Lecture 7
Transcendental Computation

Chee Yap
Courant Institute of Mathematical Sciences
New York University

Overview

We give a brief introduction to transcendental number theory, and issues of transcendental computation. Then we describe a recent result showing a first non-trivial transcendental geometric computation that is computable in the EGC sense.

- 0. Review
- I. Intro to Transcendental Number Theory
- II. A Solved Problem that Isn't: Shortest Path amidst Discs

0. REVIEW

ANSWERS and DISCUSSIONS

- REMEMBER: the prize for best exercises

- SHOW: If a reduced rational p/q is the zero of an integer polynomial $A(X) = \sum_{i=0}^{m} a_i X^i$ then $q | a_m$ and $p | a_0$
  - $*$ Corollary: if p/q is algebraic integer, then $q = 1$
  - $*$ Corollary: $\sqrt{2}$ is irrational

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What have we learned so far?

- EGC is an effective method to achieve robust numerical algorithms
- The central problem of EGC are the ZERO PROBLEMS
- EGC can be achieved for all algebraic problems
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I. Transcendental Numbers

Introduction

- What is between $\mathbb{A}$ and $\mathbb{R}$?
  - DEFINE: A transcendental number is a non-algebraic number.
  - Is e and π algebraic?
  - This is the topic of transcendental number theory
- Easier questions
  - Are there any transcendental numbers? Yes (Cantor)
  - Is e rational?
  - Whiteboard Aside: Proof that e is irrational
  - Whiteboard Aside: Proof that e is not quadratic irrational

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- Louisville's Theorem (1844)
  - ∗ If α is algebraic of degree $m > 1$ then for all $p/q \in \mathbb{Q}$, $|\alpha - (p/q)| > Cq^{-2}$
  - ∗ Proof: let $A(X)$ be minimal polynomial of α
  - ∗ Then $q^{-m} \leq |A(p/q)| = |A(p/q) - A(\alpha)| = |(p/q) - \alpha| \cdot |A'(\beta)|$
  - ∗ But $|A'(\beta)| \leq C$ for some constant depending on α
- Corollary: $\sum_{n=1}^{\infty} 2^{-n!}$ is transcendental
  - ∗ Proof: take $q = 2^{n!}$ for sufficiently large n
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PART II. A SOLVED-PROBLEM THAT ISN'T

(Joint with E.Chien, S.Choi, D.Kwon, H.Park)

Shortest Path Amidst Disc Obstacles

- Given: Points $p, q \in \mathbb{R}^2$ and a collection S of discs
- Find: shortest path from p to q which avoids the obstacles in S
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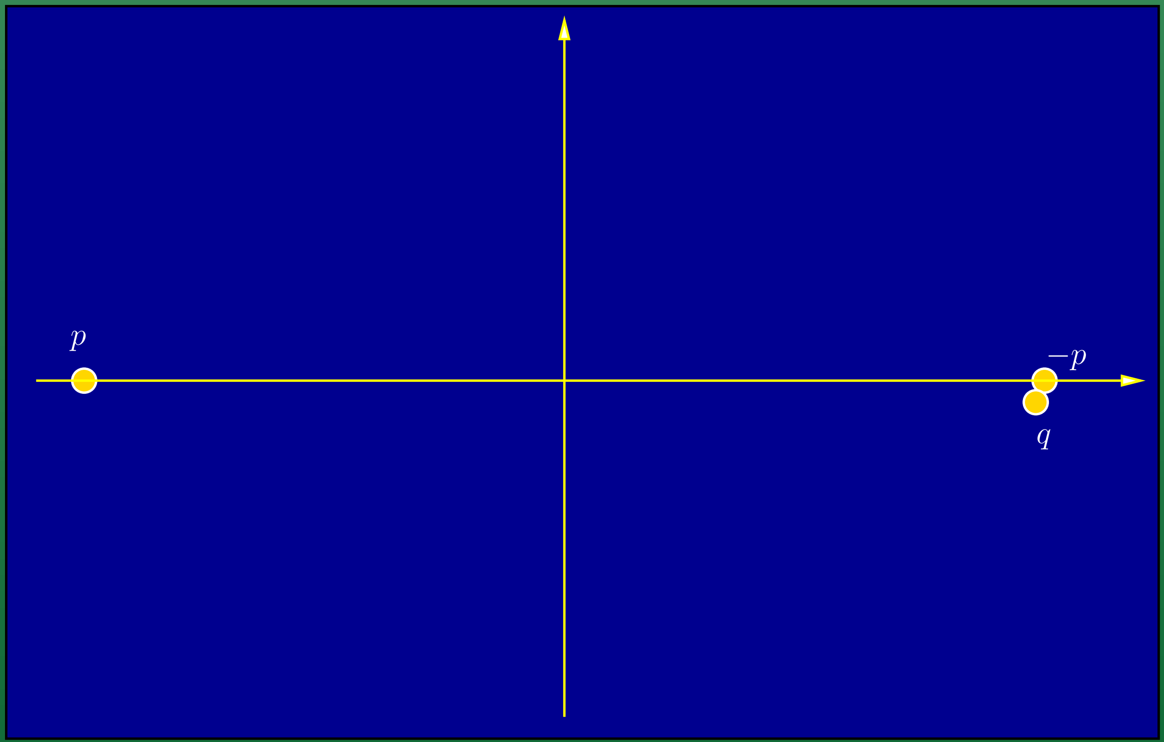
- 

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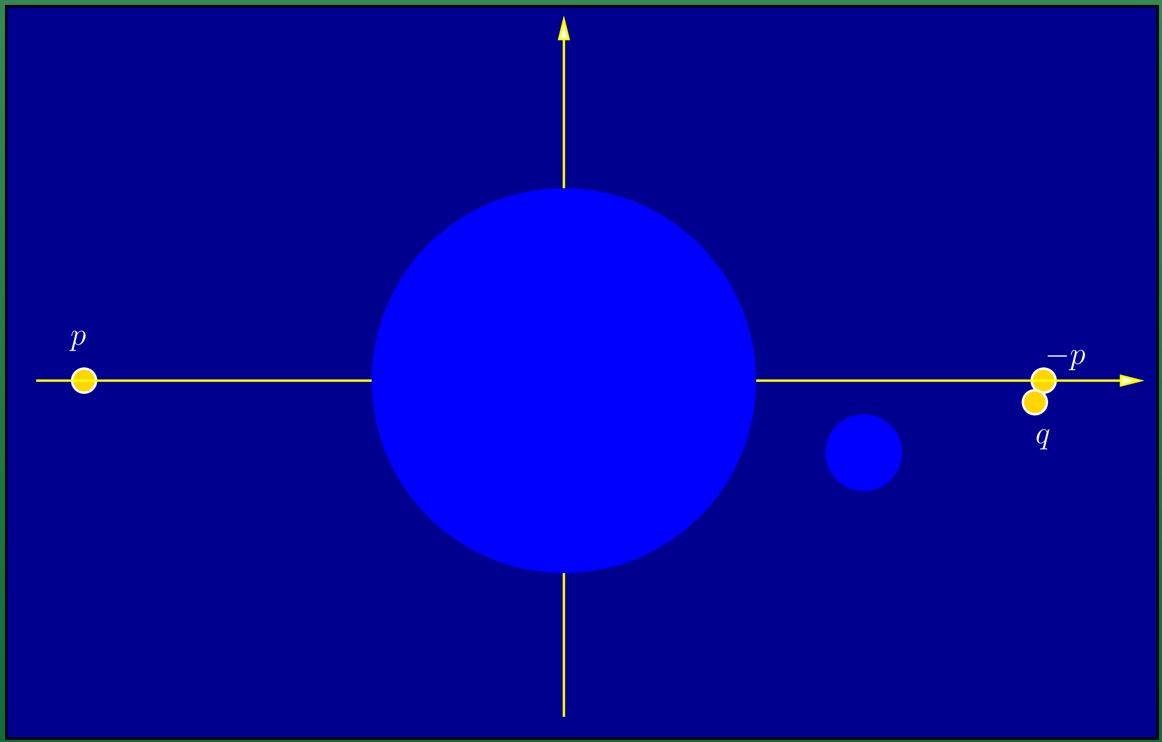

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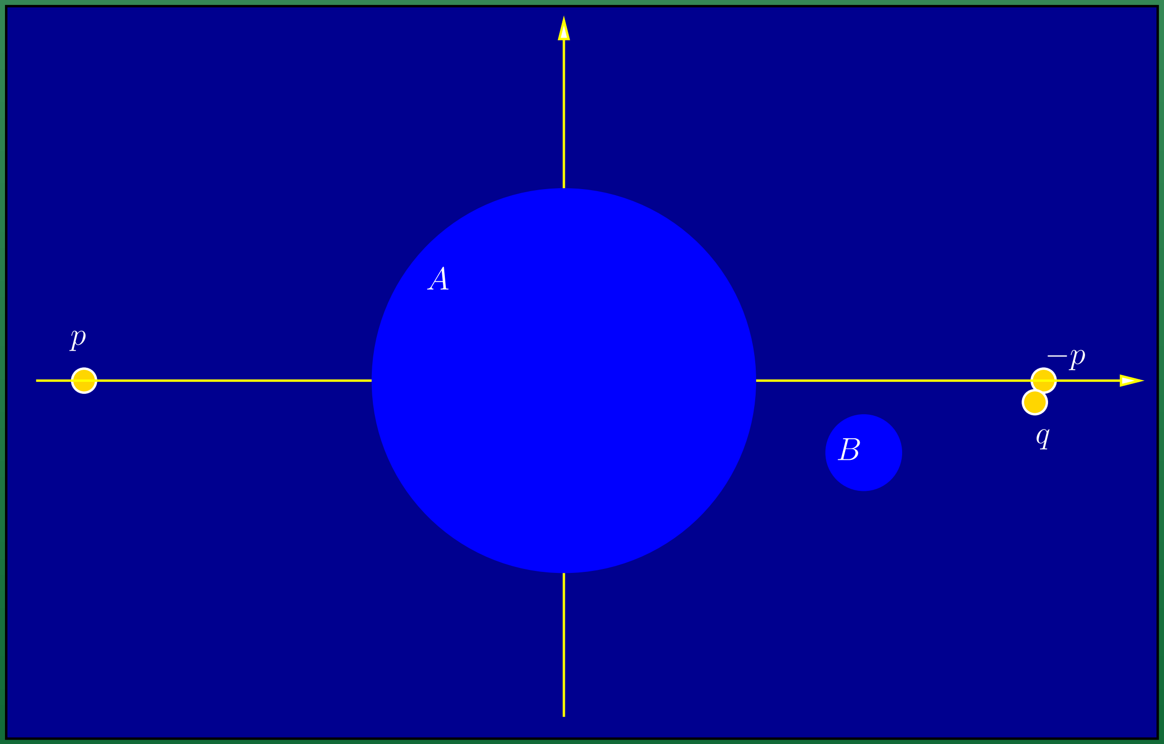
- 

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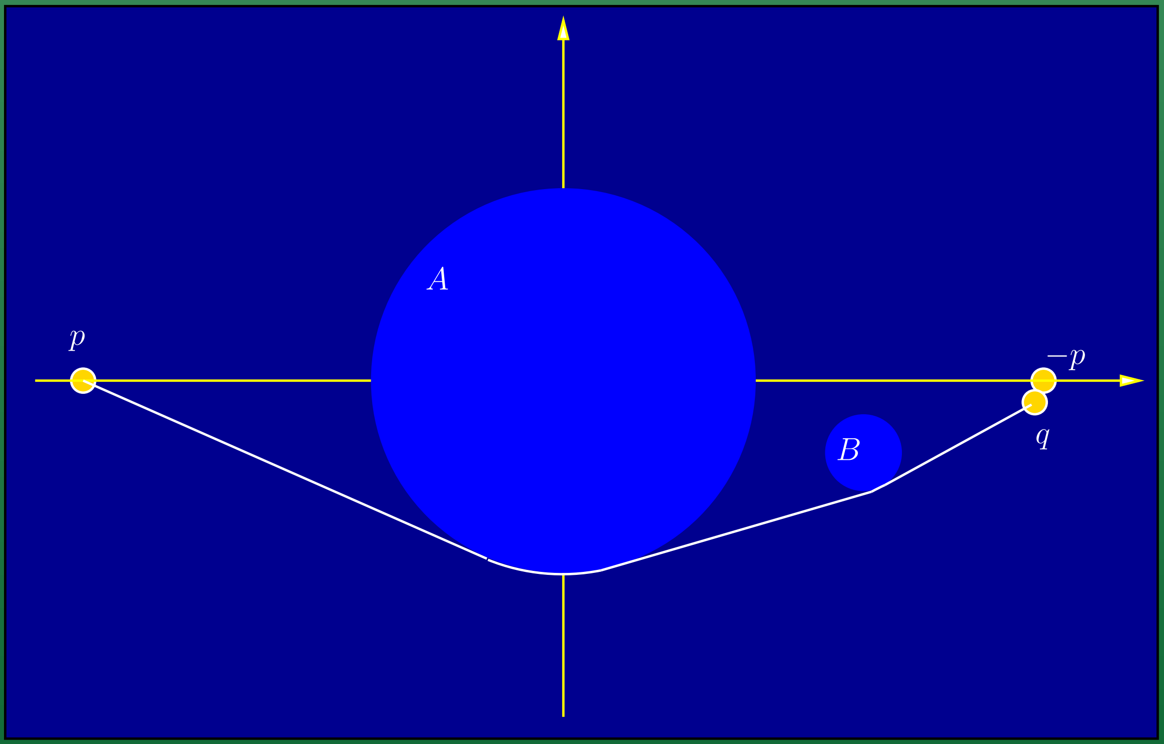

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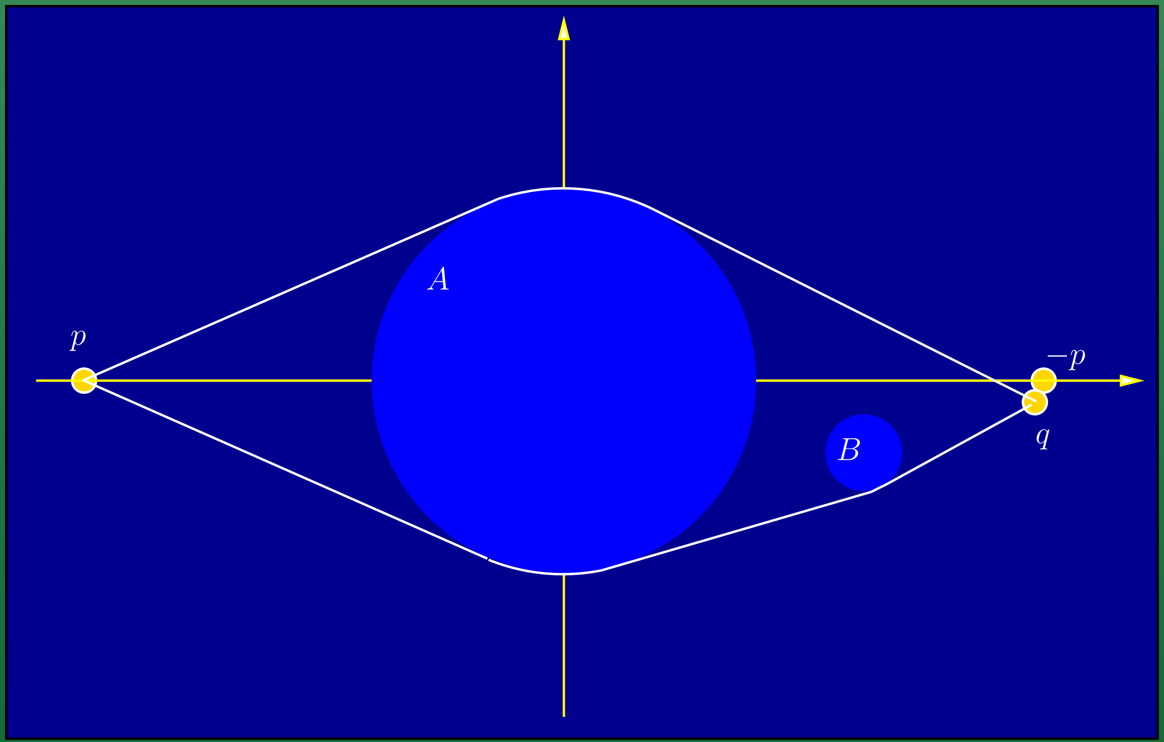

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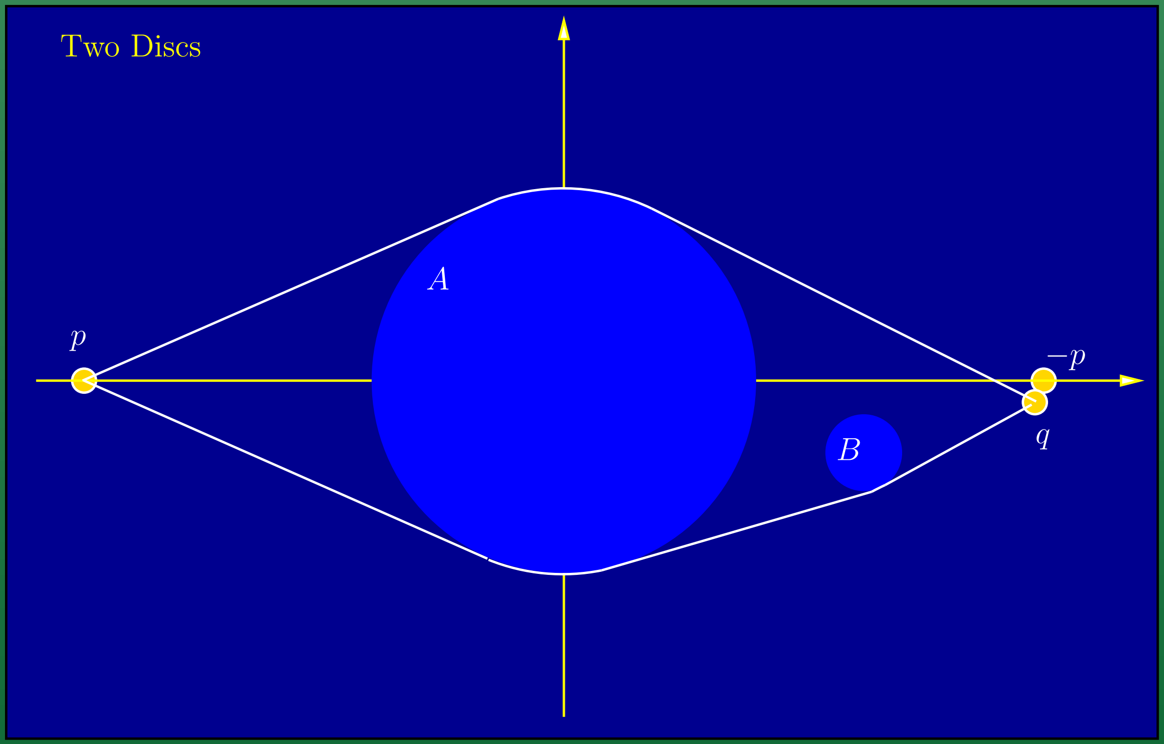
- 

Standard Solution: Reduce to Dijkstra's Algorithm

- Feasible paths: $\mu = \mu_1; \mu_2; \cdots; \mu_k$
  * μ_i is a straightline segment iff μ_{i+1} is an arc
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- Apply Dijkstra's shortest path algorithm to a combinatorial graph $G = (V, E)$
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- Real RAM model assumed!
- Length of a feasible path is

$$d(\mu) = \sum_{i=1}^{k} d(\mu_i) = \alpha + \sum_{i=1}^{m} \theta_i r_i \qquad (1)$$

  * $\alpha \geq 0$ is algebraic
  * $0 < r_1 < \cdots < r_m$ are distinct radii of discs
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- Let $d(\mu) = \alpha + \theta$, and $d(\mu') = \alpha' + \theta'$
  - * E.g., all discs have unit radius
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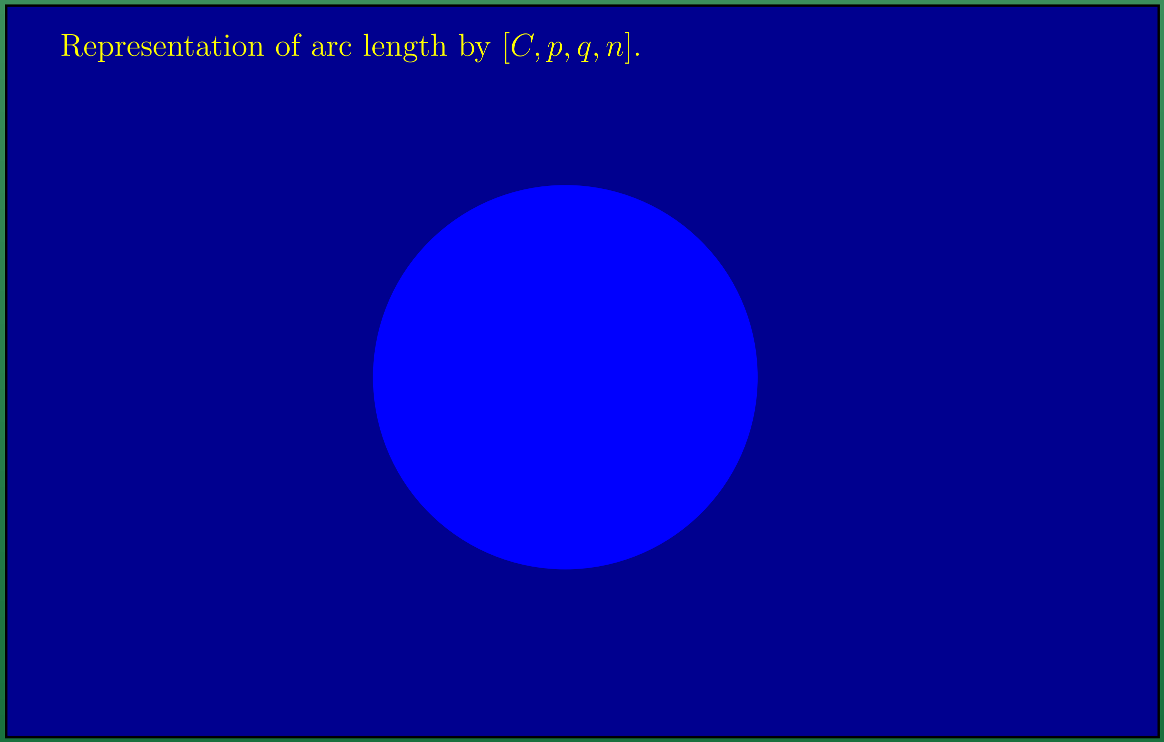

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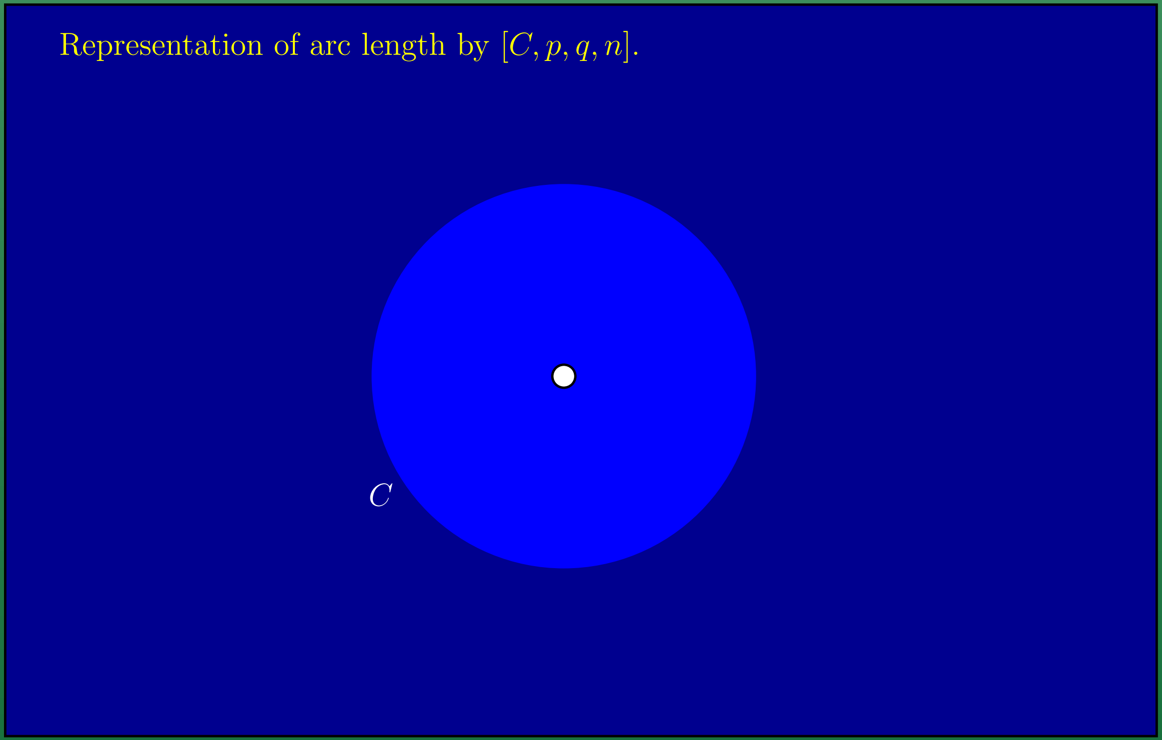

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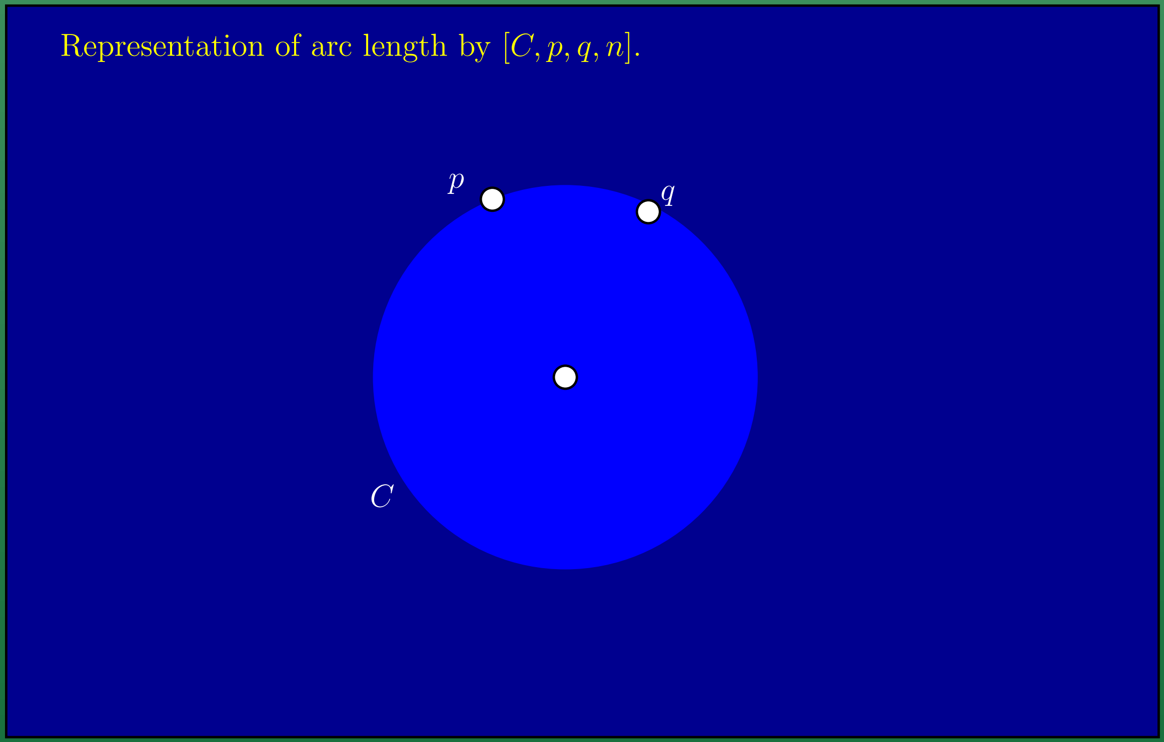

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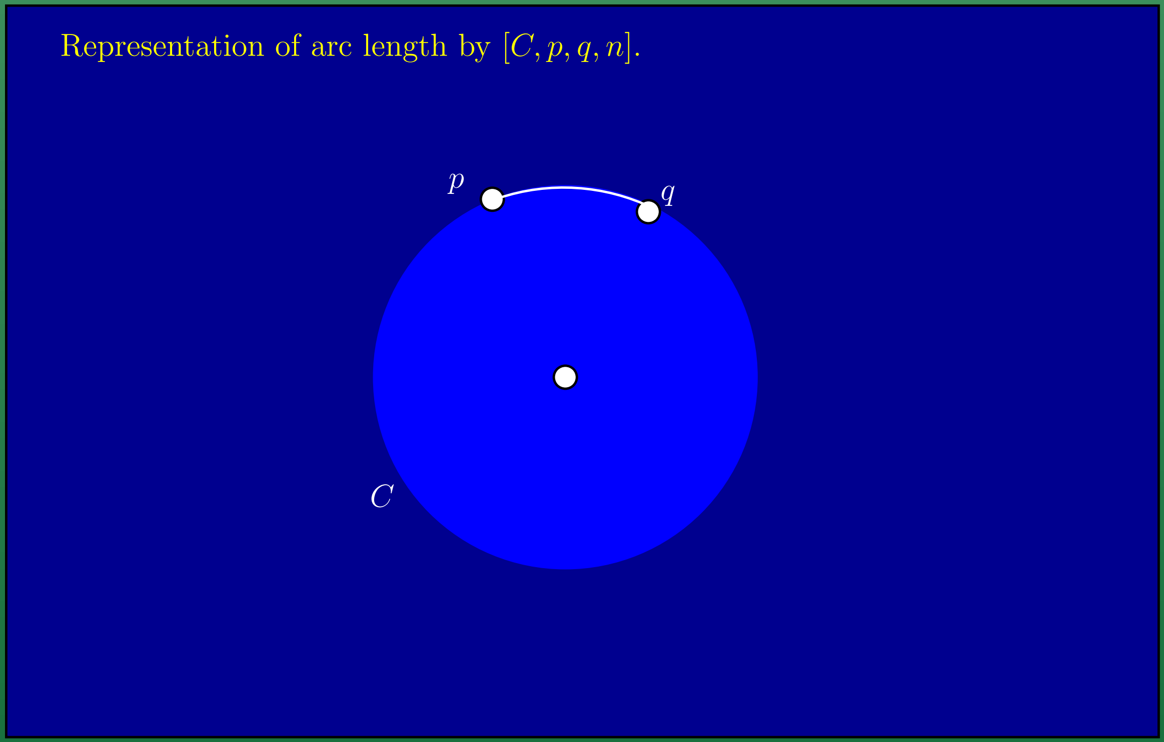

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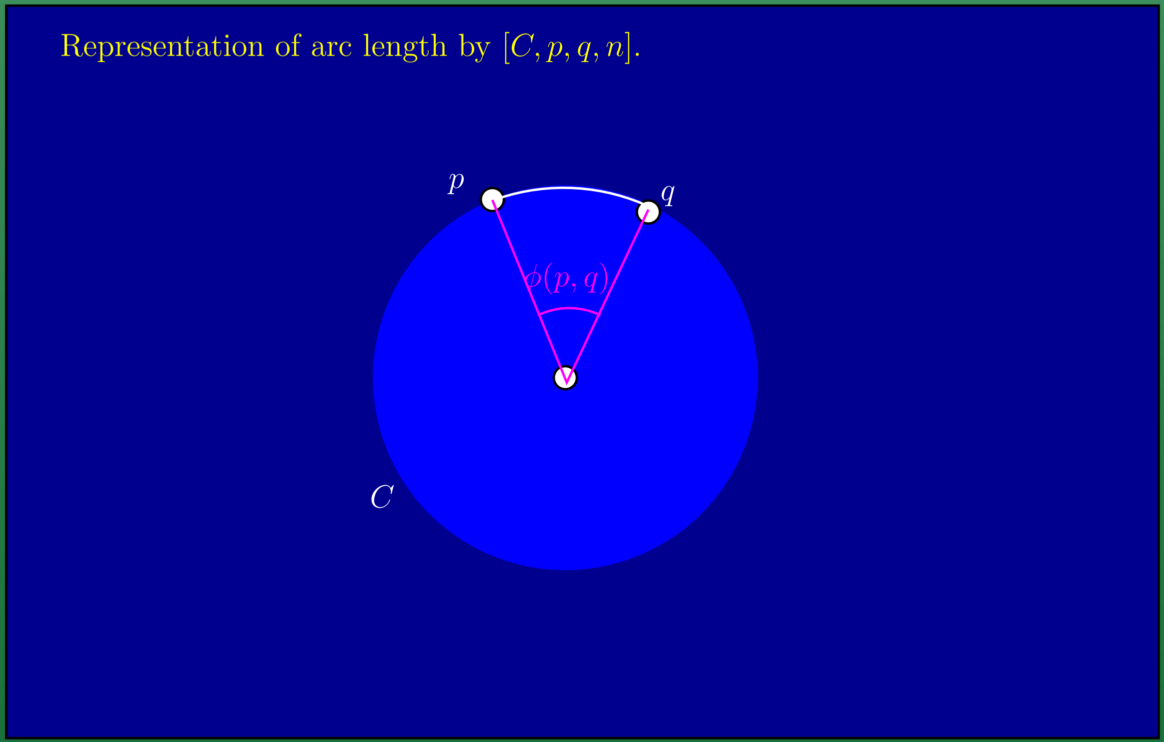

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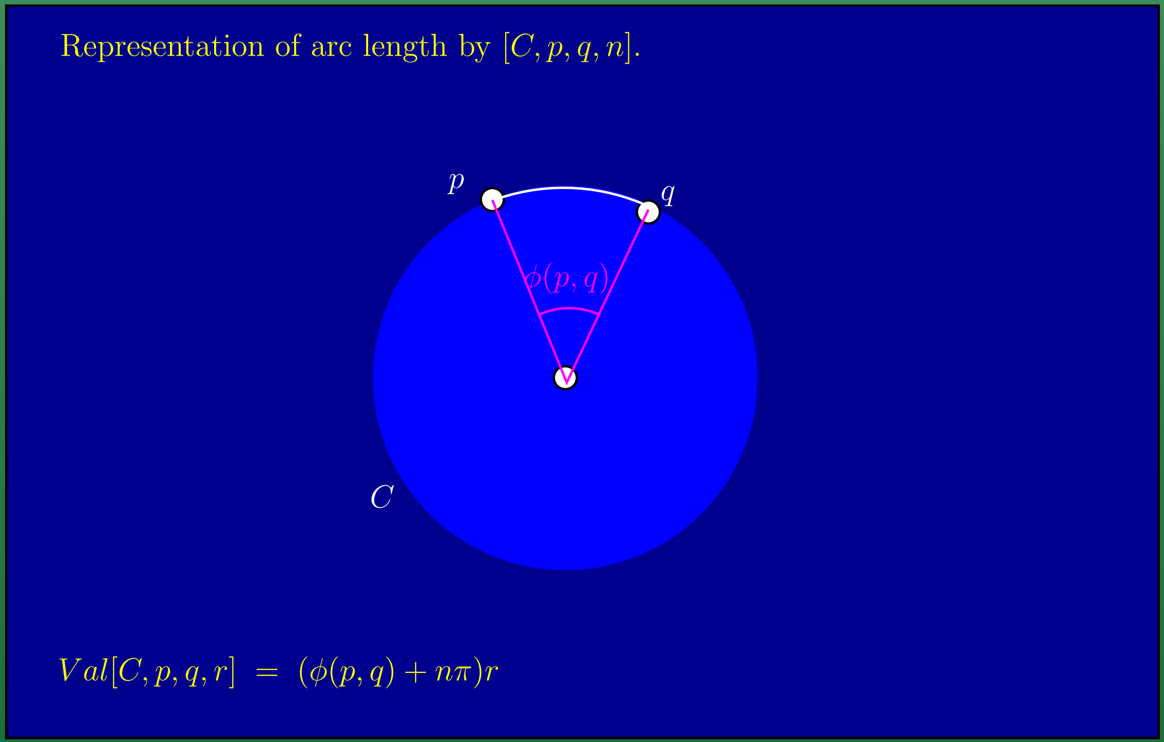

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- Let $A = [C, p, q, n]$ and $A' = [C', p', q', n']$
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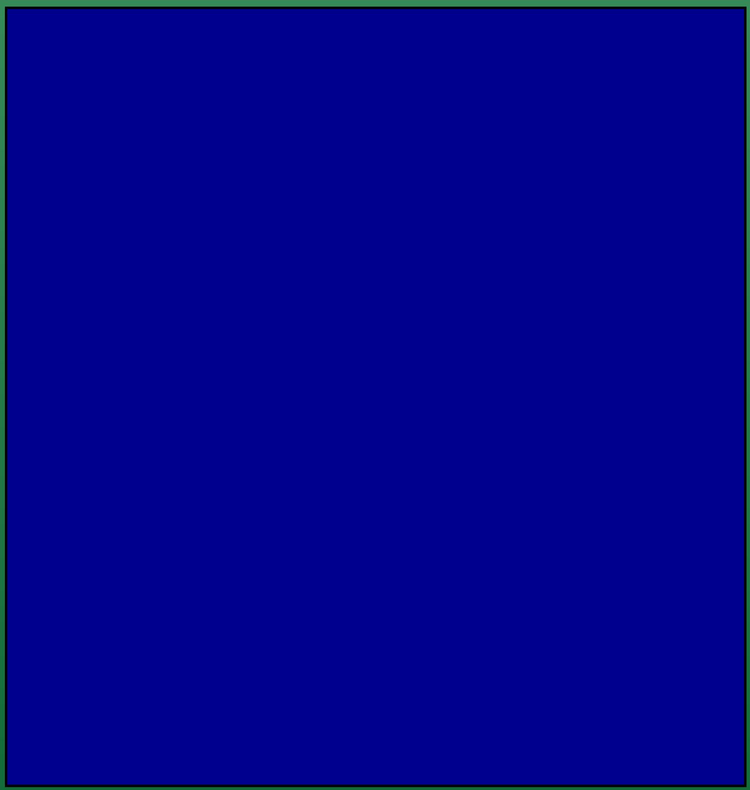

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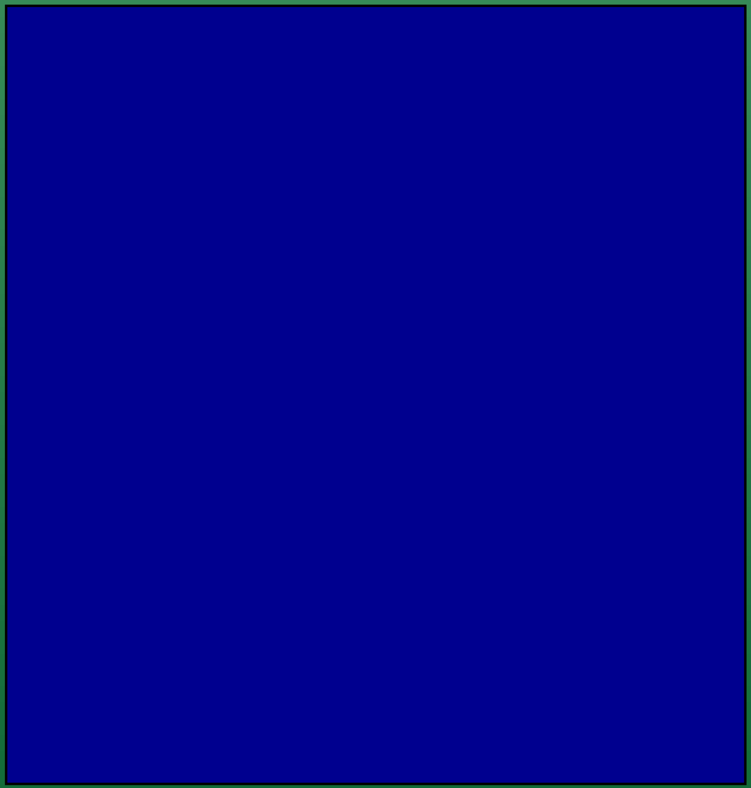

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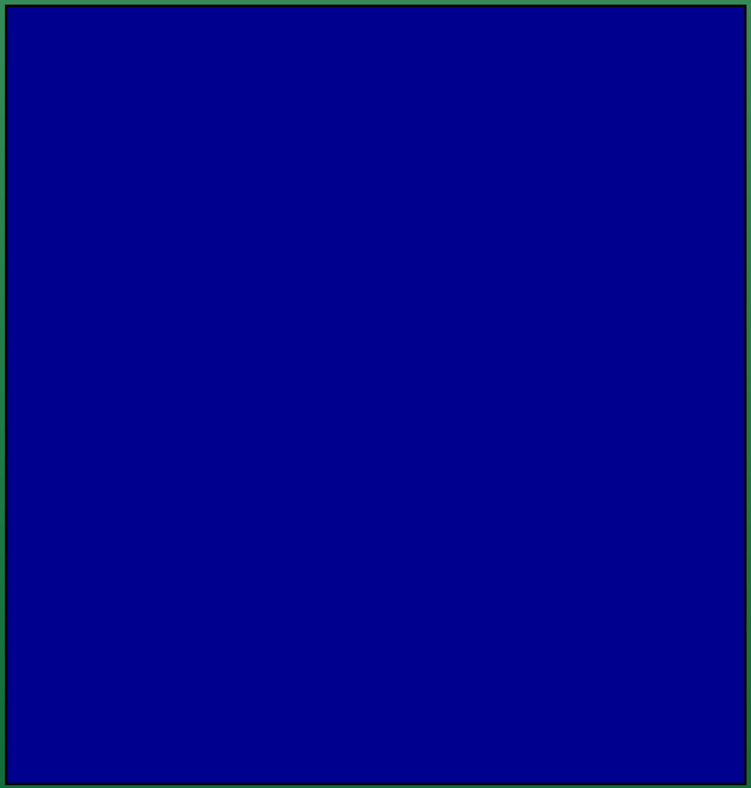

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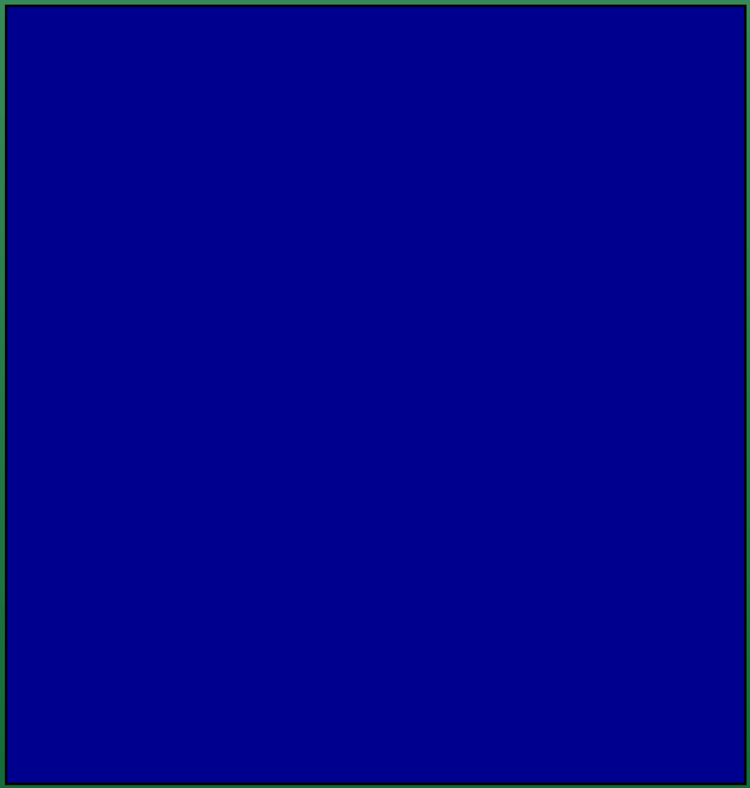
- 

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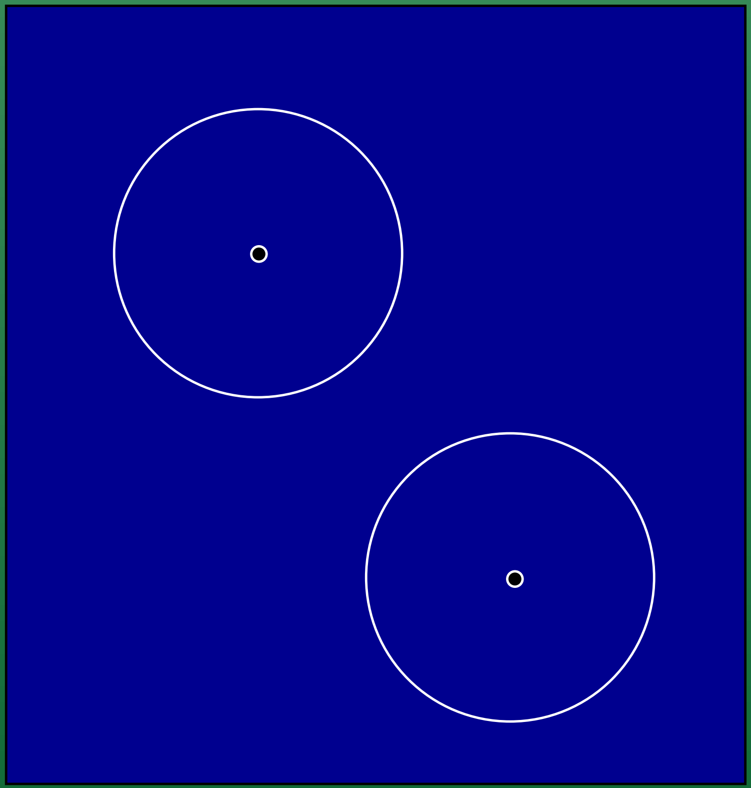

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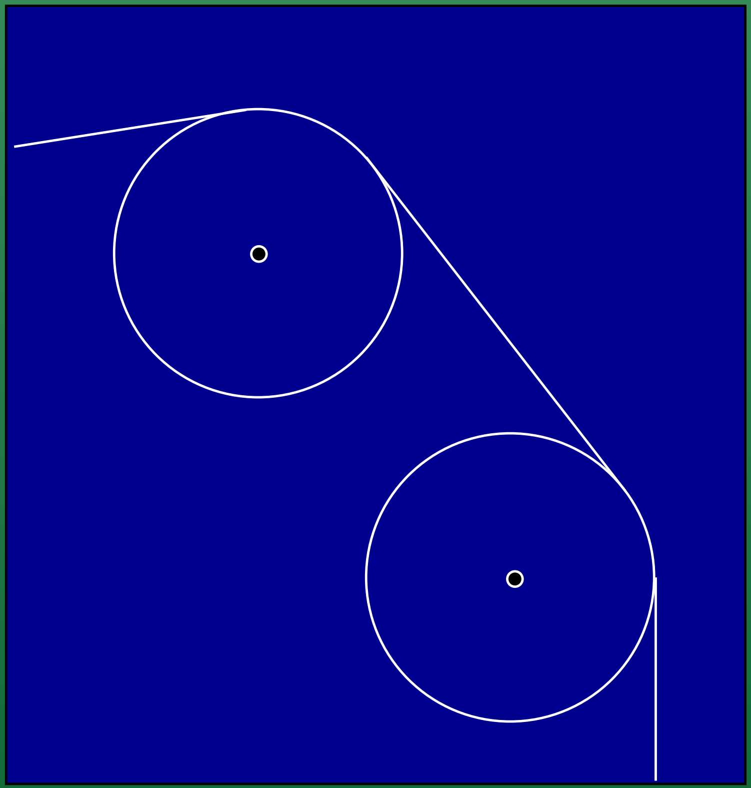

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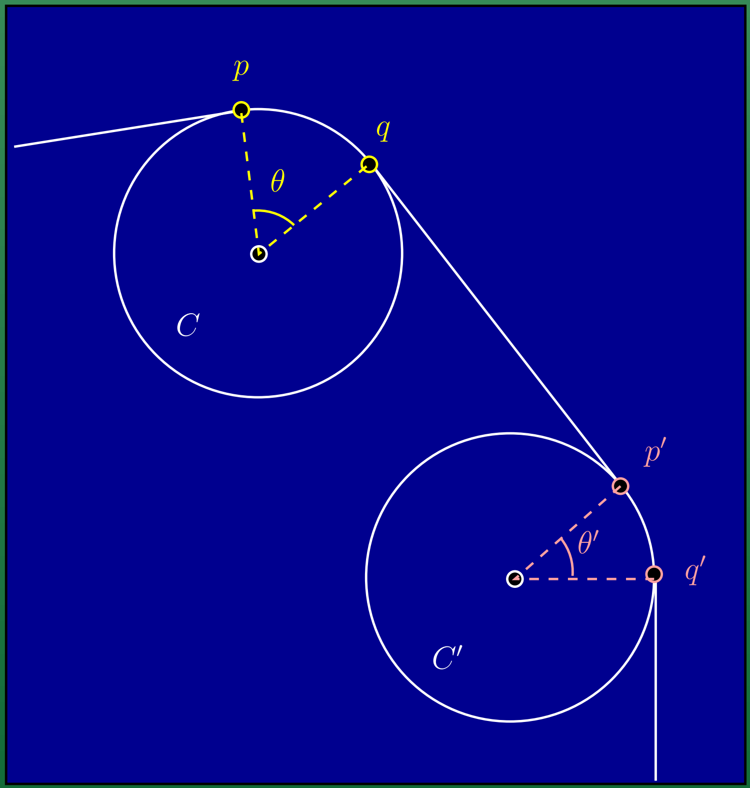

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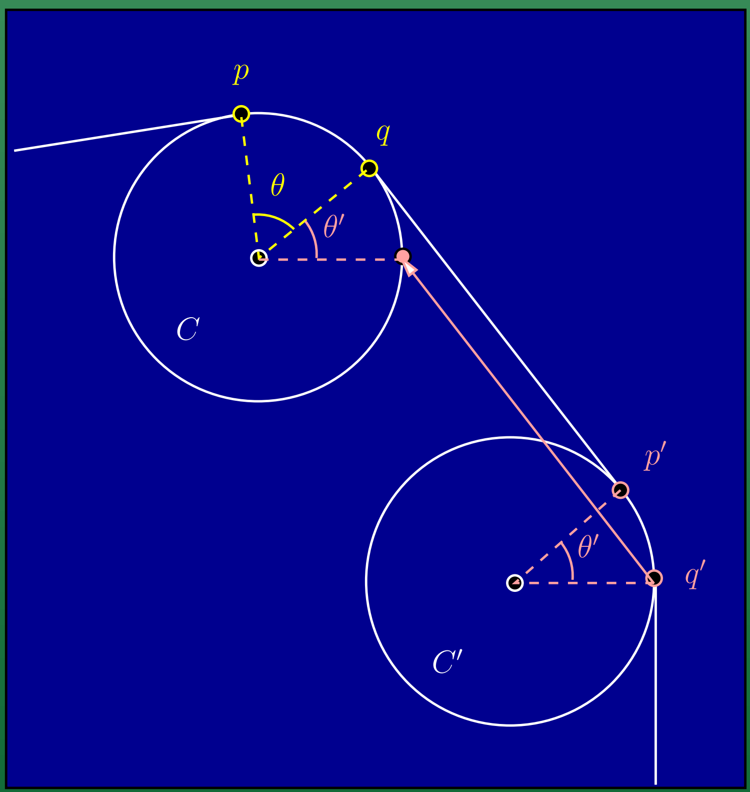

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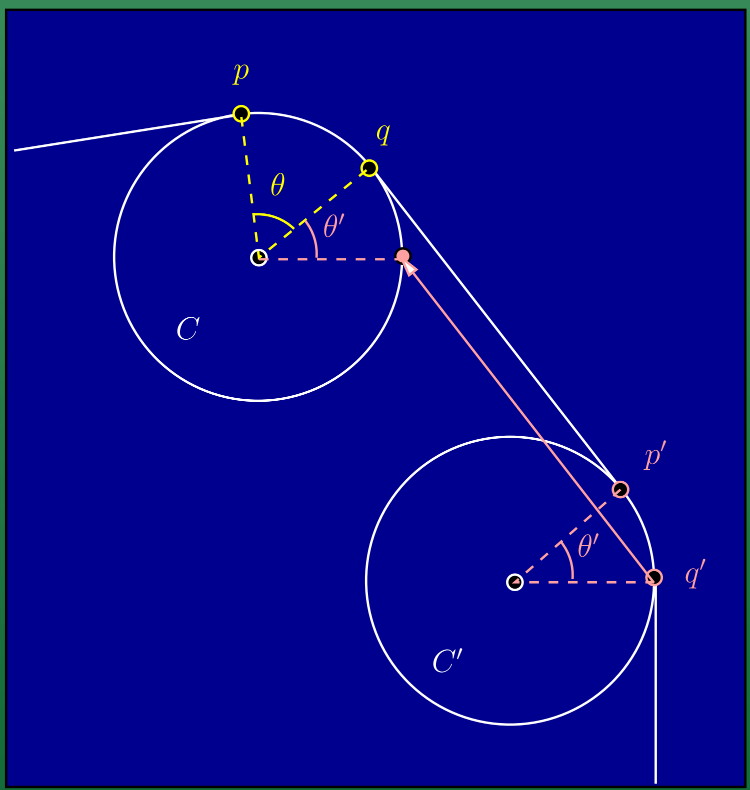

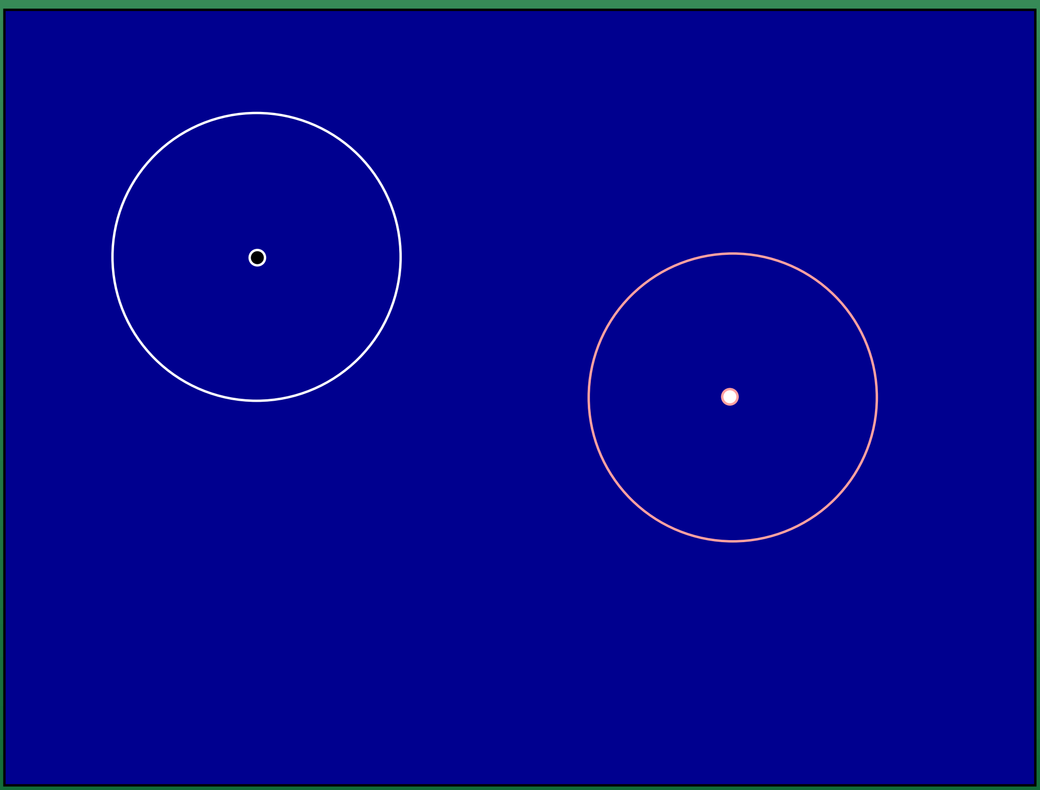

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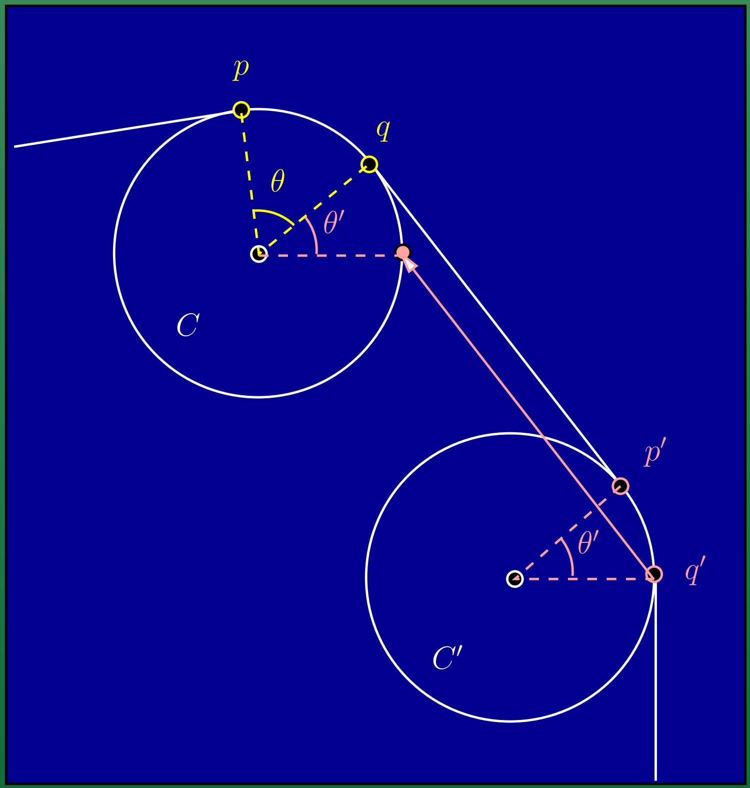

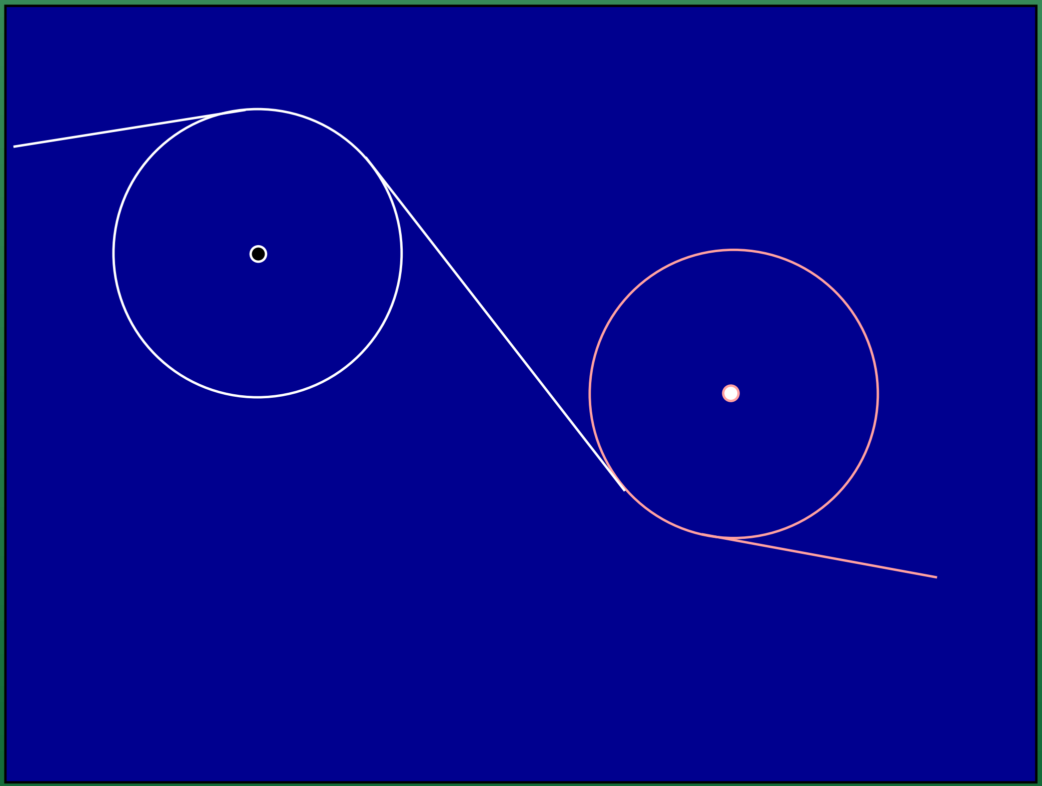

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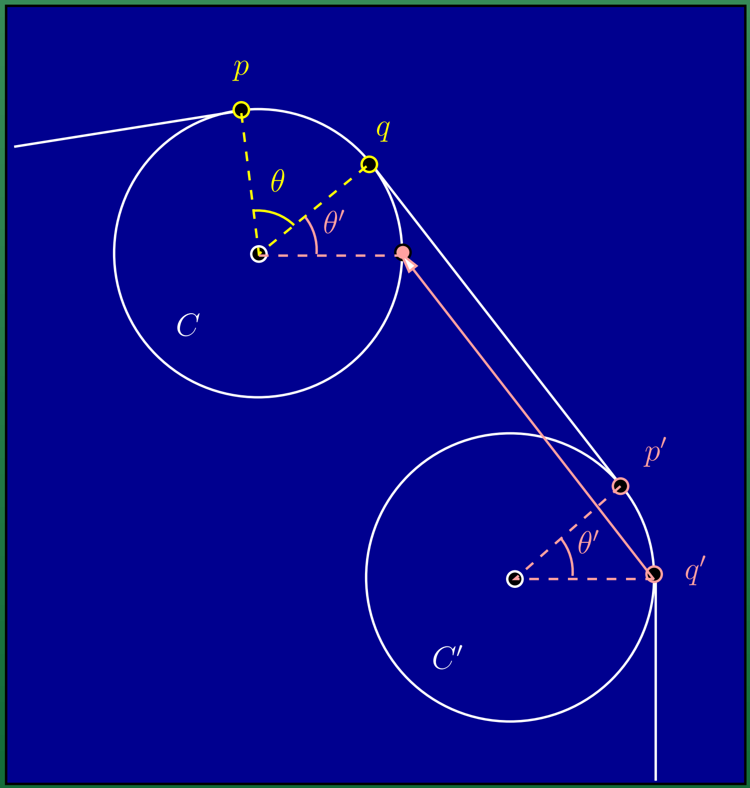

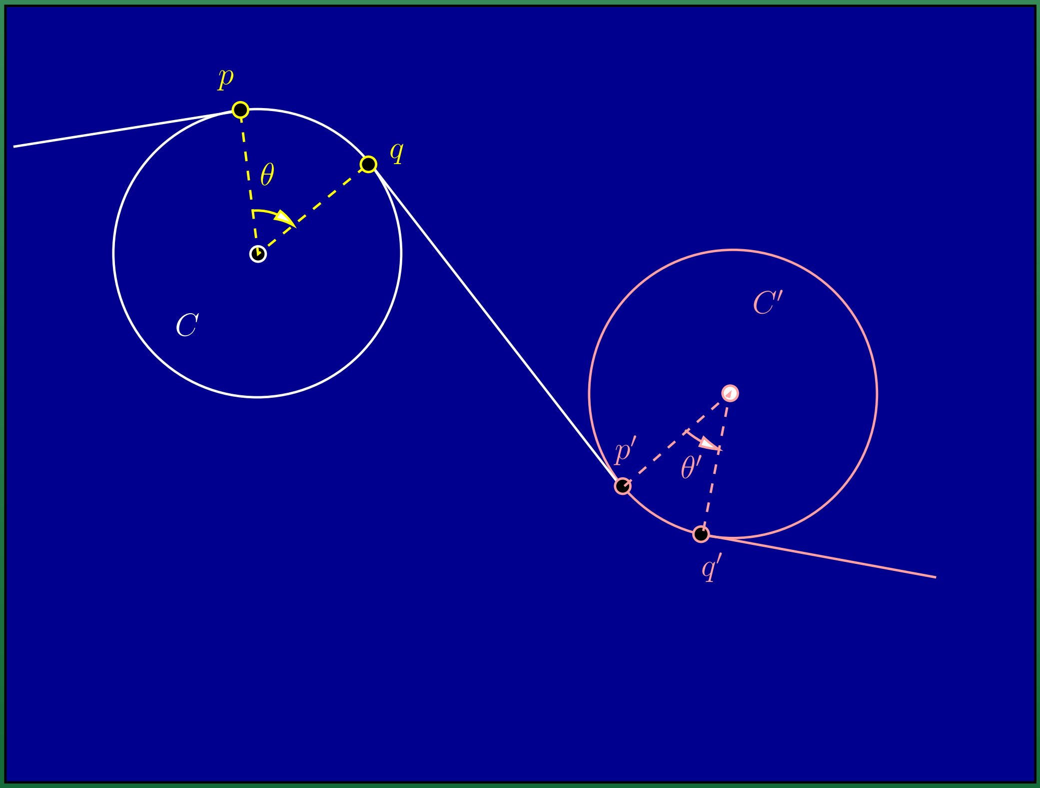

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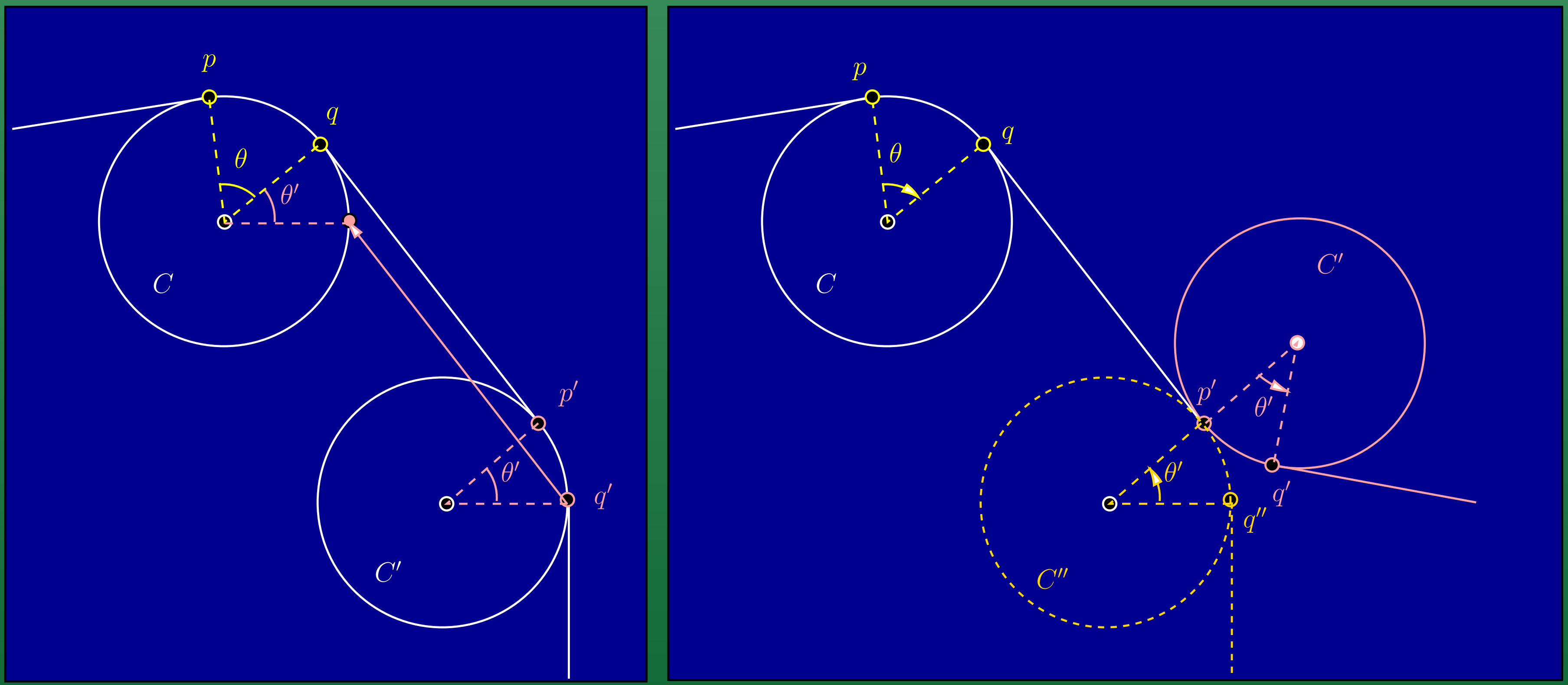

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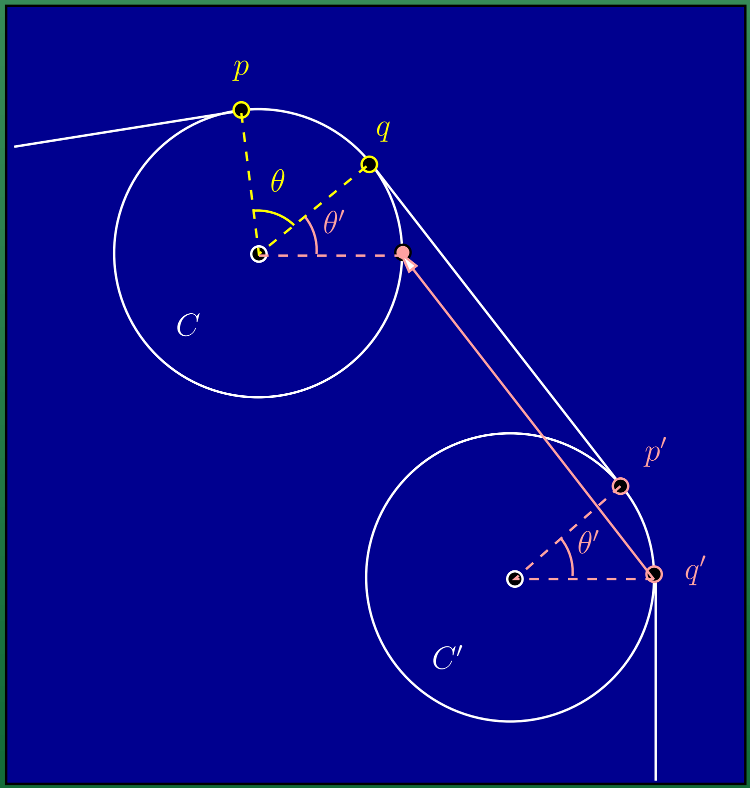

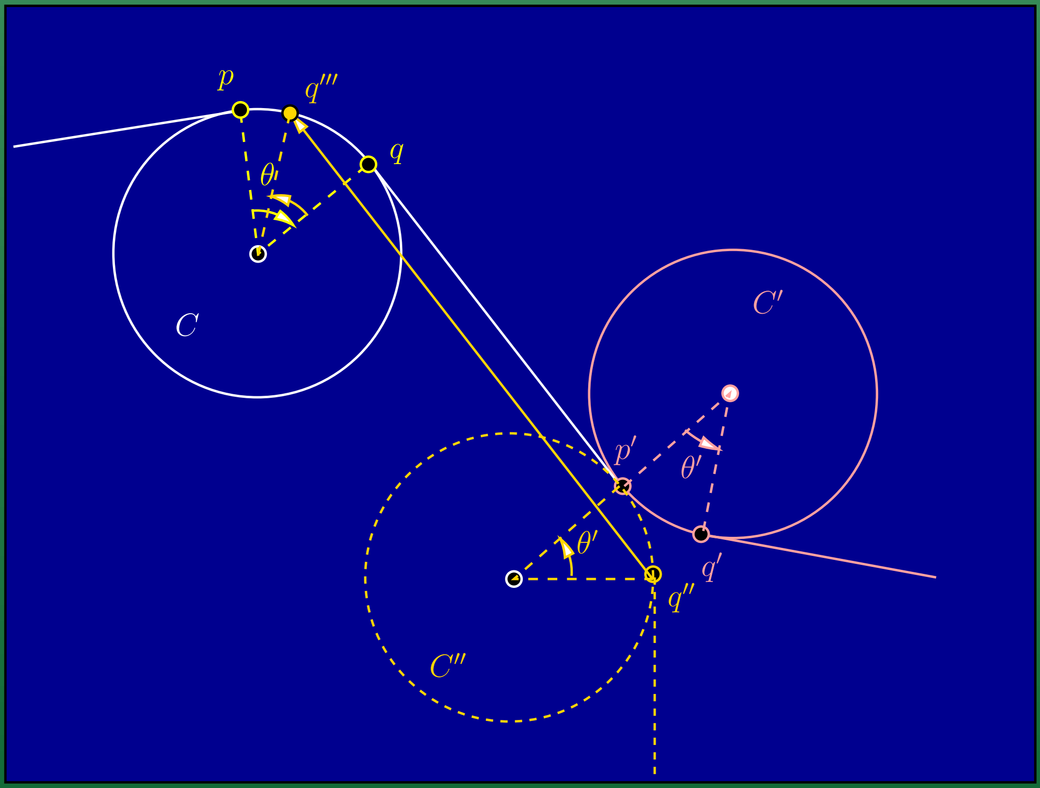

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Decidability

- THEOREM: Shortest Path for unit disc obstacles is computable.

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  - ∗ Also: Lyapunov (1955)

- Open Problems:
  - ∗ Extend to ellipse obstacles
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- Other examples of transcendental problems
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REFERENCE

- “Shortest Paths for Disc Obstacles is Computable”
  - E.Chang, S.Choi, D.Kwon, H.Park, C.Yap. 21st SoCG, 2005.

“A rapacious monster lurks within every computer, and it dines exclusively on accurate digits.”
– B.D. McCullough (2000)

THE END