Problem: Inverting the Midpoint Construction

This problem depends on the programming assignment, Polygon Midpoints. You should certainly think through that assignment before doing this problem, and since thinking through that assignment is practically equivalent to doing it, it is probably a good idea to complete that assignment before doing this problem.

A. Give an argument to show that, if k is even, then in general there does not exist a solution to the inverse problem. (Hint: Show that the last row of the coefficient matrix C can be written as a linear sum of the other rows. Explain why this proves the result.)

B. Give an argument to show that, if k is odd, then there always exists a solution to the inverse problem. (Hint: In MATLAB, construct the coefficient matrix C for some small odd k — e.g. $k = 5$ or $k = 7$ — and compute its inverse. You will see that the inverse has a very simple, regular pattern. Argue that you can use this pattern to construct the inverse of C for any odd value of k.)