Programming Assignment: Electric Charges

Chapter 5: *Linear Algebra and Probability for Computer Science Applications*

Note: The assignment is long, but the code is actually *very* short; perhaps 20 lines in total.

Suppose that A and B are electrically charged objects, located at points $\mathbf{p}_A$ and $\mathbf{p}_B$ with charges Q_A and Q_B. Then the force $\vec{F}_A(B)$ that B exerts on A is the vector

$$\vec{F}_A(B) = \frac{Q_A \cdot Q_B}{|\mathbf{p}_A - \mathbf{p}_B|^2} \cdot \frac{\mathbf{p}_A - \mathbf{p}_B}{|\mathbf{p}_A - \mathbf{p}_B|}$$

In the above product, the first factor is the magnitude of the force, which is the product of the charges divided by the distance squared; the second factor is the direction of the force, which is the direction from B to A.

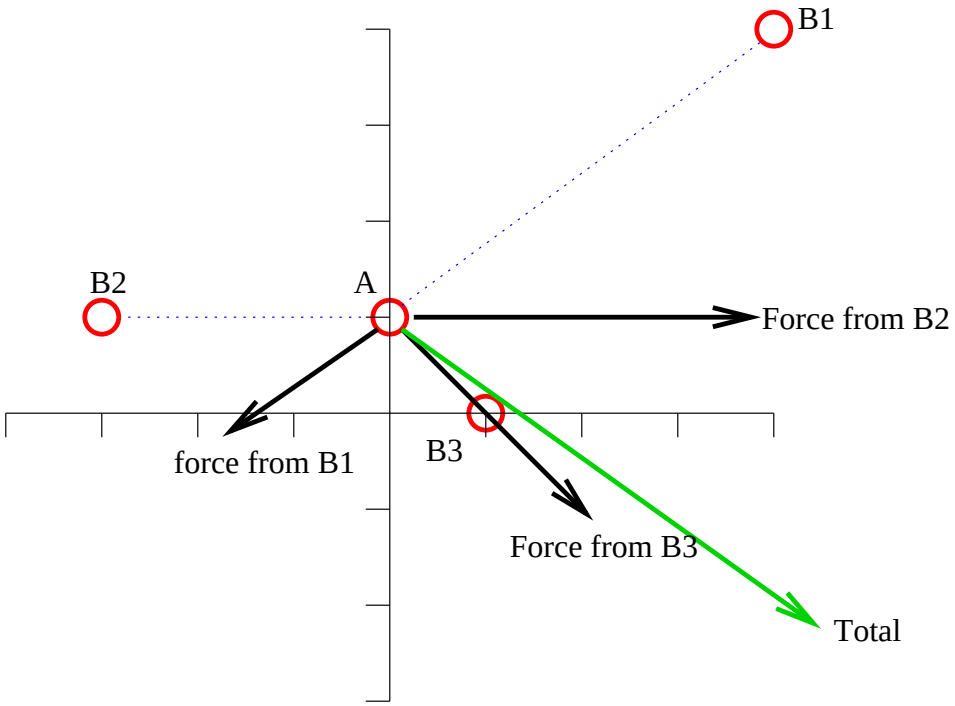
If there are several objects $B_1 \dots B_k$ exerting a force on A, then the total force on A is the sum of the forces:

$$\vec{F}_A(\{B_1 \dots B_k\}) = \sum_{i=1}^{k} \vec{F}_A(B_i)$$

If the charge on A and the position of all the charges is fixed, then the net force is a linear function of vector of charges $\langle \vec{Q} = Q_1 \dots Q_k \rangle$.

For instance, in two dimensions, we could have the following situation, illustrated in the picture.

Object	Location	Charge	$\lvert\mathbf{p}_A - \mathbf{p}_B\rvert$	Magnitude of $\vec{F}_A(B)$	$\vec{F}_A(B)$
A	$\langle 0, 1\rangle$	1		—	—
B_1	$\langle 4, 4\rangle$	50	5	$50/25 = 2$	$2 \cdot \langle -4, -3\rangle/5 = \langle -1.60, -1.20\rangle$
B_2	$\langle 1, 0\rangle$	-6	$\sqrt{2}$	$-6/2 = -3$	$-3 \cdot \langle -1, 1\rangle/\sqrt{2} = \langle 2.12, -2.12\rangle$
B_3	$\langle -3, 1\rangle$	36	3	$36/9 = 4$	$4 \cdot \langle 3, 0\rangle/3 = \langle 4.00,\ 0.00\rangle$
Total					$\langle 4.52, -3.32\rangle$



Problem 1 (50 points)

Write a function `function F = ForceMatrix(PA,PB)` where

- `PA` is a 2-dimensional column vector of the coordinates of object A of charge 1.
- `PB` is a $2 \times k$ matrix, where the ith column, `PB[:,i]` is the coordinates of object B_i.
- `F`, the value returned is the $2 \times k$ matrix with the property that for any vector of charges $\vec{\texttt{Q}}$, the value $\texttt{F} \cdot \vec{\texttt{Q}}$ is the net force on A.

For instance, in the above example, we could call

```
> PA = [0;1];
> PB = [4,1,-3; 4,0,1];
> F = ForceMatrix(PA,PB)
F =
    -0.0320    -0.3536    0.1111
    -0.0240     0.3536    0
> QB = [50; -6; 36];
> F*QB
ans =
    4.5213
   -3.3213
```

Problem 2 (5 points)

Write the following two functions: `function F = TotalForce(PA,PB,QB)` and `C = PossibleCharge(PA,PB,TF)`. In both of these `PA, PB` are the same as in problem 1. In `NetForce`, the input `QB` is a column vector of the charges on B and the value returned `F` is the total force on A, a column vector. In `PossibleCharge`, `TF` is the total force as a column vector and the value returned `C` is a possible charge vector that would give rise to that force. If there are $k > 2$ then there are multiple possible answers but your code only has to return one of these. For example, using the same values of `PA,PB,QB` we could write,

```
> F = TotalForce(PA,PB,QB)
F =
    4.5213
   -3.3213

> C = PossibleCharge(PA,PB,F)
         0
   -9.3941
   10.8000

>> TotalForce(PA,PB,C)
ans =
    4.5213
   -3.3213
```

Having done problem 1, each of these functions should consist of one quite simple line of MATLAB. The code for `TotalForce` should always work, unless A is at the same position as one of the B_i's. The code for `PossibleCharge` may fail in exceptional cases, such as your solution to problem 3.C of problem set 2.

Problem 3 (45 points)

Suppose as before there are k fixed charges $B_1 \dots B_k$ in the plane. You know the locations, but not the value of the charges, and you want to find out the value of the charges. A way to do this is as follows: You take an object A with charge 1, you put it at various points in the plane, and you measure the net force on it.

Write a function `function C = FindCharges(PA,PB,TF)` where

- `PA` is a $2 \times w$ matrix, where the ith column, `PA[:,i]` is the coordinates of the ith placement of the test charge A. The dimension w is the number of different placements you try.
- `PB` is the locations of the charges $B_1 \dots B_k$, as above.
- `F` is a $2 \times q$ matrix, where the ith column `F[:,i]` is the total force on A in its ith placement.
- The value returned `C` is the k-dimensional column vector of charges on the B_i.

Hint: Look up the Matlab `reshape` function.

For instance, in the above example, we could call

```
> PA = [0,2;1,0];
> PB = [4,1,-3; 4,0,1];
> TF(:,1) = TotalForce(PA(:,1),PB,QB);
> TF(:,2) = TotalForce(PA(:,2),PB,QB);
> C = FindCharges(PA,PB,TF)
C =
   50.0000
   -6.0000
   36.0000
```