Homework 5, due May 21.

1. Chapter 8, problem 2.

2. Chapter 8, problem 4.

3. (a) Compute the interpolating polynomial $P_L(x)$ in Lagrange form for the function $y(x) = ln(x)$, evaluated at points $x_1 = 0.1$, $x_2 = 1$, $x_3 = 2$, $x_4 = 2.9$. (b) Find a cubic polynomial $Q(x)$ such that $Q(x_1) = y(x_1)$, $Q'(x_1) = y'(x_1)$, $Q'(x_4) = y'(x_4)$ and $Q(x_4) = y(x_4)$. Compare the error of $P_L(x)$ and the $Q_L(x)$ at the point $x = 1.5$: which one approximates the function $y(x) = ln(x)$ better?

4. Suppose we want to interpolate values $y_1, y_2, \ldots y_n$ defined at points $x_1, x_2, \ldots x_n$ with a *piecewise quadratic* polynomials, $p_i(x) = a_i x^2 + b_i x + c_i$, $p_i(x)$ interpolates points (x_i, y_i) and (x_{i+1}, y_{i+1}), $i = 1 \ldots n-1$, so that the resulting interpolating functions has continuous derivative everywhere, including points x_i. Additionally, assume that $x_i = i$. Write a system of equations for a_i, b_i, c_i. Does it have a unique solution? If not, what additional data you can add to make the solution unique?