# Expected number of satisfied clauses

Note: This assumes knowledge of the propositional calculus.

A. Suppose you have a collection of $N$ clauses in the propositional logic, where each clause has three literals with three different atoms.

You assign a truth value to each atom at random, with equal probability true or false. Let random variable $K$ be the number of clauses satisfied by the assignment. What is $\text{Exp}(K)$ as a function of $N$? Justify your answer.

For example one set with $N = 5$ is

$\{$ 1. $P \vee \neg Q \vee R$.
   2. $\neg P \vee Q \vee W$.
   3. $P \vee \neg R \vee W$.
   4. $P \vee R \vee \neg W$.
   5. $Q \vee R \vee \neg W$. $\}$

If you randomly choose $P = F$, $Q = T$, $R = T$, $W = F$, then in that case 1, 2, 4, and 5 are satisfied but 3 is unsatisfied, so in that case, the value of $K$ is 4.

Hint: This is an *easy* problem; your answer should not be more than three or four sentences long. Determining the probability distribution of $K$ is difficult, and depends on the particular set of clauses. For example, $\text{P}(K = N)$ is equal to 0 only if the set of clauses is unsatisfiable, which is a hard problem (co-NP-complete). But $\text{Exp}(K)$ is the same, regardless of what the clauses are (even if the collection is just $N$ repetitions of the same clause, say.)

B. Find the probability distribution for $K$ for the above specific example of five clauses. (There is no clever way to do this; you just have to enumerate all 16 different valuations.)