Homework 2, due Thursday, February 28.

1. Find the LU factorization of the matrix

$$\begin{bmatrix} 1 & 1+\epsilon & 1 \\ 1-\epsilon & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

   For what value of ϵ the U factor has linearly-dependent rows, if LU decomposition is computed using floating-point arithmetic?

2. Prove that the product of lower triangular matrices is lower triangular, and the inverse of a nonsingular lower triangular matrix is lower triangular.

3. Is $\|A\|^{-1} = \|A^{-1}\|$? If yes, prove this, if not, give a counterexample.

4. What is the maximal increase in bandwidth one can get for LU factorization with partial pivoting for a banded matrix? Modify the LU factorization algorithm without pivoting and with partial pivoting so that it does not do unnecessary work for banded matrices. Compute the operation count for LU factorization with pivoting, back and forward substitution.

5. Modify the LU factorization, back- and forward substitution codes posted on the class page so that they do not do unnecessary work for banded matrices (the new functions should take an additional parameter, bandwidth). Compute the operation count. In the curve optimization code, replace the linear solve using matrix inversion and substitution with the LU code. Plot the time it takes to solve the linear system using a) matrix inversion as in the original code b) LU factorization, followed by forward and backward substitution c) same as b, but using functions specialized to the banded case. In Python, you can use the function `clock()` in the module `time` to time the code. use sufficiently large system sizes (starting at 1000) to obtain meaningful results. Time LU separately from back and forward substitution. Use log-log coordinates for your plots, and determine (approximately) the slope, which corresponds to the exponent p in the operation count of the form n^p.

6. Modify the curve optimization code so that it solves equation $f'' = 0$ instead of $f'''' = 0$. How does the curve appearance change? Plot condition numbers for the original matrix and the new matrix. How fast the condition number for the new matrix grows with the number of points?