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Editorial

Special Issue on Robust Geometric Algorithms and their Implementations

The robust implementation of geometric algorithms is highly nontrivial. There are several reasons for this: theoretical algorithms (i.e., algorithms on paper) traditionally assume error-free real computations, at constant time cost per operation, and data in general position. All three assumptions do not hold in the real world (i.e., algorithms on silicon). So implementors face the tasks of (1) controlling the effects of numerical errors, (2) achieving practical and efficient implementations, and (3) enumerating all degenerate possibilities and devising ways to treat them.

All three tasks are illustrated by papers in this collection. (1) The paper of Devillers and Guigue addresses the "rounding of polygons"—this problem arises because the choices we make in rounding numbers can have geometric consequences which must be controlled. (2) The paper of Emiris and Karavelas shows how to implement the predicates used in computing Apollonius diagrams efficiently and exactly—this is an issue because such predicates are time critical operations for the algorithm. (3) The paper of Schömer and Wolpert introduces the Jacobi curve technique for detecting tangential intersections—such intersections do not arise under the "data in general position" assumption.

In the last decade, we have seen the emergence of the Exact Geometric Computation (EGC) paradigm for achieving robust geometric algorithms. Unlike other approaches to nonrobustness, only the EGC approach can be encoded into a general-purpose software library which programmers can invoke to achieve full robustness for their otherwise normal programs. Today, implementors without special knowledge of robustness techniques can routinely implement efficient and robust algorithms for many problems, just by invoking libraries such as LEDA, CGAL or Core Library. This is no mean achievement: 10 years ago, the construction of any single such program would have been regarded as a challenge. What made this possible is the emergence of new techniques such as filters and effective zero bounds that bridge the gap between theory and practice.

Many challenges lie ahead in the development of EGC computation. Three key areas are represented by the current collection of papers: geometric rounding, efficient nonlinear computation, and the Zero Problem. (a) *Geometric rounding* is addressed by Devillers and Guigue. Although we have techniques for robust and efficient computation of many problems, the computed object may require high precision and often requires different numerical representation than the input numbers. For instance, the computed object may require representations of algebraic numbers. In applications, we would like such numbers to be approximated by floating point numbers. The rounding of numbers becomes complicated when they determine geometric features which must be preserved. Despite the increasingly recognized importance of geometric rounding, there are still few results. In fact, the only problem treated with any depth in the literature is the problem of rounding an arrangement of line segments on the integer grid. So the study of Devillers and Guigue is a welcome contribution to this sparse literature. (b) *Nonlinear algebraic computation* is a crucial testing ground for robustness techniques because the nonrobustness phenomenon, while bad in linear geometry, becomes much more pronounced in the nonlinear world. The paper of Schömer and Wolpert addresses one of the simplest nonlinear problems in 3-dimensions, viz., arrangement of quadric surfaces. Nonlinearity is also a key issue in the paper of Emiris and Karavelas, albeit in 2-dimensions. (c) The *Zero Problem* originally arose in logic, and concerns deciding whether a given numerical expression represents 0. Among the many deep open questions here, perhaps the most important case is whether the zeros represented by numerical expressions over the functions exp and log are decidable. The strongest positive result here is from Richardson (1997) who showed that such zeros

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are decidable if Schanuel's conjecture in transcendental number theory is true. The present paper of Richardson and Elsonbaty addresses another conjecture which would imply the efficient decidability of such zeros.

Here is a brief summary of each paper:

1. "Inner and outer rounding of Boolean operations on lattice polygonal regions" by Olivier Devillers and Philippe Guigue. This paper addresses a basic rounding problem, namely, how to do inner and outer rounding of polygonal regions. Unlike the well-studied problem of rounding line arrangements, we can distinguish three "modes" in rounding polygonal regions: rounding to an inner polygonal region, to an outer polygonal region, or to a polygonal region whose boundary is within some (Hausdorff) distance from the true boundary. The authors treat the first two modes. This terminology of "rounding mode" is suggestive of the rounding modes in IEEE floating point arithmetic. Indeed, as the authors suggest in their conclusion, what we want are geometric analogs of the IEEE standard.
2. "The predicates of the Apollonius diagram: Algorithmic analysis and implementation" by Ioannis Z. Emiris and Menelaos I. Karavelas. *Apollonius diagrams* are the Voronoi diagrams of circles. This paper focuses on the implementation of exact predicates for this computation. Their approach is based on computing with polynomials via multivariate resultants and Sturm sequences with multiprecision arithmetic. They also use arithmetic filters techniques to compute the easy cases as quickly as possible. Filters is another major technique in exact geometric computing. It should be clear that extensions and generalizations of this work is wide open.
3. "Counterexamples to the uniformity conjecture" by Daniel Richardson and Ahmed Elsonbaty. In robust algorithms, we need to guarantee some user-specified precision in the numerical output. Most practical algorithms for achieving such *guaranteed precision* rely on some form of zero bounds, called *gap functions* in this paper. If $\Omega = \{\pm, \times, \ldots\} \cup \mathbb{Z}$ is a set of complex operators, and $Expr(\Omega)$ denote the set of expressions over Ω, let $V : Expr(\Omega) \to \mathbb{C}$ be the evaluation (partial) function. A gap function has the form $g : Expr(\Omega) \to \mathbb{R}_{\geqslant 0}$ such that if $e \in Expr(\Omega)$, and $V(e)$ is well-defined and non-zero, then $|V(e)| \geqslant g(e)$. Unfortunately, provable gap functions are usually too pessimistic for algebraic expressions; when transcendental functions like exp and log are introduced into Ω, then no provable gap bounds are known. The *Uniformity Conjecture* of Richardson postulates a gap function where $-\log g(e)$ is proportional to the length of the expression e. To exclude some obvious counter-examples, we restrict e to be an "expanded" expression. There are closely related conjectures of Joris van der Hoeven called Witness Conjectures. This paper disproves the Uniformity Conjecture (and the strong form of the Witness Conjecture). The technique for generating such counter examples is interesting and has independent interest. The authors proposed a replacement for the Uniformity Conjecture.
4. "An exact and efficient approach for computing a cell in an arrangement of quadrics" by Elmar Schömer and Nicola Wolpert. Computing the intersection of two quadrics is a highly classical problem. In the last few years, a number of papers has revisited this problem in attempts to remedy various shortcomings of the classic approach of Levin (1976). In the present paper, the authors treat the slightly more general problem of computing an arrangement of a collection of quadrics. Unlike the Levin approach, the authors first project the space intersection curves (QSIC's) into the plane, and then use the standard plane sweep technique to compute the arrangement of these planar curves. What is new is that they are the first to completely treat all possibilities of this arrangement: in particular, tangential intersections are properly treated. A new technique based on (generalized) Jacobi curves is introduced to handle such intersections. If $f = 0$ and $g = 0$ are two plane curves, the *Jacobi curve* is defined as $h_1 = f_x g_y - f_y g_x = 0$. In case the intersection of $f = 0$ and $g = 0$ at a point p with multiplicity 2, the curve $h_1 = 0$ will intersect f (and also g) transversally at p. Tangency can then be detected by numerical means. If the multiplicity of intersection is larger than 2, then higher analogs of h_1 must be used.

In conclusion, we believe that each of these papers represents the tip of a trove of new body of results. The ultimate goal of making EGC computation as widely accessible as current day floating-point computation will depend on such discoveries.

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