Programming Assignment: Drawing Ellipses

Chapter 2: *Linear Algebra and Probability for Computer Science Applications*

The points on an ellipse with center $p = \langle p_x, p_y \rangle$ with semi-axes of length a and b, where the semi-axis of length a is oriented at angle θ from the x axis, is given by the parameterized curve

$$\langle p_x, p_y \rangle + \langle a\cos(\theta)\cos(\phi) - b\sin(\theta)\sin(\phi), a\sin(\theta)\cos(\phi) + b\cos(\theta)\sin(\phi) \rangle,\ \phi \in [0, 2\pi]$$

Part A

Write a MATLAB function `Ellipse(p,a,b,theta`) where `p` is a column vector; `a`, `b`, and `theta` are floating points and `theta` is in radians. The function

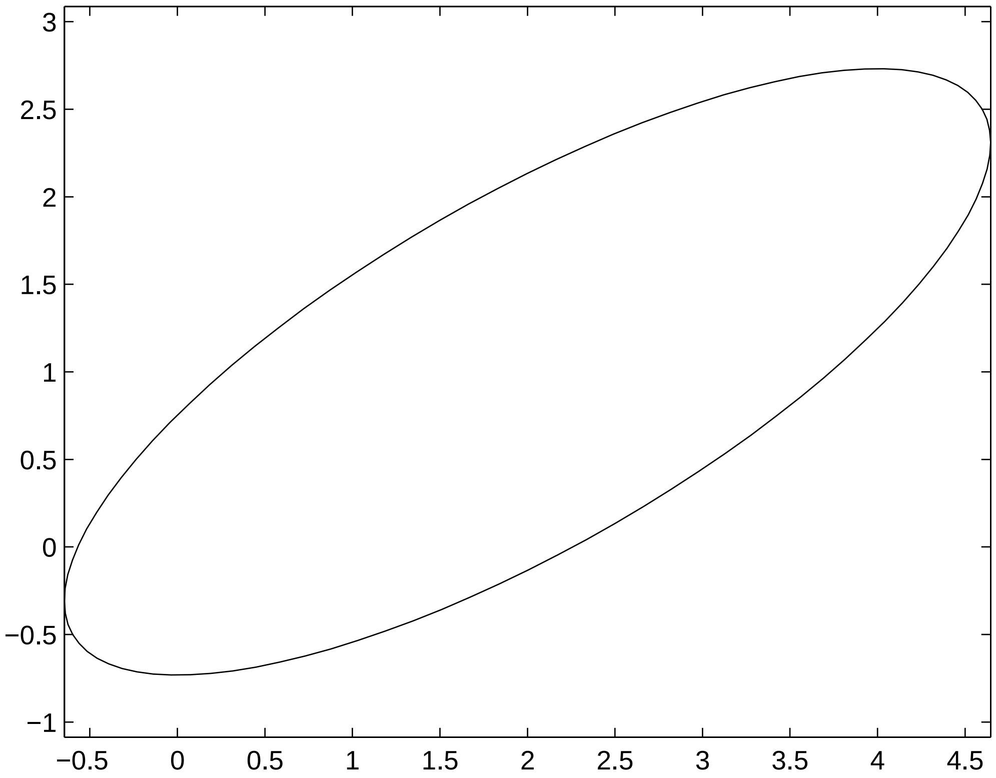
i. Computes the 101 points on the ellipse corresponding to the values $\phi = (i-1) * 2\pi/100, i = 1 \dots 101$ and puts these coordinates in a 2×101 matrix, which it returns as its value;

ii. Draws a picture of the ellipse in black.

For instance, the function call `Ellipse([2;1],3,1,pi/6)` generates the figure shown below, and returns a 2×101 matrix whose first 7 columns are:

```
Columns 1 through 7

  4.5981    4.5616    4.5149    4.4584    4.3921    4.3164    4.2316
  2.5000    2.5514    2.5967    2.6357    2.6682    2.6942    2.7135
```



You should try to write the function as "straight-line" code, with no explicit loops at the level of the MATLAB code. Hint: Use the `repmat` function to turn the point `p` into a 2 × 101 array. Of course the MATLAB built in functions you are calling have such a loop in the background, but these are presumably much more efficiently implemented than an explicit loop in the MATLAB code. There will be a penalty of 2 points out of 100 (total for the entire assignment) for writing an explicit loop.

For all assignments involving drawing geometric pictures, you should use the plotting call `axis equal` to use equal scales on the x and y axes; otherwise, the picture tends to come out visually squooshed.

Part B

Write a MATLAB function `TwoEllipses(p1,a1,b1,theta1, p2,a2,b2,theta2)` that

i. Draws the two ellipses in black.

ii. Draws a blue line between the two points on the two ellipses that are furthest apart, and a red line between the two points that are closest together. (Break ties arbitrarily.) If the two points that are closest together are less than 0.1 apart, then it draws an asterisk there rather than a red line.

iii. Returns two values: the distance between the two closest points and the distance between the two most distant points.

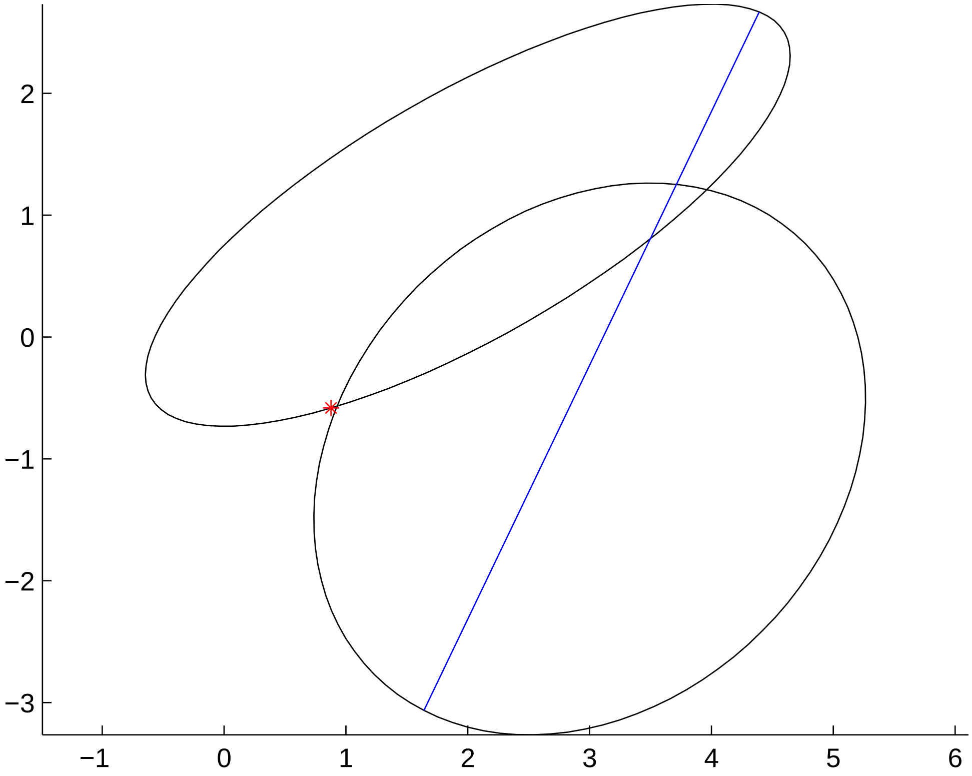
When you want to do multiple calls to `plot` in the same figure, you need to call `hold on`; otherwise, each call to `plot` erases the previous ones.

You may use explicit loops in this part of the assignment.

For example the function call

```
>> [Short,Long]=TwoEllipses([2;1],3,1,pi/6,[3;-1],2,2.5,3*pi/4)
Short =
    0.0437
Long =
    6.3588
```

generates the following picture:



2
1
0
−1
−2
−3
−1
0
1
2
3
4
5
6