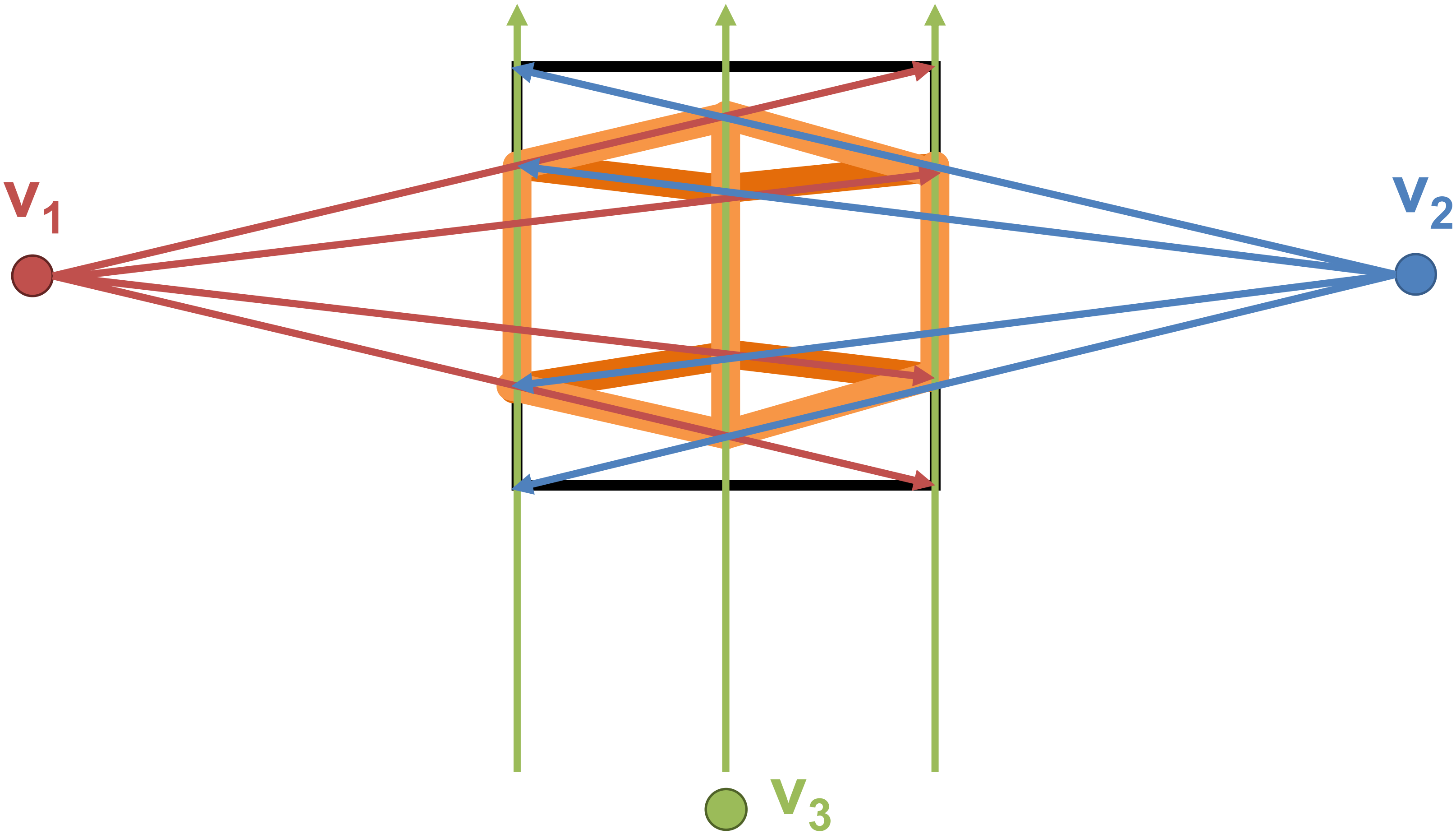
Can solve for focal length, principal point

Slide credit: S. Lazebnik

Directions and vanishing points

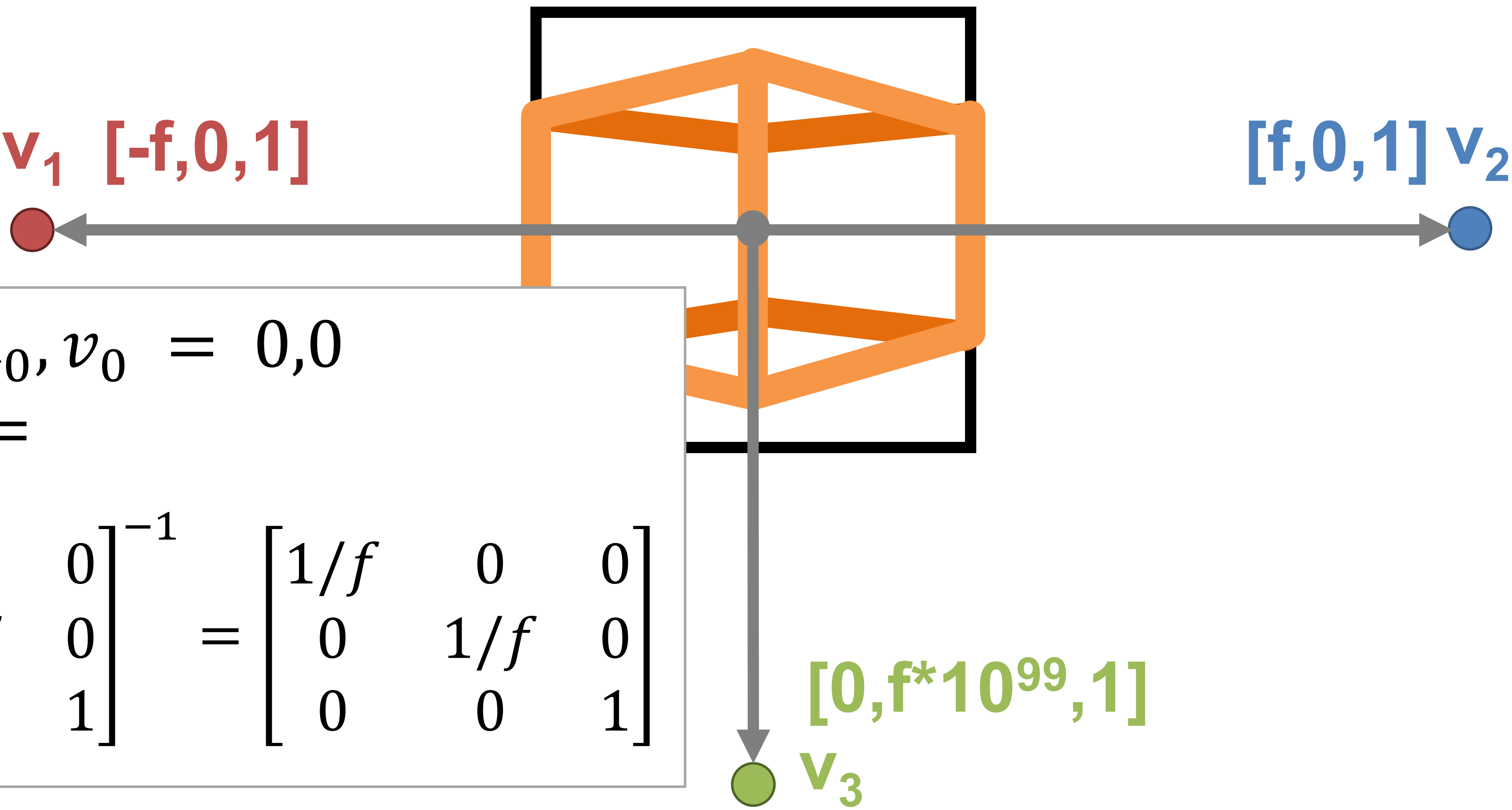


Directions and vanishing points



Directions and vanishing points

If $\boldsymbol{v}$ vanishing point, and $\boldsymbol{K}$ the camera intrinsics, $\boldsymbol{K}^{-1}\boldsymbol{v}$ is the corresponding direction.



Set $u_0, v_0 = 0{,}0$

$K^{-1} =$

$$\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

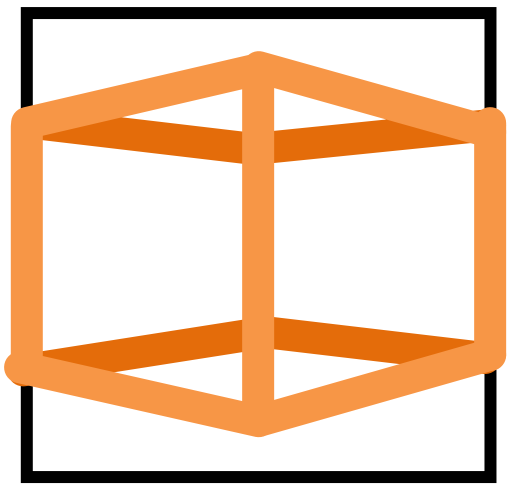
Directions and vanishing points

If I normalize each $\boldsymbol{K}^{-1}\boldsymbol{v}_i$, I get:

$$\left[-\frac{1}{\sqrt{2}}, 0\,\frac{1}{\sqrt{2}}\right], \left[\frac{1}{\sqrt{2}}, 0\,\frac{1}{\sqrt{2}}\right], [0,1,0]$$

v_1 [-f,0,1]

$K^{-1}v_1 = [-1,0,1]$



[f,0,1] v_2

$K^{-1}v_2 = [1,0,1]$

$K^{-1}v_3 = [0, 10^{99},1]$

[0,f*10^{99},1]

v_3

$$K^{-1} = \begin{bmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation from vanishing points

Know that $\lambda_i \boldsymbol{v_i} = \boldsymbol{KRe_i}$ and have **K**, but want **R**

So: $\lambda \boldsymbol{K^{-1}} \boldsymbol{v}_i = \boldsymbol{Re_i}$

What does $\boldsymbol{Re_i}$ look like?

$$\boldsymbol{Re_1} = [\boldsymbol{r_1} \quad \boldsymbol{r_2} \quad \boldsymbol{r_3}] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \boldsymbol{r_1}$$

The ith column of R is a scaled version of

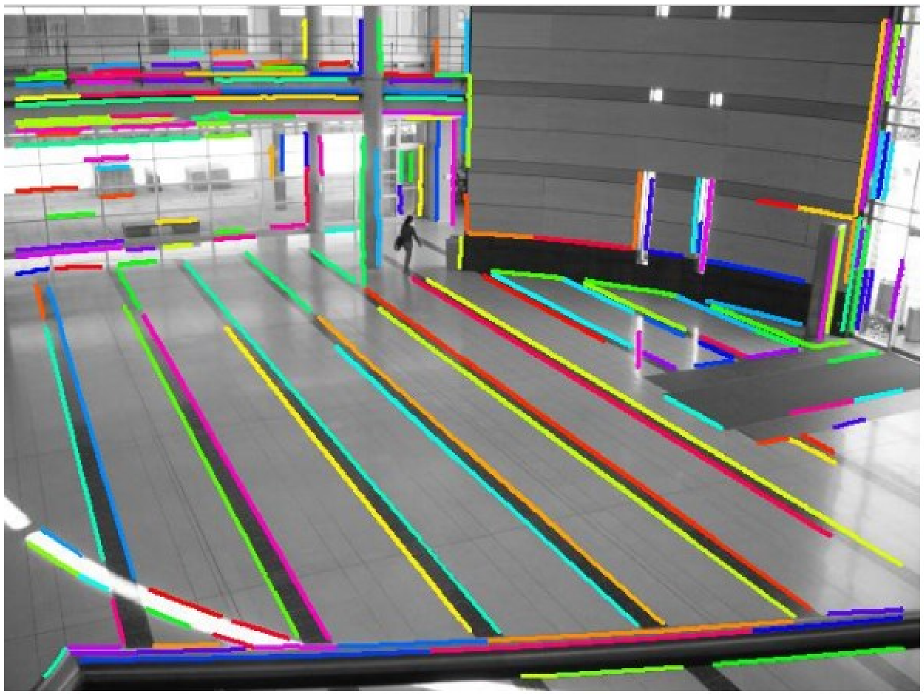
$\boldsymbol{r_i} = \lambda \boldsymbol{K^{-1}} \boldsymbol{v_i}$

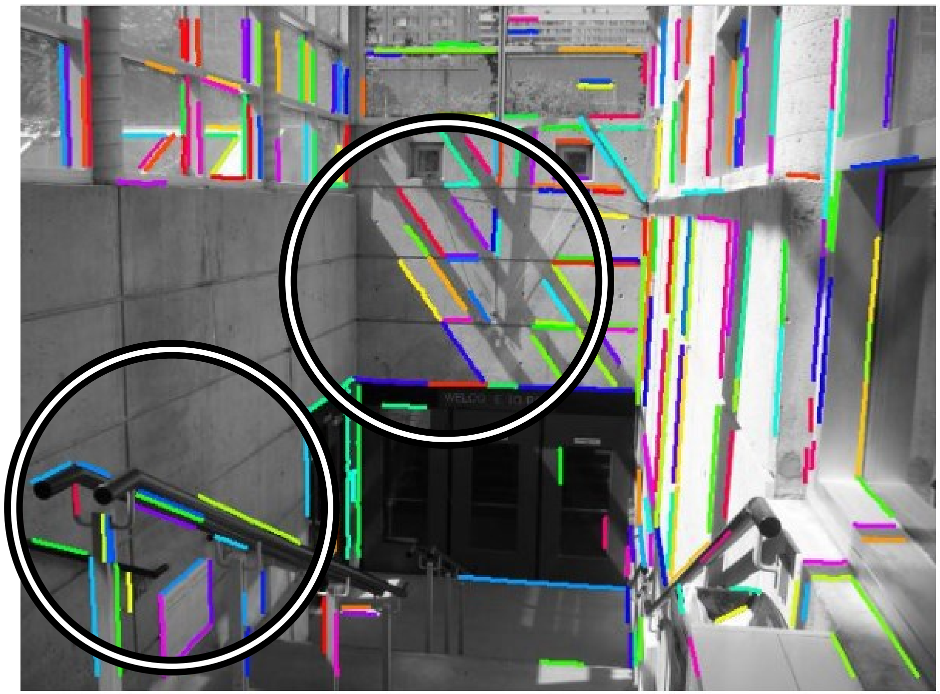
Calibration from vanishing points

- Solve for K (focal length, principal point) using 3 orthogonal vanishing points
- Get rotation directly from vanishing points once calibration matrix known
- Pros:
  - Could be totally automatic!
- Cons:
  - Need 3 vanishing points, estimated accurately, AND orthogonal with at least two finite!

Slide credit: S. Lazebnik

Finding Vanishing Points





What might go wrong with the circled points?

Image credit: J.P. Tardif

Finding Vanishing Points

- Find long edges $E = \{e_1, \ldots, e_n\}$
- All $\binom{n}{2}$ intersections of edges $v_{ij} = e_i \times e_j$ are potential vanishing points
- Try all triplets of popular vanishing points, check if the camera's focal length, principal point "make sense"
- **What are some options for this?**

Finding Vanishing Points

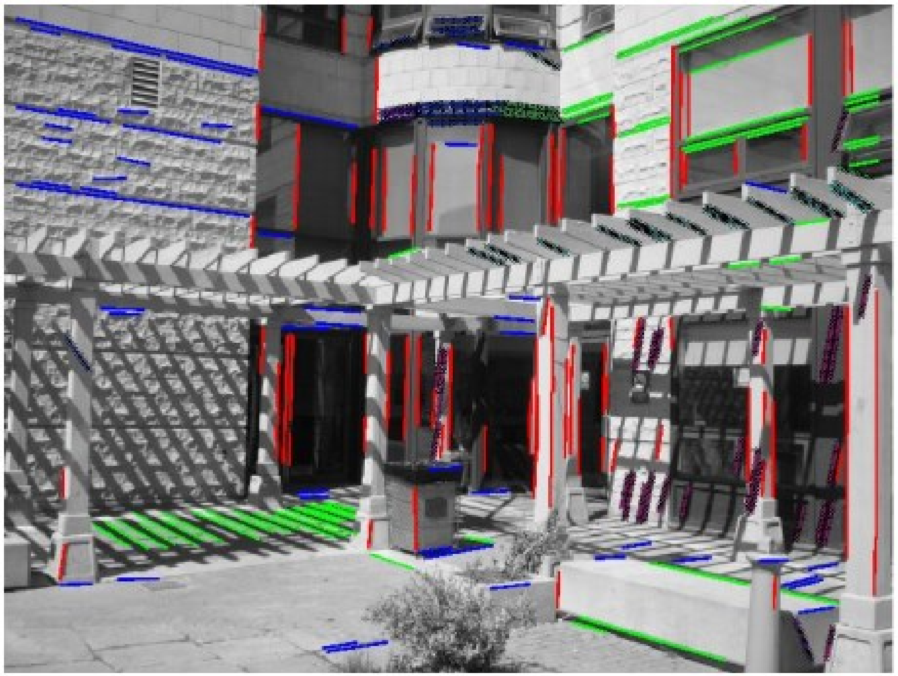
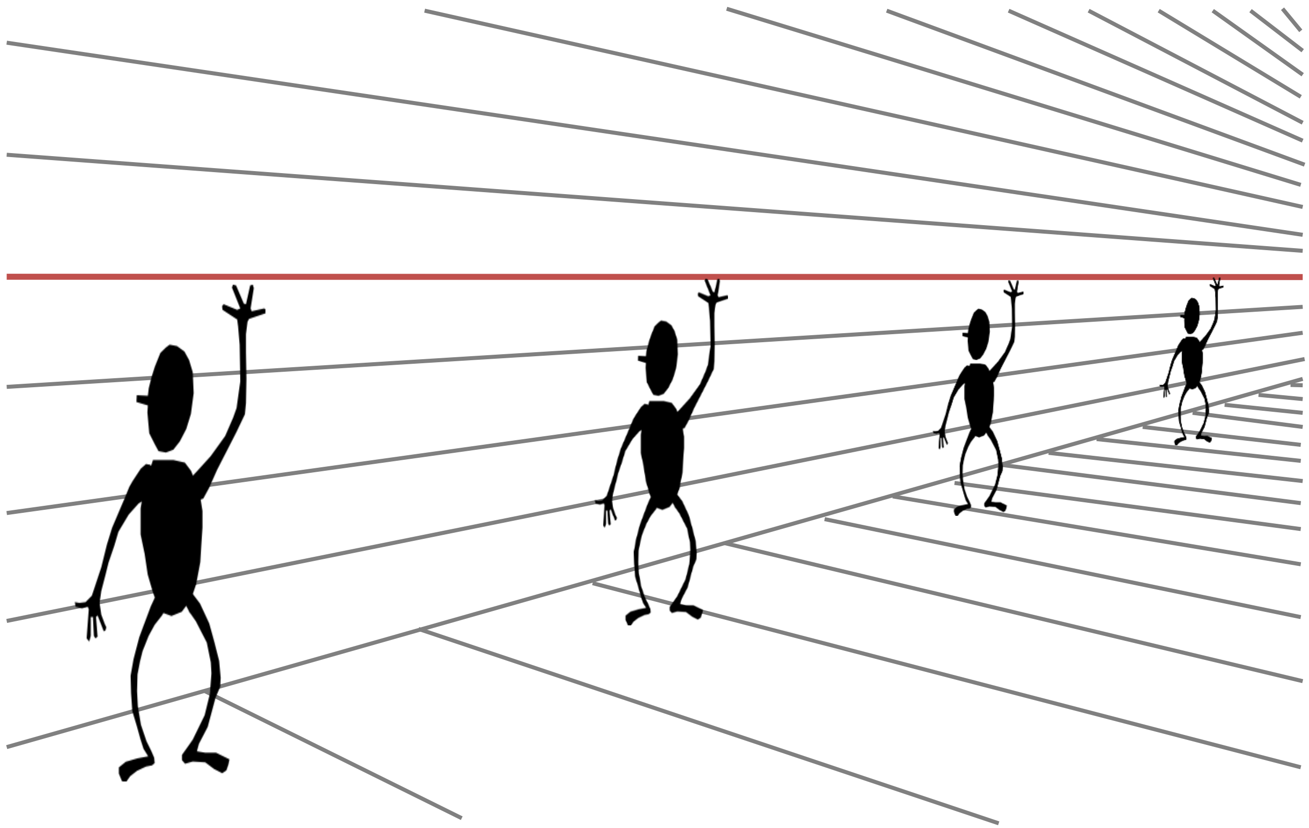


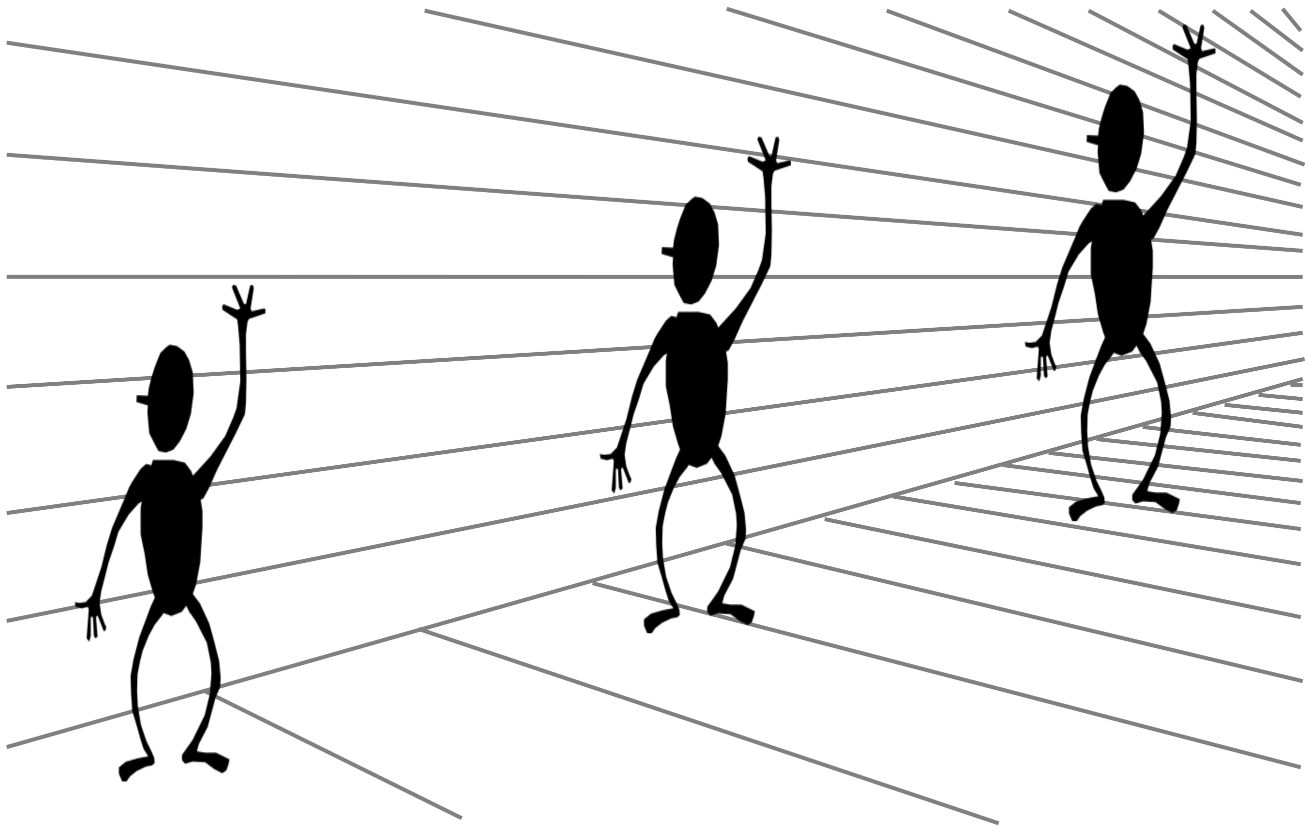
Image credit: J.P. Tardif

Measuring height



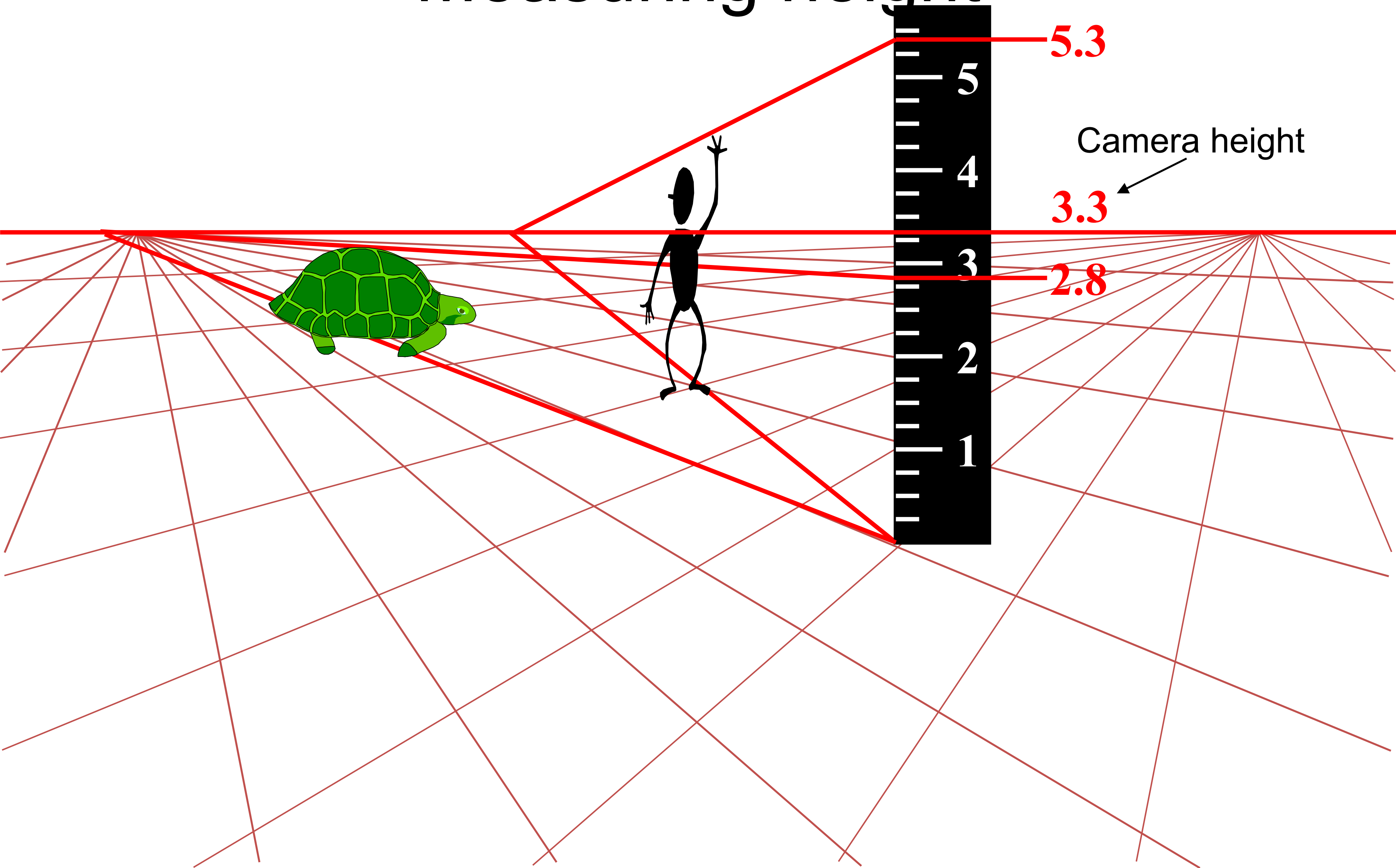
Slide by Steve Seitz

Measuring height

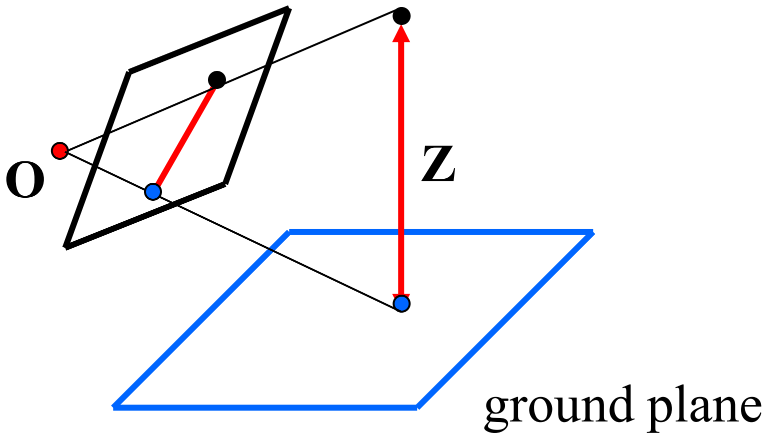


Slide by Steve Seitz

Measuring height



Measuring height without a ruler



Compute Z from image measurements: We'll need more than vanishing points to do this

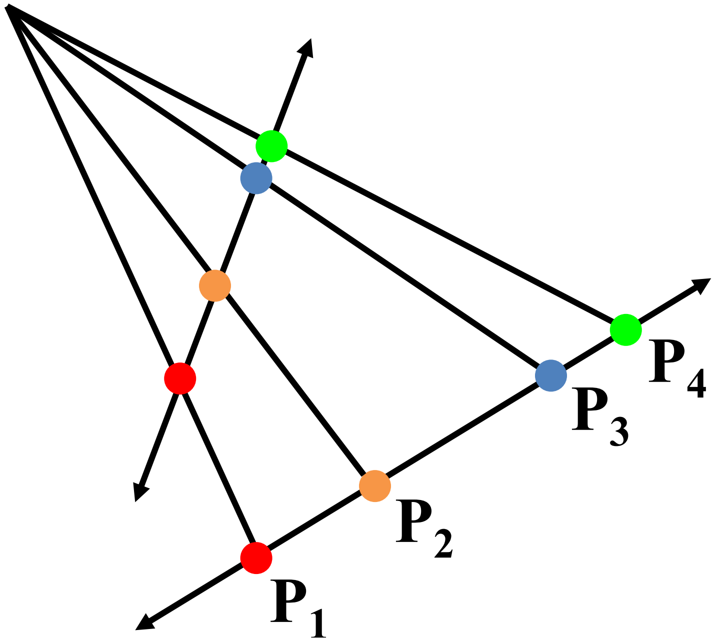
Projective invariant

- We need to use a *projective invariant*: a quantity that does not change under projective transformations (including perspective projection)

Slide credit: S. Lazebnik

Projective invariant

- We need to use a *projective invariant*: a quantity that does not change under projective transformations (including perspective projection)
- The cross-ratio of four points:

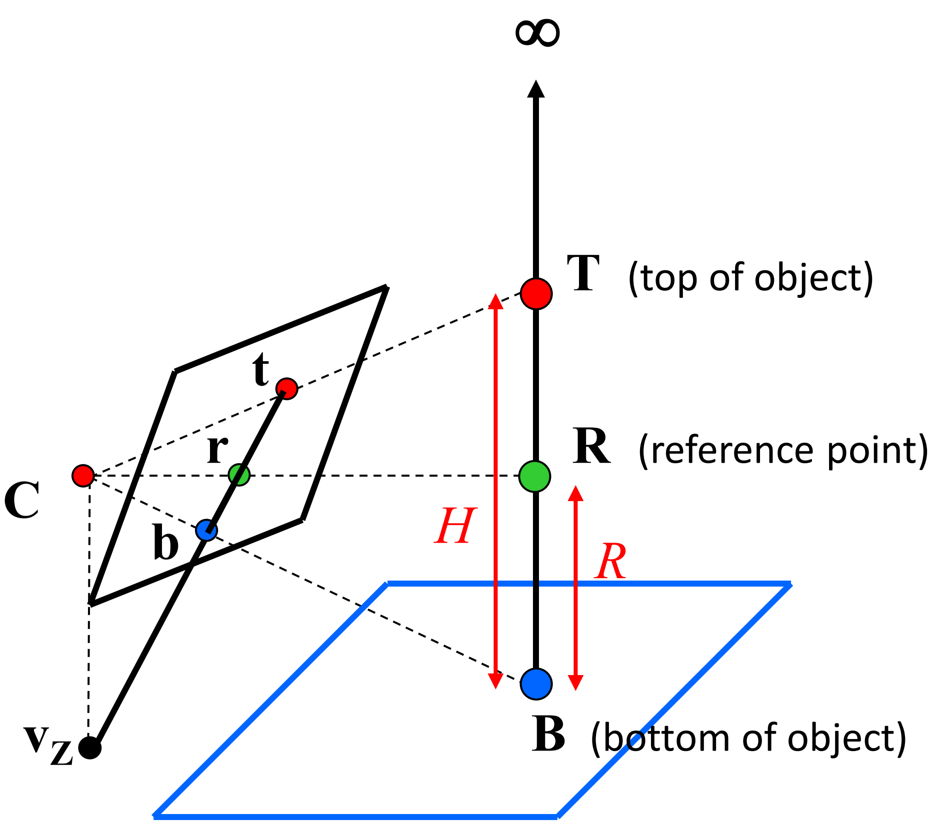


$$\frac{\|P_3 - P_1\| \, \|P_4 - P_2\|}{\|P_3 - P_2\| \, \|P_4 - P_1\|}$$

This is one of the cross-ratios (can reorder arbitrarily)

Slide credit: S. Lazebnik

Measuring height



$$\frac{\|\mathbf{T}-\mathbf{B}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{R}-\mathbf{B}\|\,\|\infty-\mathbf{T}\|} = \frac{H}{R}$$

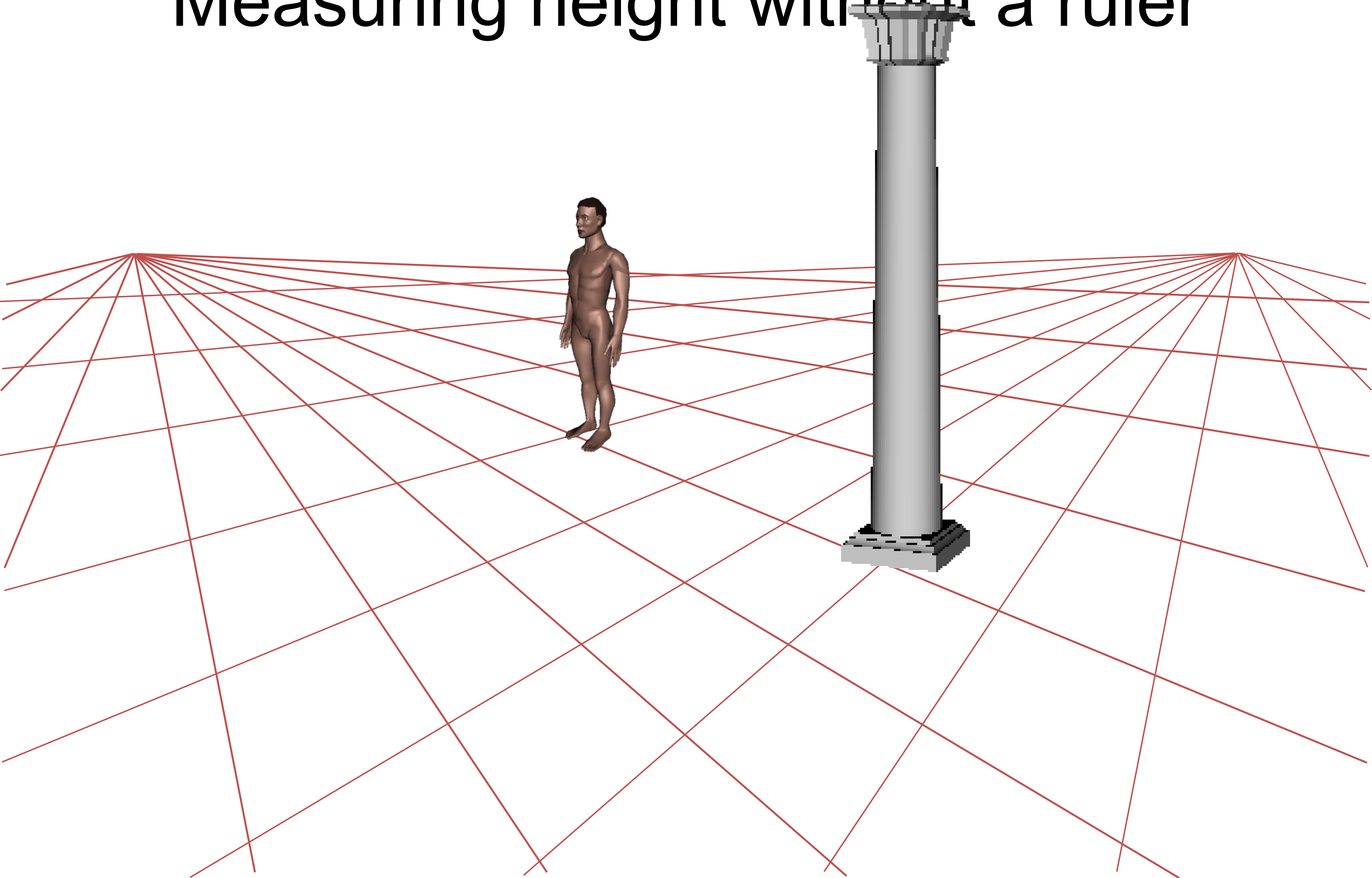
scene cross ratio

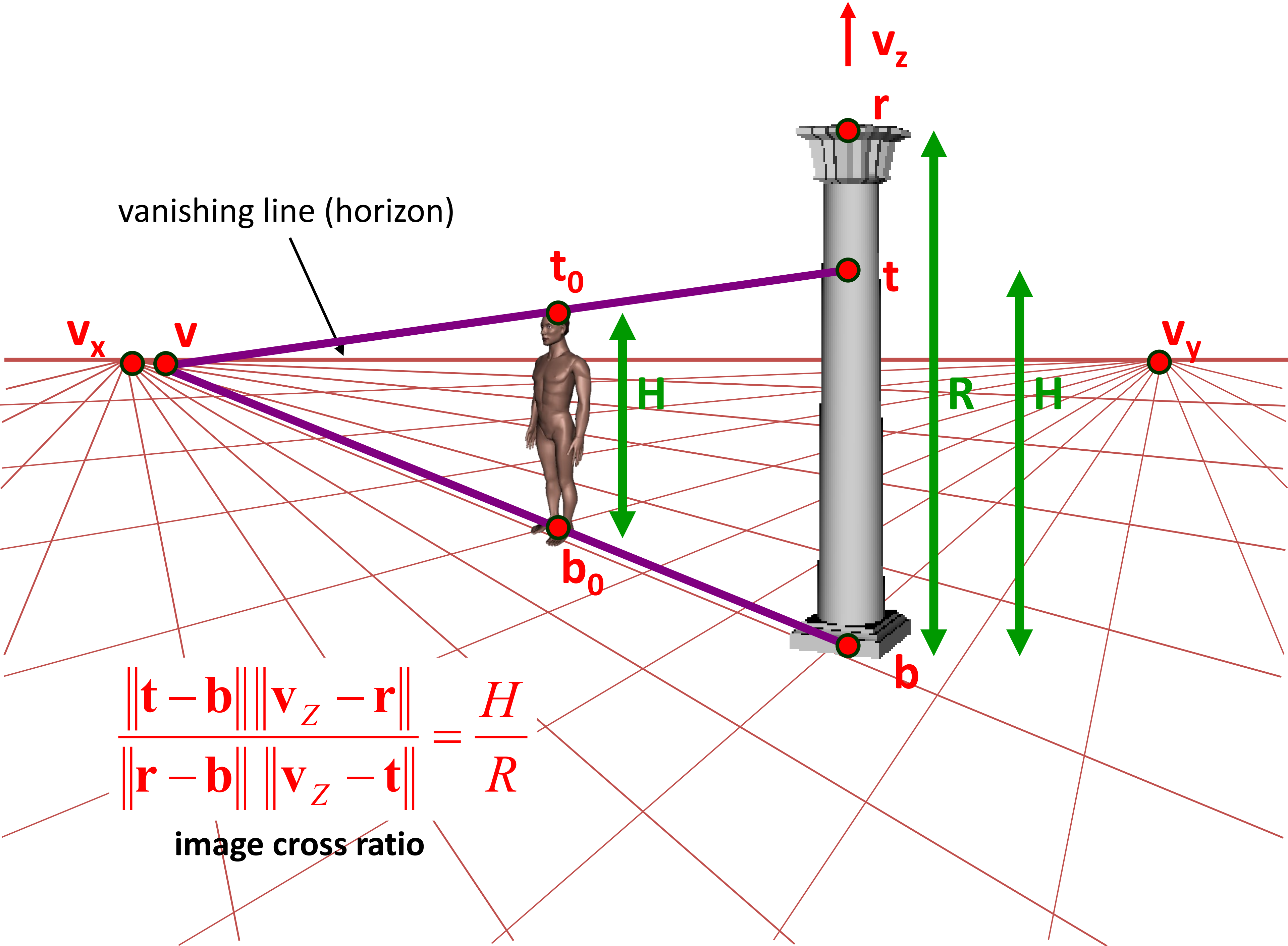
$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|} = \frac{H}{R}$$

image cross ratio

Slide credit: S. Lazebnik

Measuring height without a ruler



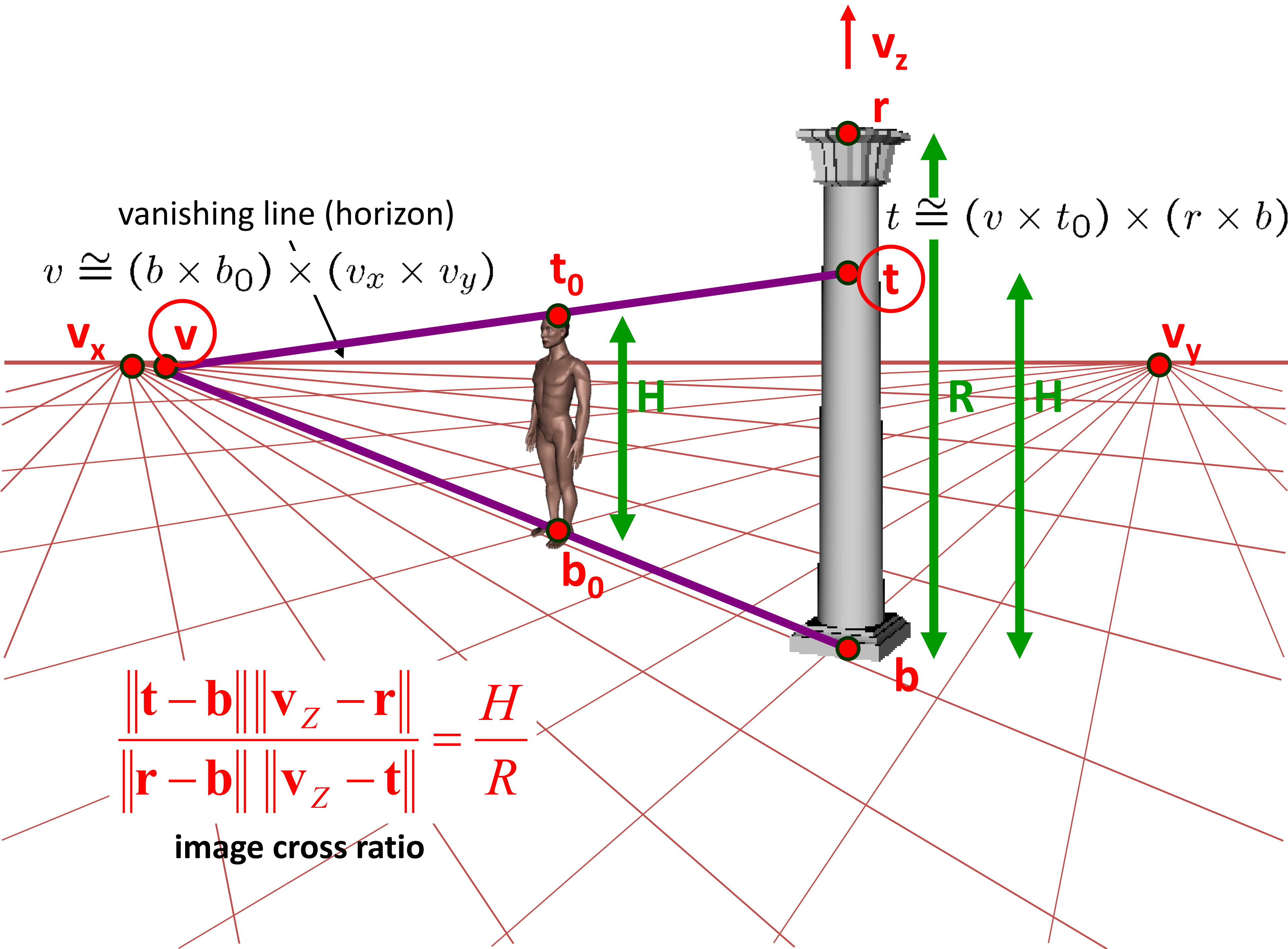


$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

Remember This?

- Line equation: $ax + by + c = 0$
- Vector form: $\boldsymbol{l}^T \boldsymbol{p} = 0$, $\boldsymbol{l} = [a, b, c]$, $\mathbf{p} = [x, y, 1]$
- Line through two points?
  - $\boldsymbol{l} = \boldsymbol{p_1} \times \boldsymbol{p_2}$
- Intersection of two lines?
  - $\boldsymbol{p} = \boldsymbol{l_1} \times \boldsymbol{l_2}$
- Intersection of two parallel lines is at infinity



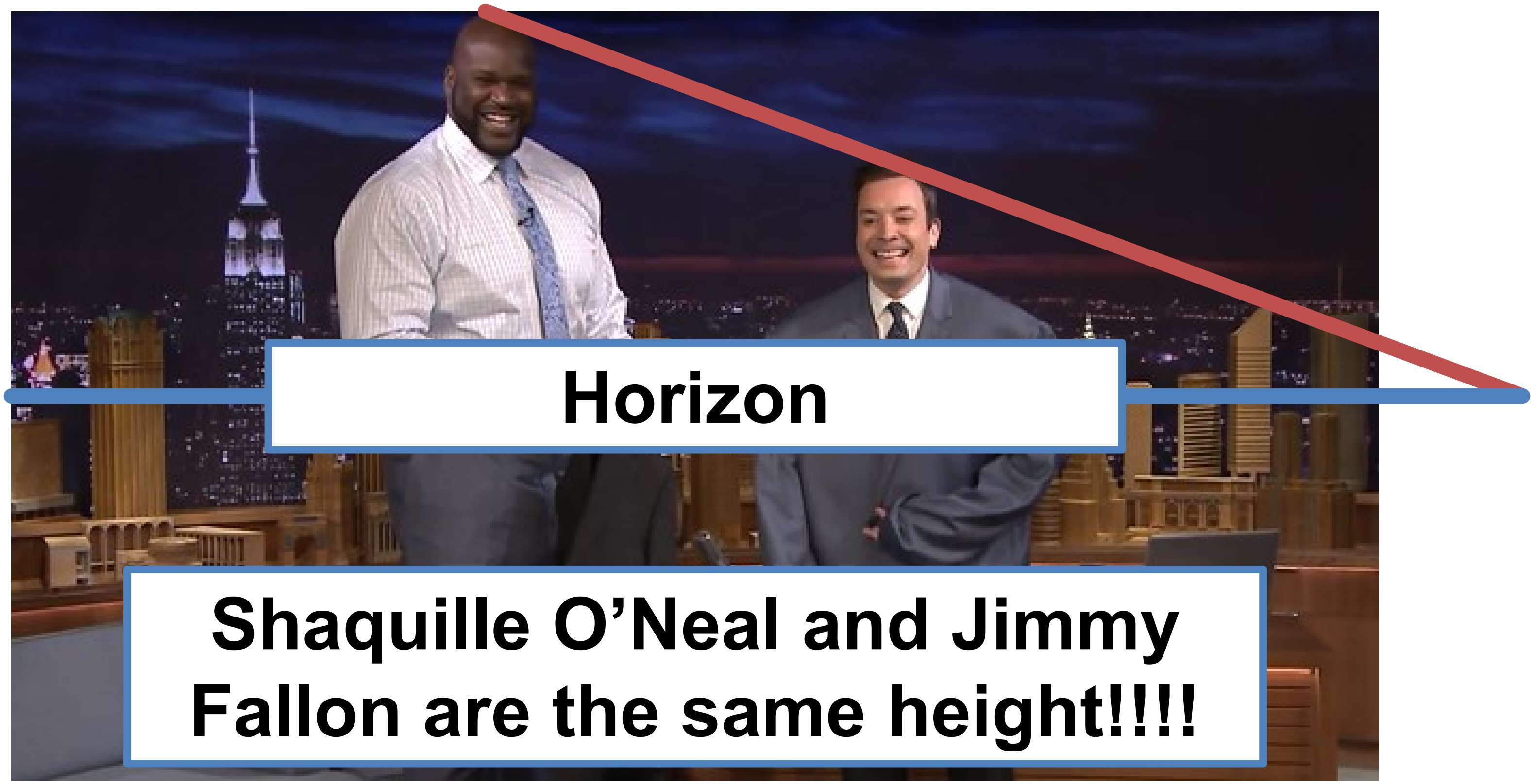
v_z
r
vanishing line (horizon)
$v \cong (b \times b_0) \times (v_x \times v_y)$
$t \cong (v \times t_0) \times (r \times b)$
t_0
t
v_x
v
v_y
H
R
H
b_0
b
$\dfrac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|} = \dfrac{H}{R}$
image cross ratio

Example Gone Wrong

Know length of red → can figure out height of blue because they intersect at vanishing point v

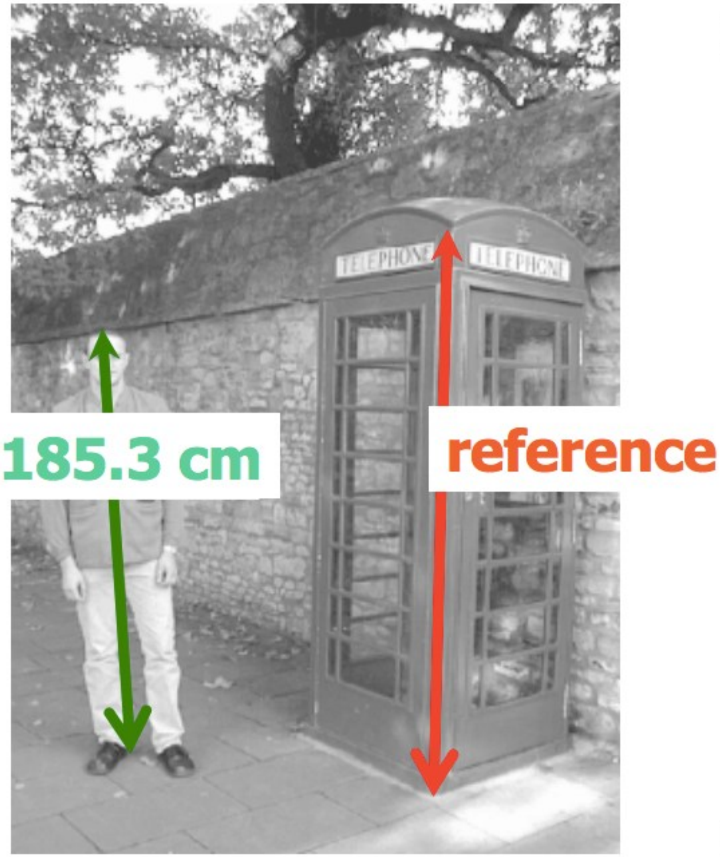
Wrong! Any two lines always intersect!
Need to point to same 3D direction / VP.

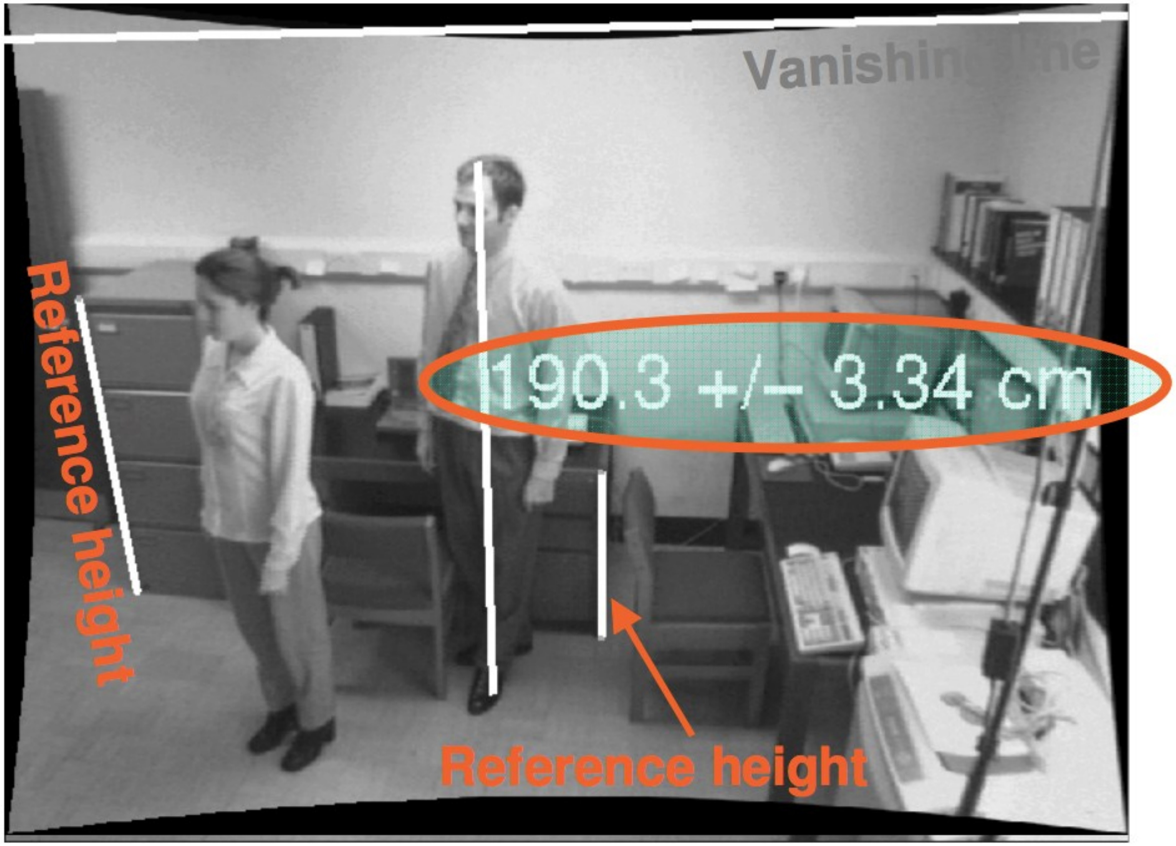
Example Gone Wrong



Wrong! Need to connect feet to the horizon (at infinity – thank homogenous coordinates), and then to Jimmy's head.

Examples





A. Criminisi, I. Reid, and A. Zisserman, [Single View Metrology](), IJCV 2000
Figure from [UPenn CIS580 slides]()

Slide credit: S. Lazebnik

Another example

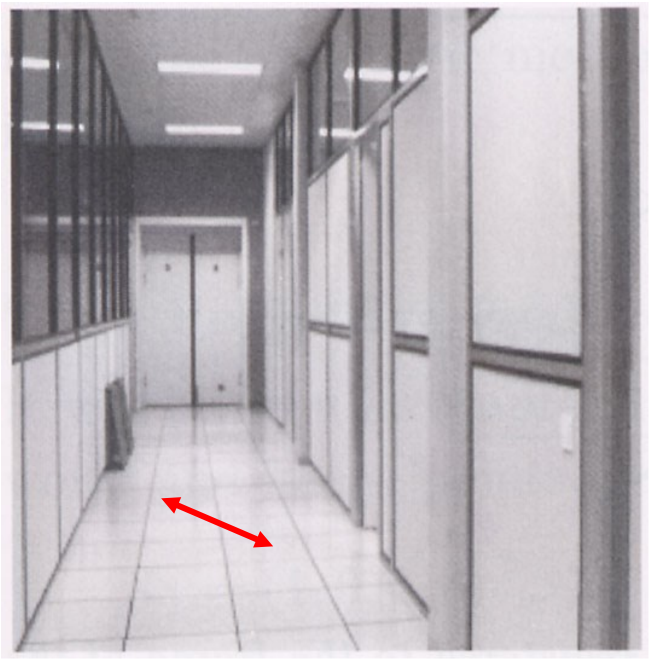
- Are the heights of the two groups of people consistent with one another?

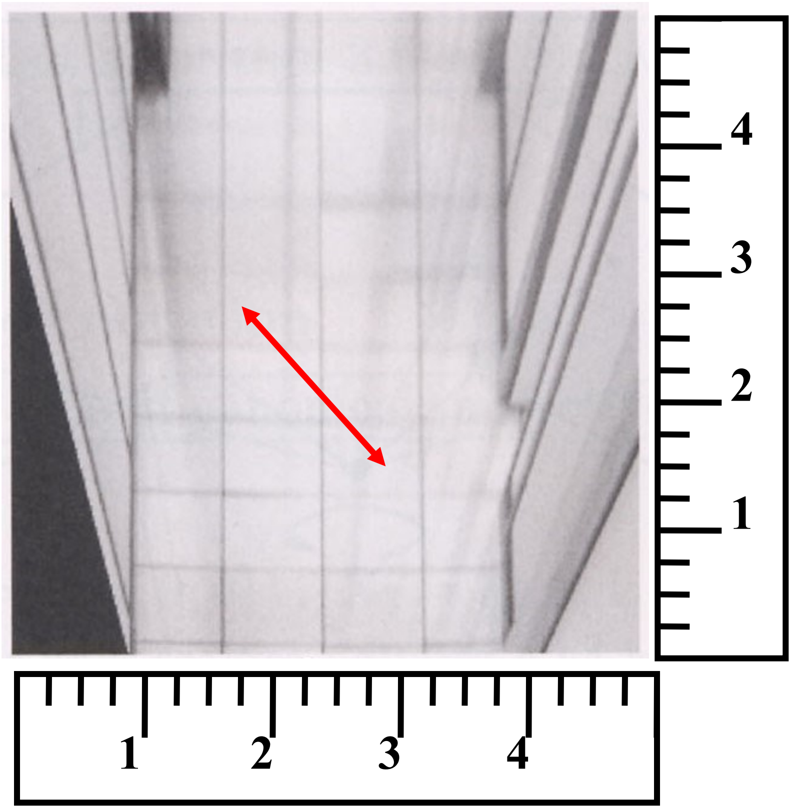
Piero della Francesca*, Flagellation,* ca. 1455

A. Criminisi, M. Kemp, and A. Zisserman, [Bringing Pictorial Space to Life: computer techniques for the analysis of paintings](), *Proc. Computers and the History of Art*, 2002

Slide credit: S. Lazebnik

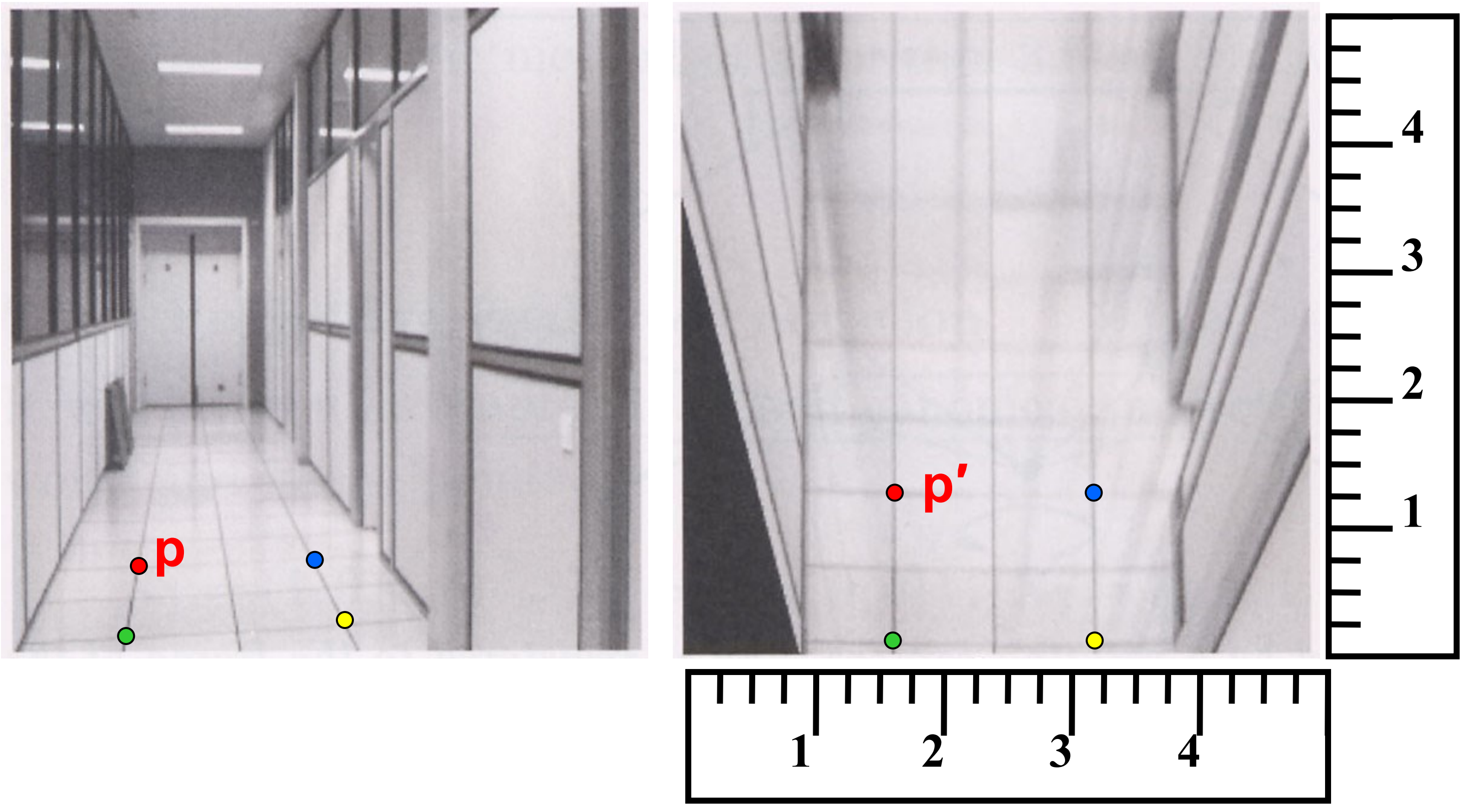
Measurements on planes





Slide credit: S. Lazebnik

Measurements on planes



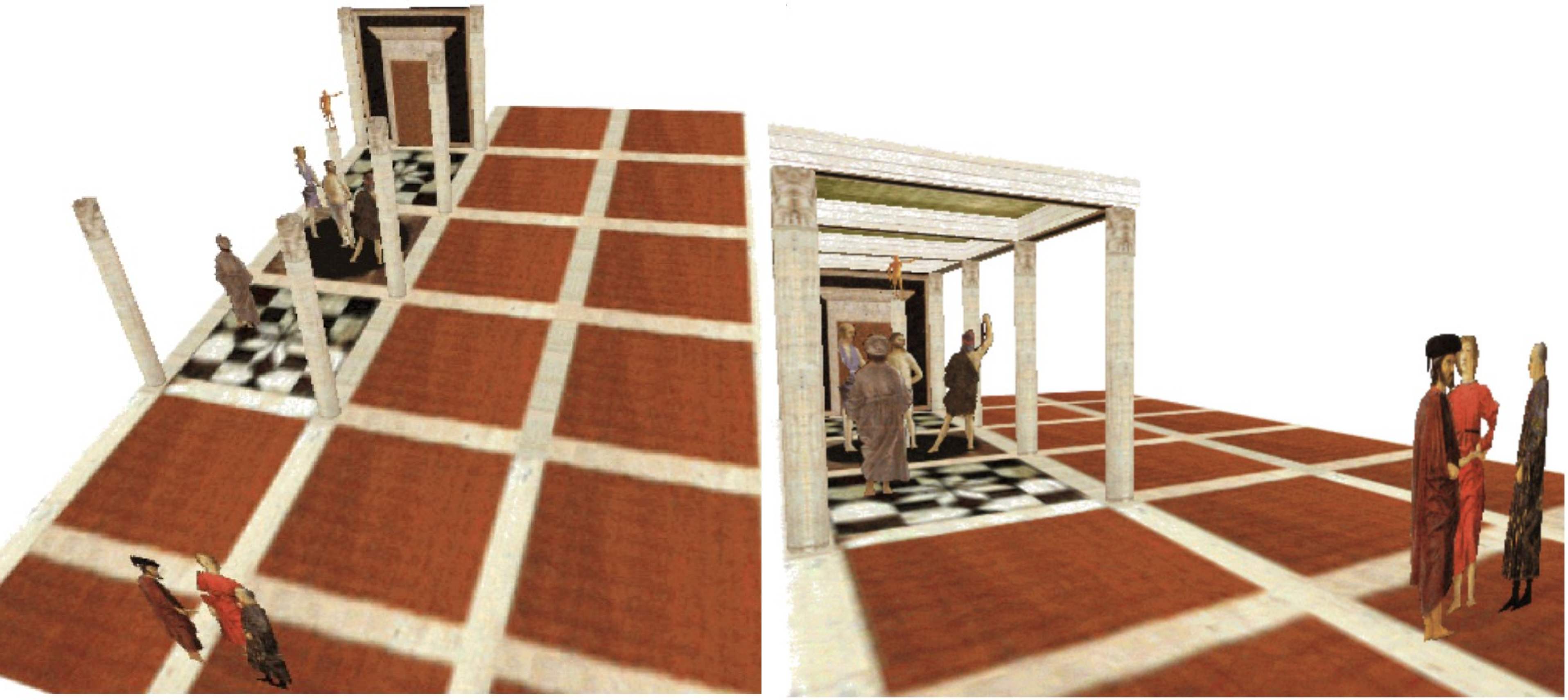
Slide credit: S. Lazebnik

Image rectification: example

Piero della Francesca*, Flagellation,* ca. 1455

Slide credit: S. Lazebnik

Application: 3D modeling from a single image



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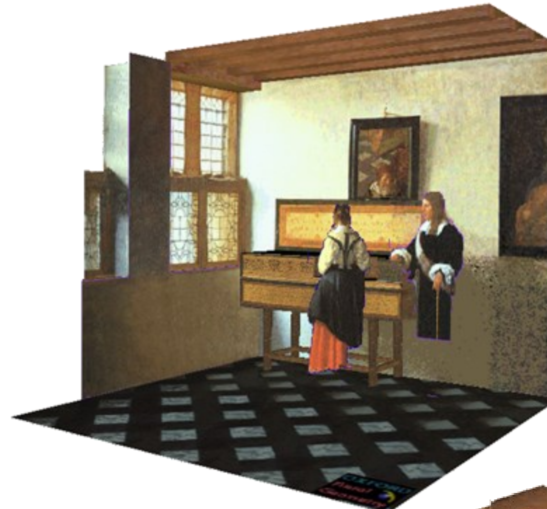
Application: 3D modeling from a single image

J. Vermeer, *Music Lesson*, 1662



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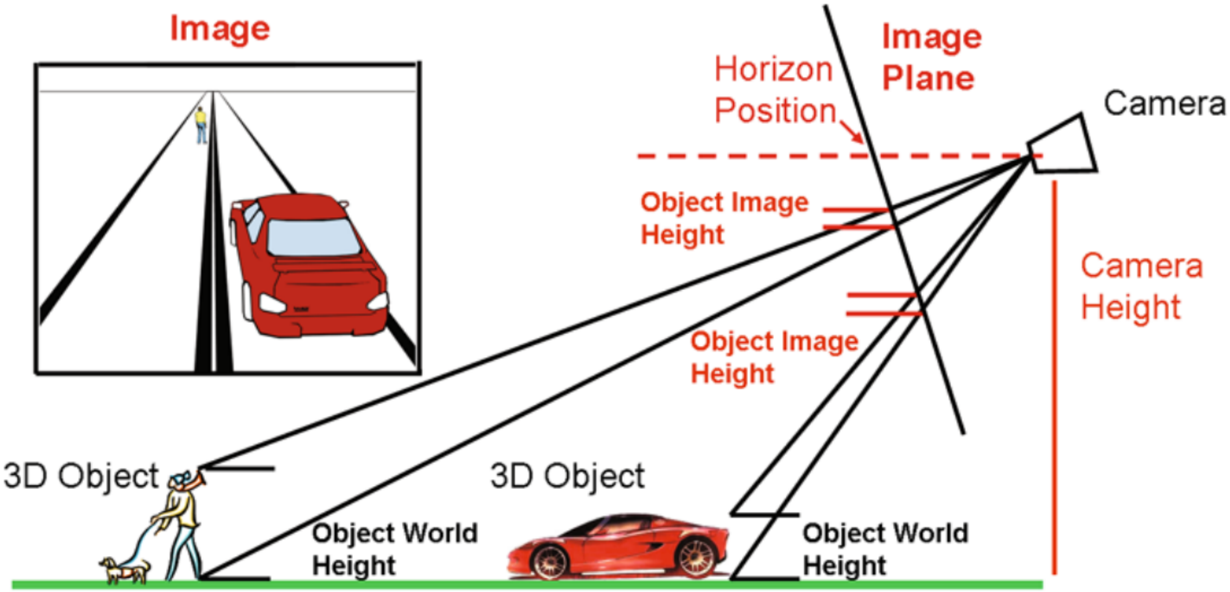
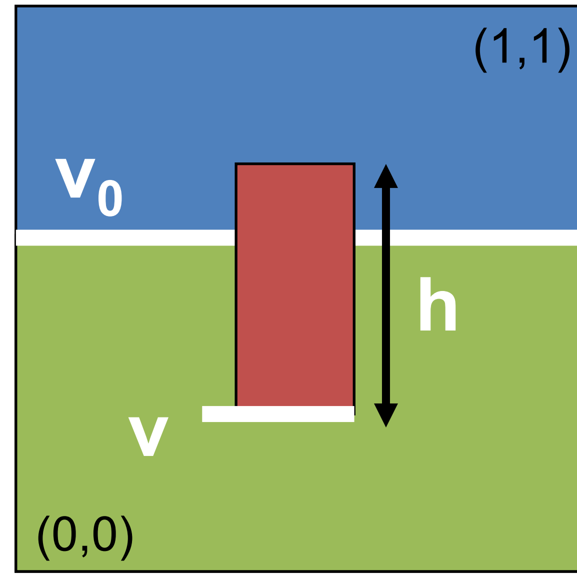
Slide credit: S. Lazebnik

Application: Object Detection

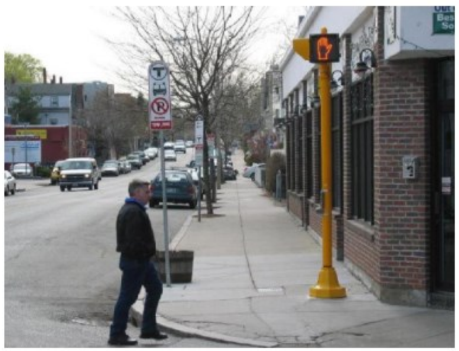




"Reasonable" approximation: $$y_{object} \approx \frac{h y_{camera}}{v_0 - v}$$

Diagram Credit: D. Hoiem

Application: Object detection



(a) input image

Diagram Credit: D. Hoiem

Application: Object detection

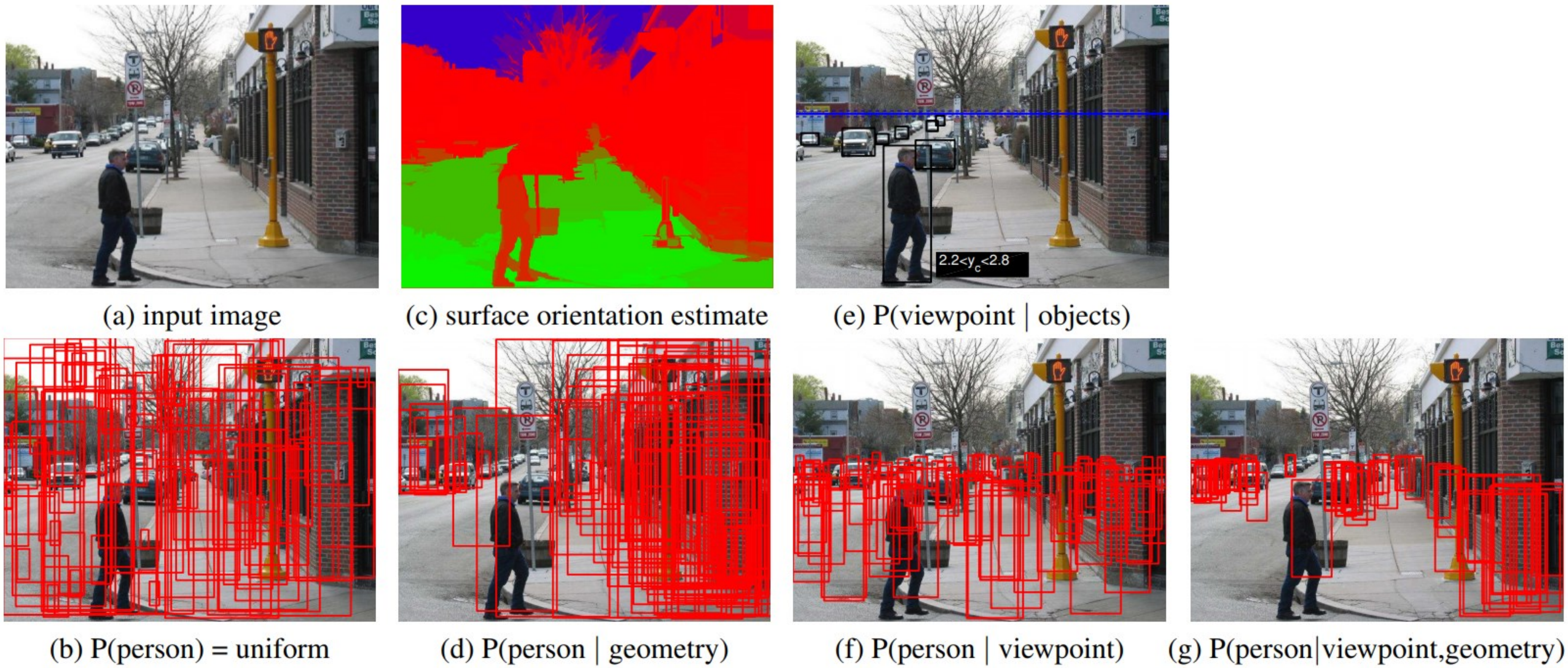
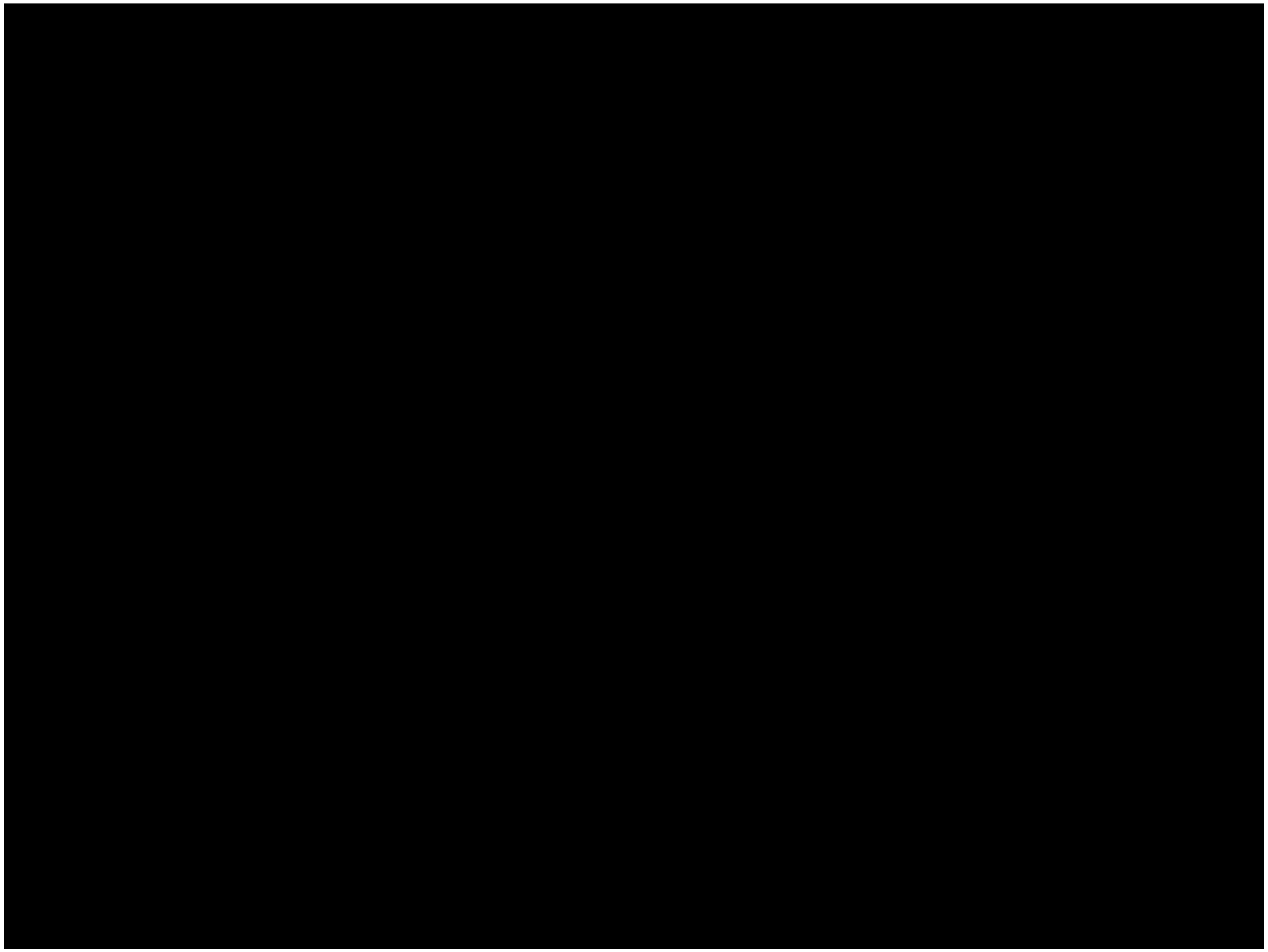


Diagram Credit: D. Hoiem

Application: Image Editing



K. Karsch and V. Hedau and D. Forsyth and D. Hoiem, [Rendering Synthetic Objects into Legacy Photographs](), *SIGGRAPH Asia* 2011

Application: Estimating Layout

V. Hedau, D. Hoiem, D. Forsyth
Recovering the spatial layout of cluttered rooms ICCV 2009