Multi-model Estimation in the Presence of Outliers

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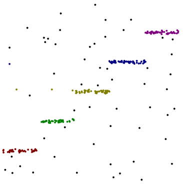
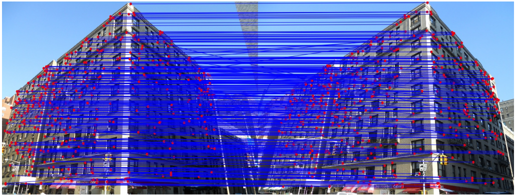
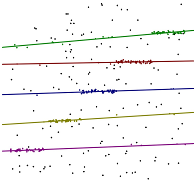
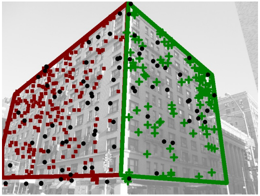
Big picture

How do we find models in data (discrete data points) that contains multiple models and outliers?

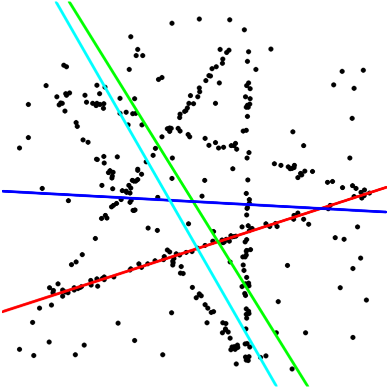
Lots of applications in vision: geometric figure fitting, planar surface detection; motion segmentation; etc.

Agenda

1. Outlier-Robust Single Model Case (RANSAC)
2. Very brief coverage of the Multiple Model Case
3. Evaluation of Multi-Model Estimation Algorithms

Traditional approaches

Traditional approaches fail in the presence of multiple models and outliers; objective functions fail to provide sensible goals.



Ordinary Least Squares (O.L.S.), Total Least Squares (T.L.S.) (via PCA), Least Median of Squares (LMedS), Random Sample Consensus (RANSAC)

Two definitions

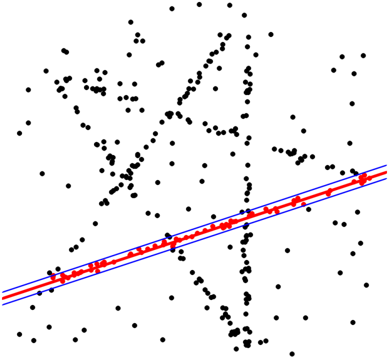
-Minimum sample set (*MSS*): set of data points with the minimum cardinality required to estimate a model.

-Consensus set of a model μ: the set of points that fit a model sufficiently well.

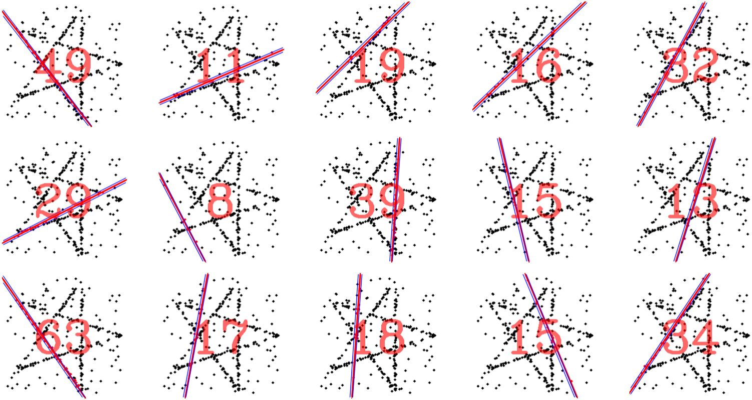
$$CS(\mu, DataPoints, \epsilon) = \{p \in DataPoints : R(\mu, p) < \epsilon\}$$

R is the error function and ϵ is the *inlier threshold*.

Intuitively: $|CS|$ as a function to optimize; picking minimum sample models to generate models.



Some Minimum Sample Sets



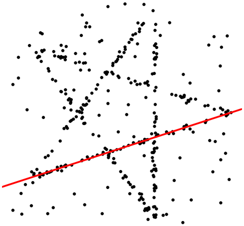
RANSAC

```
RANSAC(DataPoints, M, ε):
    bestModel = None; maxCSSize = 0
    for i in range(M):
        MSSModel = Estimate(getMSS())
        CS = {d ∈ DataPoints : R(MSSModel, d) < ε}
        if |CS | > maxCSSize:
            bestModel = MSSModel; maxCSSize = |CS|
    Return bestModel
```

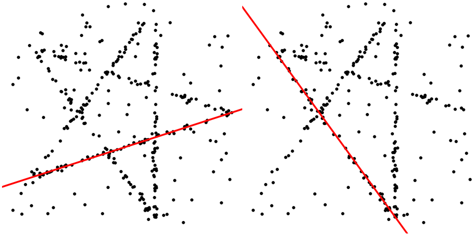
Issues?

- Need some parameters
- Not guaranteed to succeed
- More subtly: $|CS|$ is a heuristic, not a justifiable function to optimize.

The multi-model case?



The multi-model case?



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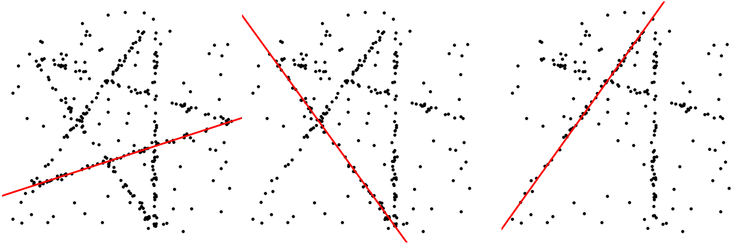
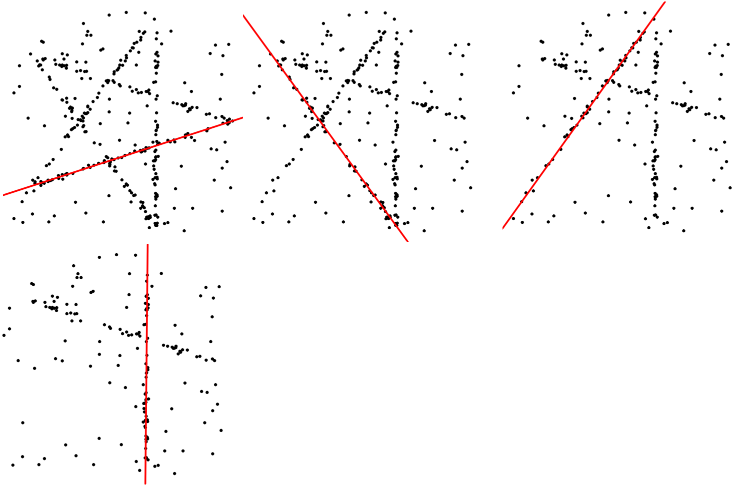
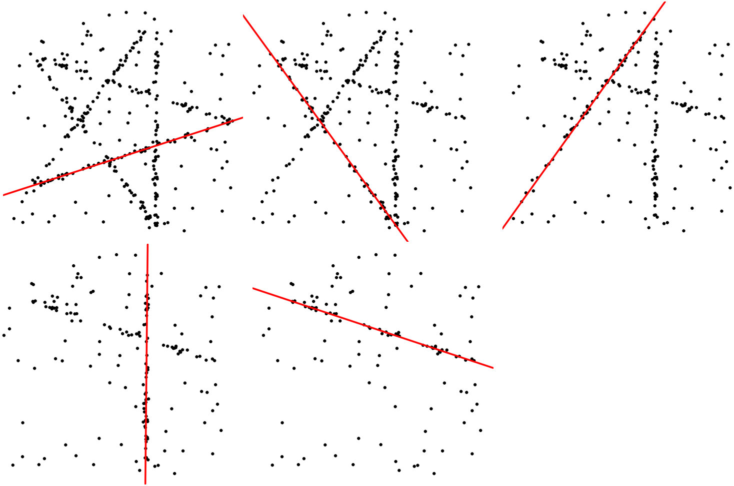


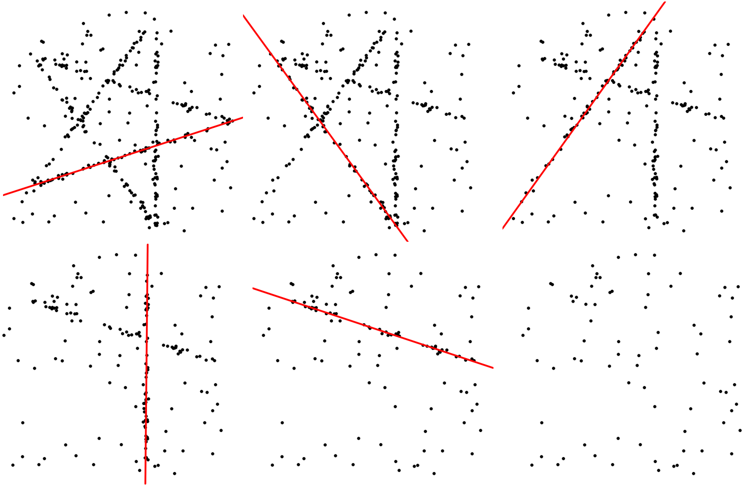
The multi-model case?



The multi-model case?



The multi-model case?



This is conventionally referred to as Sequential RANSAC.

Can we do better?

Can we do any better than Sequential RANSAC?

- Do RANSAC in parallel: MultiRANSAC (2005).
- Analyze histograms: Residual Histogram Analysis (RHA) (2006).
- Form an alternative representation for data points using minimum sample models: J-linkage (2008) and Merging J-linkage (2010), Kernel Fitting (2009).
- Energy minimization: PEARL (2010).

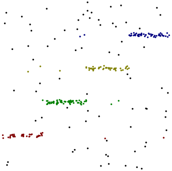
Evaluated: Sequential RANSAC, MultiRANSAC, RHA, J-linkage, Merging J-linkage, Kernel Fitting

How do we choose?

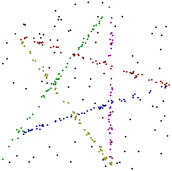
Lots of methods; how do we choose?

- How much a-priori knowledge do the algorithms need?
- How fast are the algorithms?
- **How well do the algorithms work?**

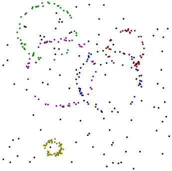
Data Sets



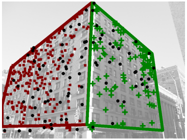
Stairs4



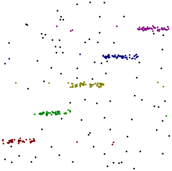
Star5



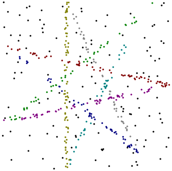
Circles5



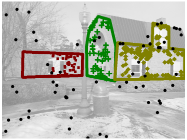
Planes2



Stairs5



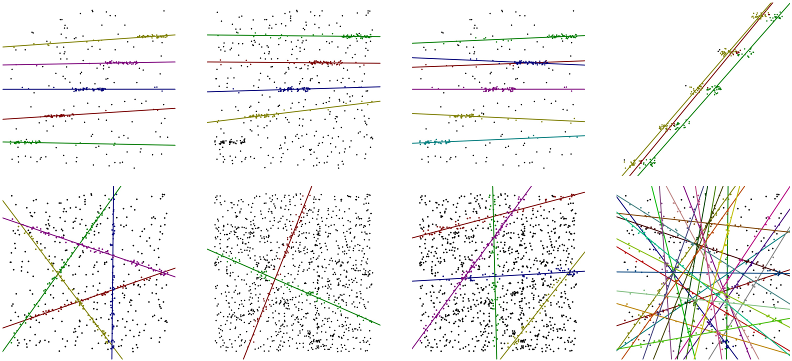
Star7



Planes3

Evaluating Results

Need automated evaluation that can satisfy competing goals: able to handle degenerate configurations; readily comprehensible.



Scoring Functions

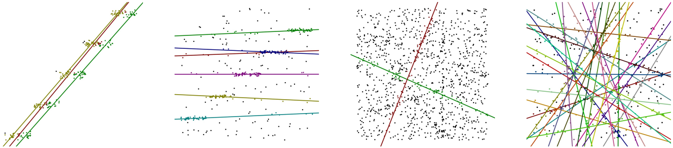
Settled on treating the task as a classification problem.
Classification metric:

$$\frac{\# \textit{Points Correctly Classified}}{\# \textit{Total Points}}$$

Need to define a number of things:

- When is a point correct? (Inlier/outlier, model-aware criteria)
- What points are we looking at? (ground-truth inliers only, inliers and outliers)
- How do we establish a mapping between estimated and ground-truth models? (Maximum-intersection, Strongest-intersection)

Scoring functions, continued



Inlier/Outlier:	**0.98**	**0.93**	**0.59**	0.65
Many-to-1:	0.2680	**0.93**	**0.57**	0.63
N-str-to-1:	0.2680	0.82	**0.52**	0.61
N-str-to-1 Inl.:	0.27	0.82	0.09	**0.95**

Tests

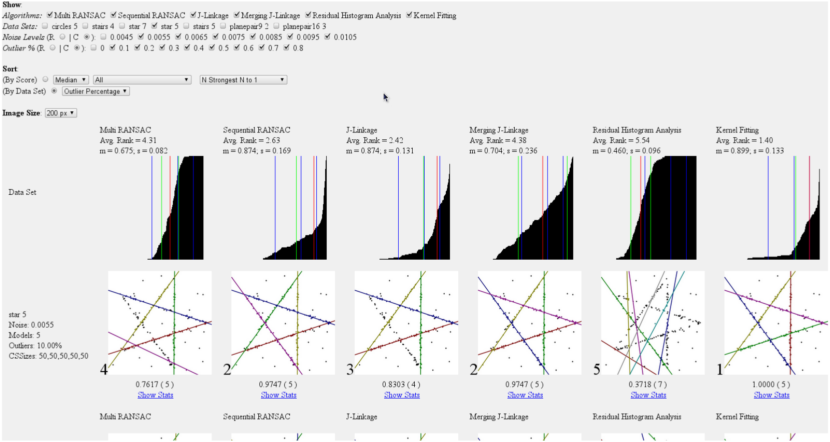
Test set = data set + outlier setting + noise setting

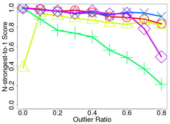
- 7 data sets
- 9 gross outlier percentages (0% - 80%)
- 4 or 7 noise scales

Ran each of 6 algorithms on each of 387 test sets 15 times. Tests took $\sim$33 computer days

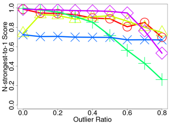
Interpreting results

Looked at results manually, made graphs

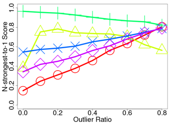




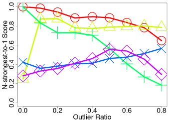
(a) Stairs4



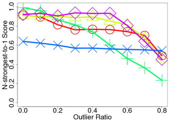
(b) Star5



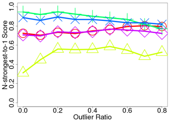
(c) Planes2



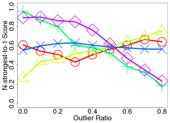
(d) Stairs5



(e) Star7

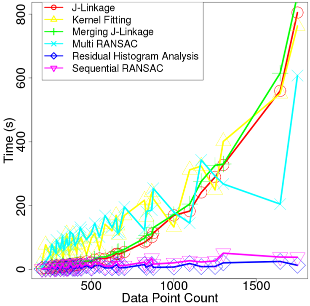


(f) Planes3



(g) Circles 5

Runtimes



# of points	SeqRS	MulRS	JL	MJL	RHA	KF
411	16.2	88.2	19.9	22.8	10.5	22.8
822	22.1	122.8	87.7	98.4	15.0	213.9
1645	37.3	202.2	559.7	616.3	24.5	529.0
	$O(N)$	$O(N)$	$O(N^2)$	$O(N^2)$	$O(N)$	$O(N^2)$

Results and Conclusions

Concrete suggestions: Sequential RANSAC is a strong first choice since it is $O(N)$, and generally effective; J-Linkage and Kernel Fitting work better, but are $O(N^2)$.

Future work:

- More algorithms
- Motion segmentation task
- Real-world geometric figure-fitting

Selected References

1. Model Estimation:
   - T.-J. Chin, H. Wang, and D. Suter. Robust fitting of multiple structures: The statistical learning approach. In ICCV 2009.
   - M. Fischler and R. Bolles. Random sample consensus: A paradigm for model fitting with applications to image analysis and automated cartography. In CACM 1981.
   - D. Fouhey, D. Scharstein, and A. Briggs. Multiple plane detection in image pairs using J-linkage. In ICPR 2010.
   - W. Zhang and J. Kosecká. Nonparametric estimation of multiple structures with outliers. In *Dynamical Vision, ICCV 2005 and ECCV 2006 Workshops*.
   - M. Zuliani, C.S. Kenney, and B.S. Manjunath. The multiRANSAC algorithm and its application to detect planar homographies. In ICIP 2005.
2. Evaluations:
   - S. Choi, T. Kim, and W. Yu. Performance evaluation of RANSAC family. In BMVC 2009.
   - R. Tron and R. Vidal. A benchmark for the comparison of 3-d motion segmentation algorithms. In CVPR, 2007.