Homework 2, due Monday, March 24.

1. Chapter 3, problems 1,2,11.

2. Find the LU factorization of the matrix

$$\begin{bmatrix} 1 & 1+\epsilon & 1 \\ 1-\epsilon & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

   For what value of ϵ the U factor has linearly-dependent rows, if LU decomposition is computed using floating-point arithmetic?

3. Prove that the product of lower triangular matrices is lower triangular, and the inverse of a nonsingular lower triangular matrix is lower triangular.

4. Modify the LU factorization, back- and forward substitution codes posted on the class page so that they do not do unnecessary work for banded matrices (the new functions should take an additional parameter, bandwidth). Compute the operation count. Plot the time it takes to solve a banded linear system with random entries with 5 nonzero entries per row (2 on each side of the diagonal) using a) matrix inversion b) LU factorization, followed by forward and backward substitution c) same as b, but using functions specialized to the banded case. In Python, you can use the function `clock()` in the module `time` to time the code. use sufficiently large system sizes (starting at 1000) to obtain meaningful results. Time LU separately from back and forward substitution. Use log-log coordinates for your plots, and determine (approximately) the slope, which corresponds to the exponent p in the operation count of the form n^p.