Homework 3, due Thursday, April 3.

1. Write down a system for fitting the function $f(t) = c_0 + c_1 t + c_2 e^(t)$ to data points (t_i, y_i), $i = 1 \dots n$. Modify the least-squares curve code to solve this problem. Make sure that degenerate cases when the problem does not have a solution or has multiple solutions are detected. Extra credit: in the case of multiple solutions, find a way to choose a reasonable unique solution.

2. Show that the Housholder matrix used in QR decomposition $H(v) = I - 2\frac{vv^T}{(v \cdot v)}$, where v is a nonzero vector, has the following properties: a) $H^T H = I$, b) $H = H^T$ (and, as a consequence $H^2 = I$). Find a vector v for which this matrix annihilates all but the first entry of the vector $e = [1, 1, \dots 1]^T$, i.e. $He = [a, 0, \dots 0]^T$.

3. Write down the definition of a quadratic Bezier segment. (a) What is the geometric meaning of the middle control point? (b) If we want to construct a smooth curve out of quadratic segments what conditions the control points should satisfy?

4. Modify the code posted on the web page to draw smooth curves made out of quadratic Bezier segments. If the user specifies points (x_i, y_i), $i = 0 \dots n$, use points $(x_0, y_0), (x_2, y_2), (x_4, y_4)$ as Bezier segment endpoints and even points as middle control points for quadratic segments. If the number of points is even, do not draw the last segment.

5. Suppose we interpolate n points (t_i, y_i) with a piecewise degree 5 polynomial $P(t)$ (that is, on each interval between points the curve is a polynomial). What is the maximal number of continuous derivatives can we have at t_i? Prove your answer.