Phong Shading

Computer Graphics
Guest Lecture
10/30/07
Graphics vs. Physics

Color of an object

Intensity/Color of light ray

Intensity/Color of point light source

Reflection Spectrum:
Fraction of light of each frequency that gets reflected

Radiance Distribution

Intensity spectrum of a point light source
Some physics...

- Radiometry = measuring light
- Photon: quantum of light with position, direction of propagation and a wavelength $\lambda$

  (assumption - light propagates in straight lines in isotropic medium)

- speed $c$ that depends on refractive index of medium
- frequency $f = c/\lambda$: invariant to refractive index
- energy $q = hf = hc/\lambda$, $h \sim 1 \text{e-34}$ Plank’s constant
• **Light** = Radiant Energy

• measured in **joules**, \( Q = \sum_i \frac{hc}{\lambda_i} \)

• **Spectral Radiant Energy**

• sun vs. single-wavelength laser

• amount of radiant energy per unit wavelength interval at wavelength \( \lambda \)

• measured in \((\text{joules/nm})\), \( Q_\lambda = \frac{dQ}{d\lambda} \)

• **Radiant flux** (Radiant power)

• power = energy per unit time

• time rate of flow of radiant energy

• measured in \((\text{joules/sec, watts})\), \( \Phi = \frac{dQ}{dt} \)
• **Radiant Flux Density**
  
  • radiant flux per unit area at a point on a surface
  
  • measured in \((\text{watts/m}^2)\)
  
  • arriving flux - Irradiance
  
  \[ E = \frac{d\Phi_{in}}{dA} \]
  
  • leaving flux - Radiant exitance
  
  \[ M = \frac{d\Phi_{out}}{dA} \]

![Diagram of irradiance and radiant exitance](image-url)
Solid Angles

Reminder:
A regular angle $\theta = \frac{l}{R}$
circle has $2\pi$ radians

differential solid angle can be assigned a direction. Unit: steradian (full sphere = $4\pi$)
Radiance: Measure of Light

**Radiance (Luminance):** Number of photons (or energy) arriving per time at a small area perpendicular to the ray from a particular direction

\[
L(\omega) = \frac{d^2 \Phi \cos \theta}{dA d\omega} = \frac{\text{watt}}{m^2 \text{steradians}}
\]

*Physical quantity equivalent to psychological concept of brightness as observed by us.*
Local illumination model

Describes interaction of the light and the surface

- light can be reflected, absorbed and transmitted
- most important: reflection
- reflection can be ideal specular, diffuse and anything between
- reflection equation relates the outgoing radiance in some direction to the incoming radiance
• Once again, radiance:

\[ L(\omega, \theta) = \frac{d^2 \Phi}{\cos \theta dA d\omega} = \frac{\text{watt}}{m^2 \text{steradians}} \]

• and irradiance (flux density incoming energy per area)

\[ E = \frac{d\Phi_{\text{in}}}{dA} \]

\[ E(\omega, \theta) = L(\omega, \theta) \cos \theta d\omega = \frac{\text{watt}}{m^2} \]
"BRDF"
Bidirectional Reflectance Distribution Function

\[
f(\omega_i, \omega_r) = \frac{L_r(\omega_r, \theta_r)}{E_i(\omega_i, \theta_i)}
\]

unit: steradians\(^{-1}\)

reflected radiance exiting a surface in direction \(r\)

irradiance incident on the surface from direction \(i\)
Reflection equation

the outgoing radiance in direction $r$ is the sum of the radiances due to radiance from all incoming directions:

$$L_r(\omega_r) = \int f_r(\omega_i, \omega_r) \ L_i(\omega_i) \cos \theta_i \ d\omega_i$$

the integral is over the upper hemisphere

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Reflection geometry

Two main components:

- Light source characteristics
- Position
- Intensity

For each frequency (color), different intensity can be specified for different colors.

Surface properties:
- Reflectance for each frequency (color)

Reflection geometry equation:

\[
R = 2(N \cdot V)N - V
\]

\(N\): normal
\(L\): direction to the light source
\(V\): direction to the eye
\(R\): reflected direction

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Snell’s law

If a surface separates two media with different refraction indices (e.g. air and water) the light rays change direction when they go through.

**Snell’s law:** the refracted ray is stays in the plane spanned by the normal and the direction of the original ray. The angles between the normal and the rays are related by
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
where \( n_1 \) and \( n_2 \) are refraction indices.

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Snell’s law

If a surface separates two media with different refraction indices (e.g. air and water) then the light rays change direction when they go through.

Snell’s law: the refracted ray is stays in the plane passed by the normal of the ray.

The angles between the normal of the ray and the incident and refracted rays are.

\[ T = -nV + \left( n(V, N) - \sqrt{1 - n^2(1 - (V, N)^2)} \right) N \]

\[ n = \frac{n_1}{n_2} \]
Illumination model

Two main components:

- light source characteristics
  - position
  - intensity for each freq. (color)
    often, different intensity can be specified for different colors
  - directional distribution

- surface properties
  - reflectance for each freq. (color)
  - different reflectance can be specified for diffuse and specular light
And now graphics...

!! Midterm Evaluation
Local lighting model

Describes interaction of the light with the surface.

Almost never truly based on physics: perception plays a greater role.

Visible light: electromagnetic waves, with wavelengths 400nm (violet) - 700nm (red); intensity can vary over many orders of magnitude.

Computer model: only three “frequencies”: RGB, intensity varies over a small range, typically only 255 discrete values/ color.
Perception

Perceptual response to light with an arbitrary spectrum

\[ \text{Perceptual response produced by a linear combination of 3 primary wavelengths. (R,G,B)} \]
Highlights

• Avoid surfaces that look dull/lifeless
• Change as object moves
• Provides visual information about shape and motion
• A simple model: Phong Shading
• Developed by Bui-Tong Phong, in 1973 dissertation “Illumination for Computer Generated Pictures”
• Largely empirical
• Fast and realistic enough
A simple model

In the model commonly used in graphics applications, there are several components:

- **Diffuse reflection**: intensity does not depend on the direction to the viewer.

- **Specular**: simulates reflective surfaces and specular highlights depending on the direction to the viewer.

- **Ambient**: a crude approximation to the illumination created by the light diffusely reflected from surfaces.
Ambient Component

\[ L_a = I_a s_a = [r_a, g_a, b_a] \]

- Scattered light in the environment
- Seems to come from all directions
- When ambient light hits a surface, it’s scattered in all directions
- Usually given a dim constant value (e.g. .2)
- Example: a greenish object might have an ambient value of \([r_a, g_a, b_a] = [.1, .3, .1]\)
Diffuse (Lambertian) Component

- **Lambert’s Cosine Law**: Color intensity is proportional to the cosine of the angle between surface normal and direction to light source.

\[
L_d \propto \cos \theta \\
L_d \propto (n \cdot L)
\]

\[
L_d = I_d s_d (n \cdot L) \\
L_d = I_d s_d \max(0,(n \cdot L)) \\
(\text{or } = I_d s_d |n \cdot L|)
\]

\[
L_d = [r_d,g_d,b_d] |n \cdot L|
\]

Does not depend on view direction = scatters the same way in all directions

Diffuse (Lambertian) Component

- Light comes from one direction
- Brighter if perpendicular to surface
- Once hits surface, scattered equally
- Example: Paper
Specular (Phong) Component

Highlights - part of surface where $\alpha=0$ brightest highlight. There’s also brightness when $\alpha$ small.

$$r = 2(l \cdot n)n-l$$

Need a function s.t. it’s high when $r=e$ and gradually falls off as $r$ gets away from $e$.

$$L_s = l_s s_s (e \cdot r)\max(0, e \cdot r)^p$$

$l_s s_s (e \cdot r)$ too large area, can be negative

$$L_s = \lbrack r_s, g_s, b_s \rbrack\max(0, e \cdot r)^p$$
Specular (Phong) Component

- Light comes from a particular direction
- Bounces off surface at a preferred direction
- “Shininess”
- Mirror - light from source reaches the eye only if $r=e$
Specular reflection

$p = 0$
$p = 1$
$p = 2$

$p = 10$
$p = 25$
$p = 100$
Metal vs. plastic

Natural look of metallic surfaces is difficult to simulate, but the first approximation is obtained using proper highlight color.

For plastic objects, highlights are close in color to the color of the light. For metals, to the color of the surface. Assuming white lights, for plastic set \( k_{\text{spec}} = [c, c, c] \), where \( c \) is a constant, for more metallic look set \( k_{\text{spec}} = k_{\text{diff}} \).
Attenuation

- In real life: Radiance goes down by $1/r^2$
- Ex: Stars vs. Sun

- Due to approximations inverse-square law results in very dark images
- Solution: Let the user choose the attenuation factors $(a,b,c)$

$$1/(a + br + cr^2)$$

Default - (1,0,0) - no attenuation
Complete Equation

\[ L_d = [r_d, g_d, b_d] \max(0, (n \cdot L)) \]
\[ L_a = [r_a, g_a, b_a] \]
\[ L_s = [r_s, g_s, b_s] \max(0, (e \cdot r)) \]
\[ A = \frac{1}{(a + br + cr^2)} \]

Total Radiance for a light \( L = A(L_a + L_d + L_s) \)

For multiple lights \( L_i \), with own colors \([L_{r_i}, L_{g_i}, L_{b_i}]\)

\[ L = A([r_a, g_a, b_a] + \sum_i([L_{r_i}, L_{g_i}, L_{b_i}] ( [r_d, g_d, b_d] \max(0, (n \cdot L_i)) + [r_s, g_s, b_s] \max(0, (e \cdot r_i)^p )))) \]
Few points to note...

• No shadow support.

• No second or higher order reflections (thus the ambient term)

• For fast computation assume
  
  • Light sources are point sources, placed infinitely away

  • Viewer is also at infinity.
For your homework

• Given globally:
  • viewing direction (camera)
  • light direction

• Given per vertex : [x,y,z, nx, ny, nz]

• Use the light model equation
  \[
  [r,g,b] = ([r_a,g_a,b_a] + \sum_i ([L_r,L_g,L_b] [r_d,g_d,b_d] \max(0,(n \cdot L_i)) + [r_s,g_s,b_s] \max(0,(e \cdot r_i)^p)))
  \]

• to get to [x,y,z,r,g,b]