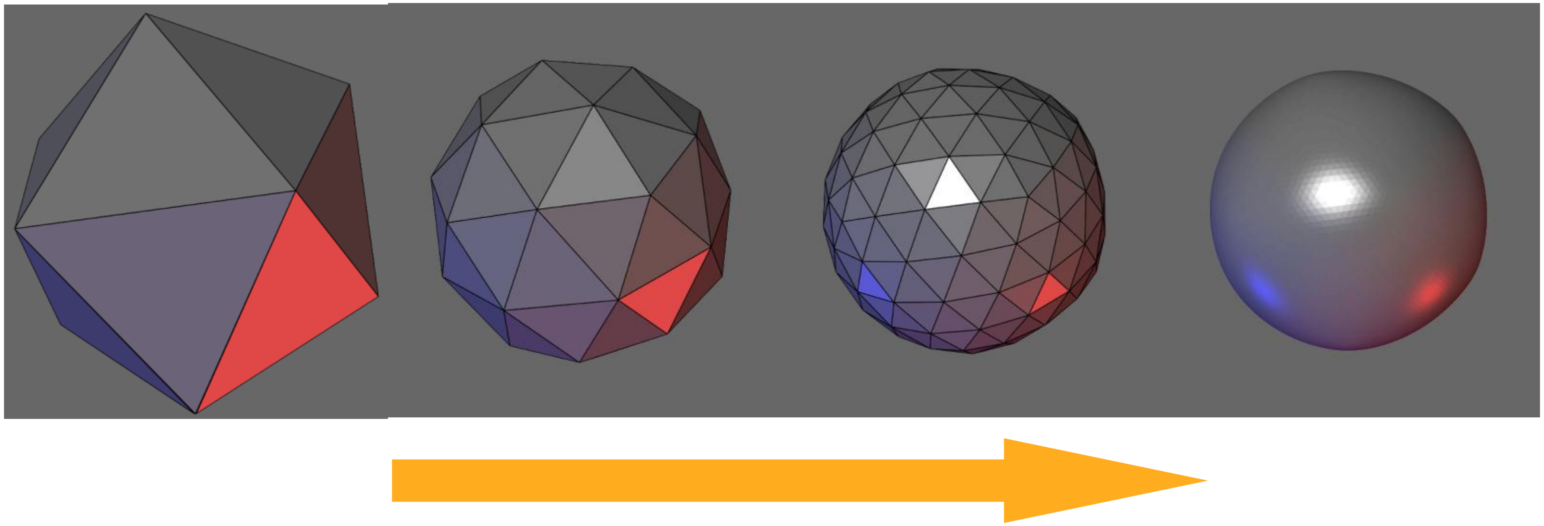
Example: Loop Scheme

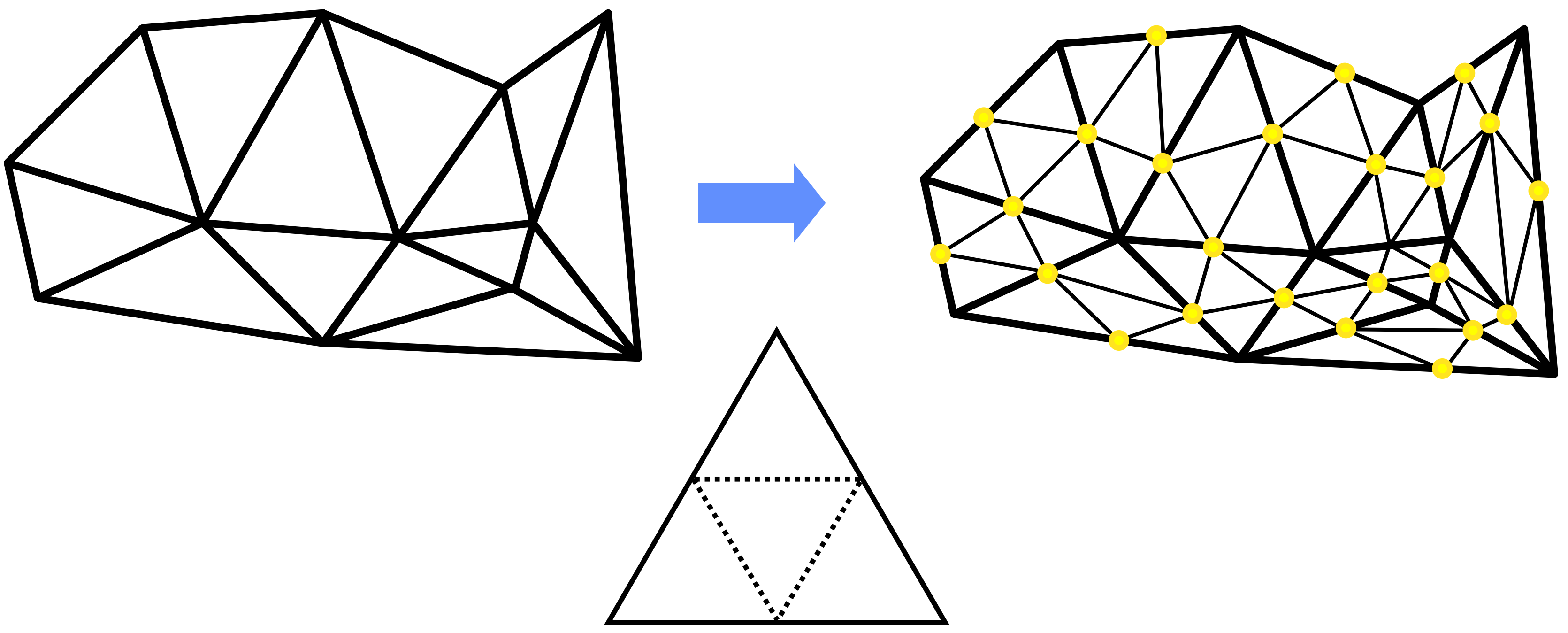
What makes a good scheme?



- recursive application leads to a smooth surface

Example: Loop Scheme

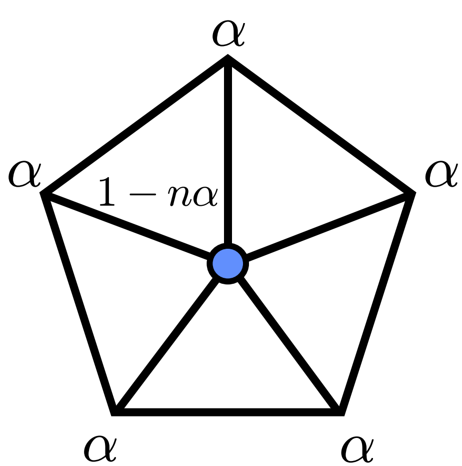
Refinement rule

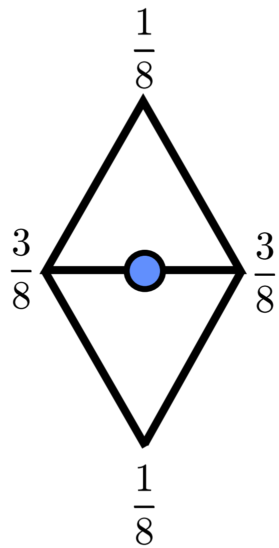


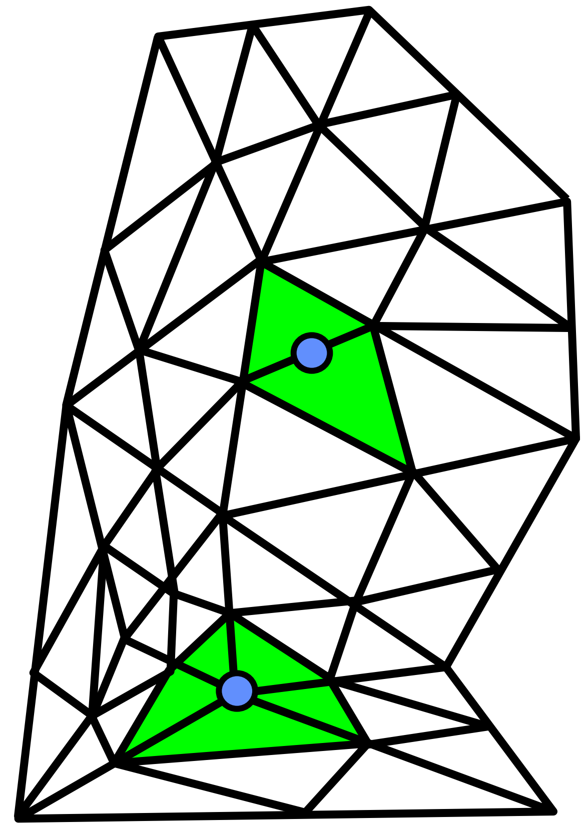
Example: Loop Scheme

Two geometric rules:

- **even (update old points)**
- **odd (insert new)**





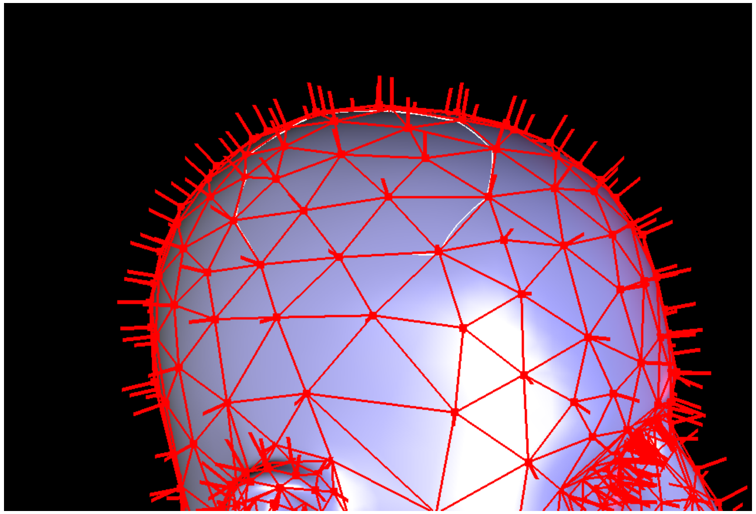


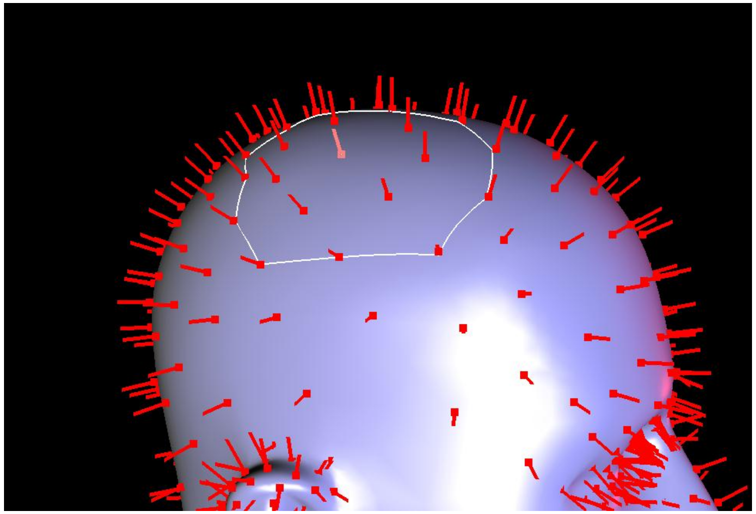
$\alpha = 3/8n$, $n > 3$, $\alpha = 3/16$, if $n=3$

Control Points

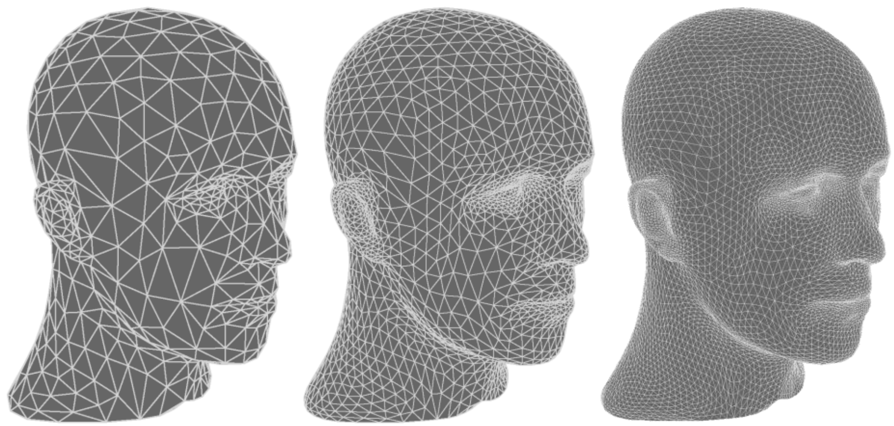
Vertices of initial mesh

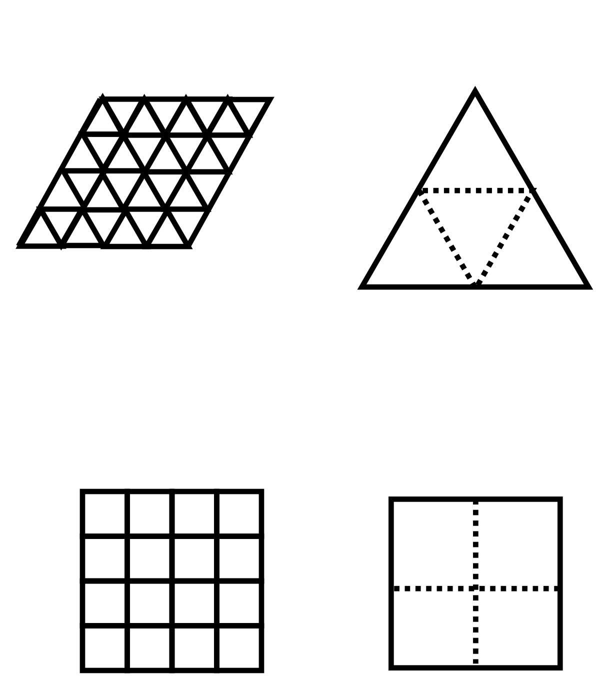
- define the surface
- each influences finite part of surface





Triangles and Quads

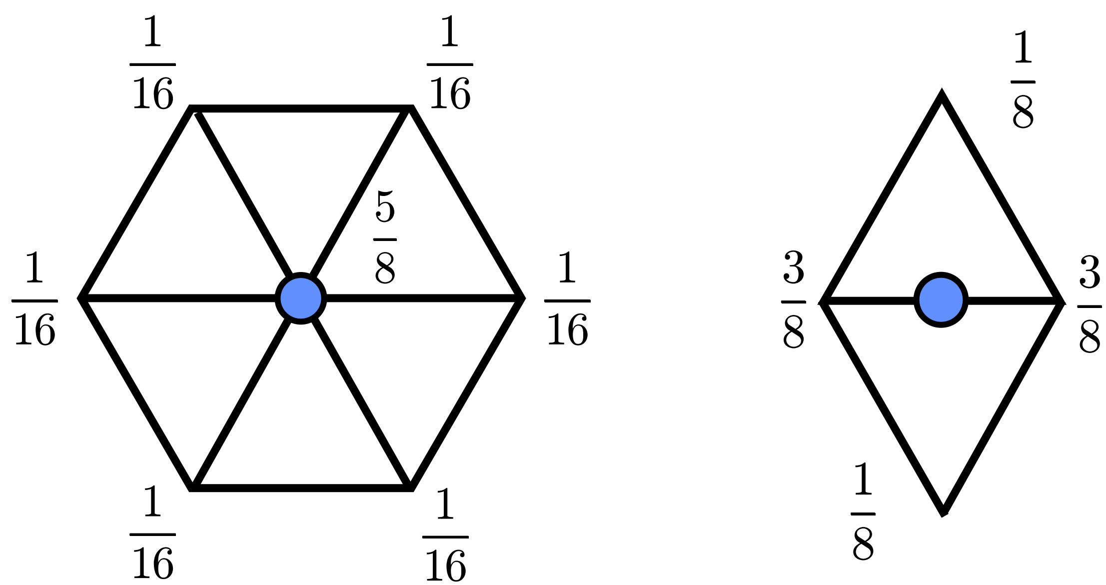




Subdivision and Splines

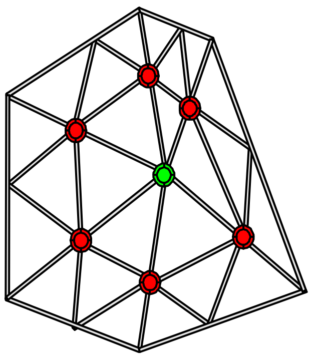
Uniform splines

- can be computed using subdivision
- quartic box spline rules:

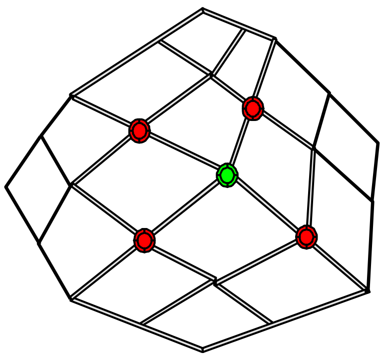


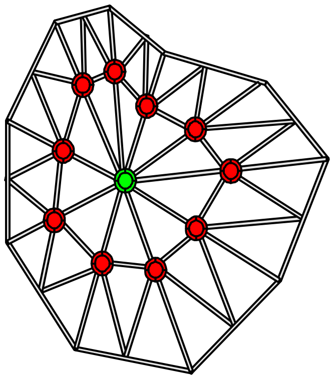
Extraordinary Vertices

Triangle meshes **Quad meshes**

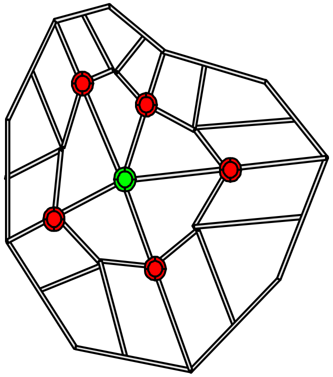


regular





extraordinary

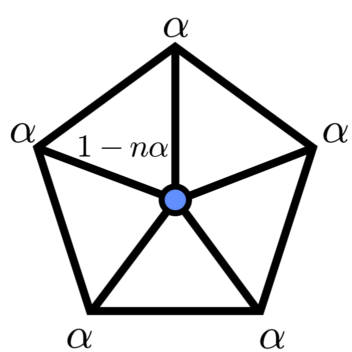


Constructing the Rules

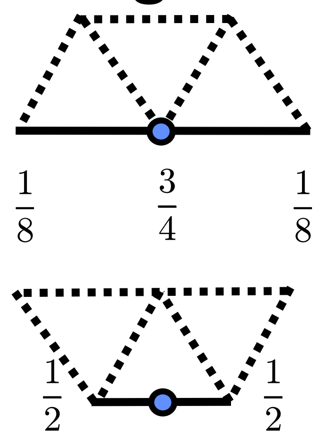
Start with spline rules

- **define rules for:**

Extraordinary vertices



Boundarie
s

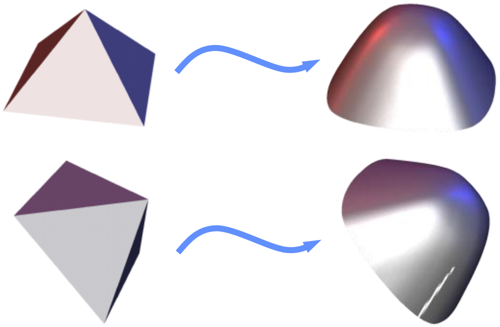


Creases etc.

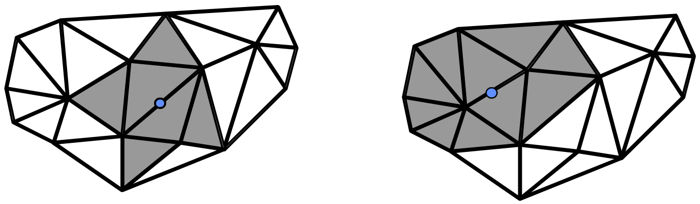
• • •

Constructing the Rules

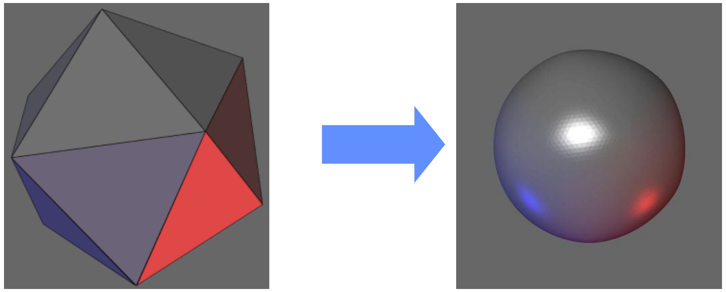
- invariance under rotations and translations



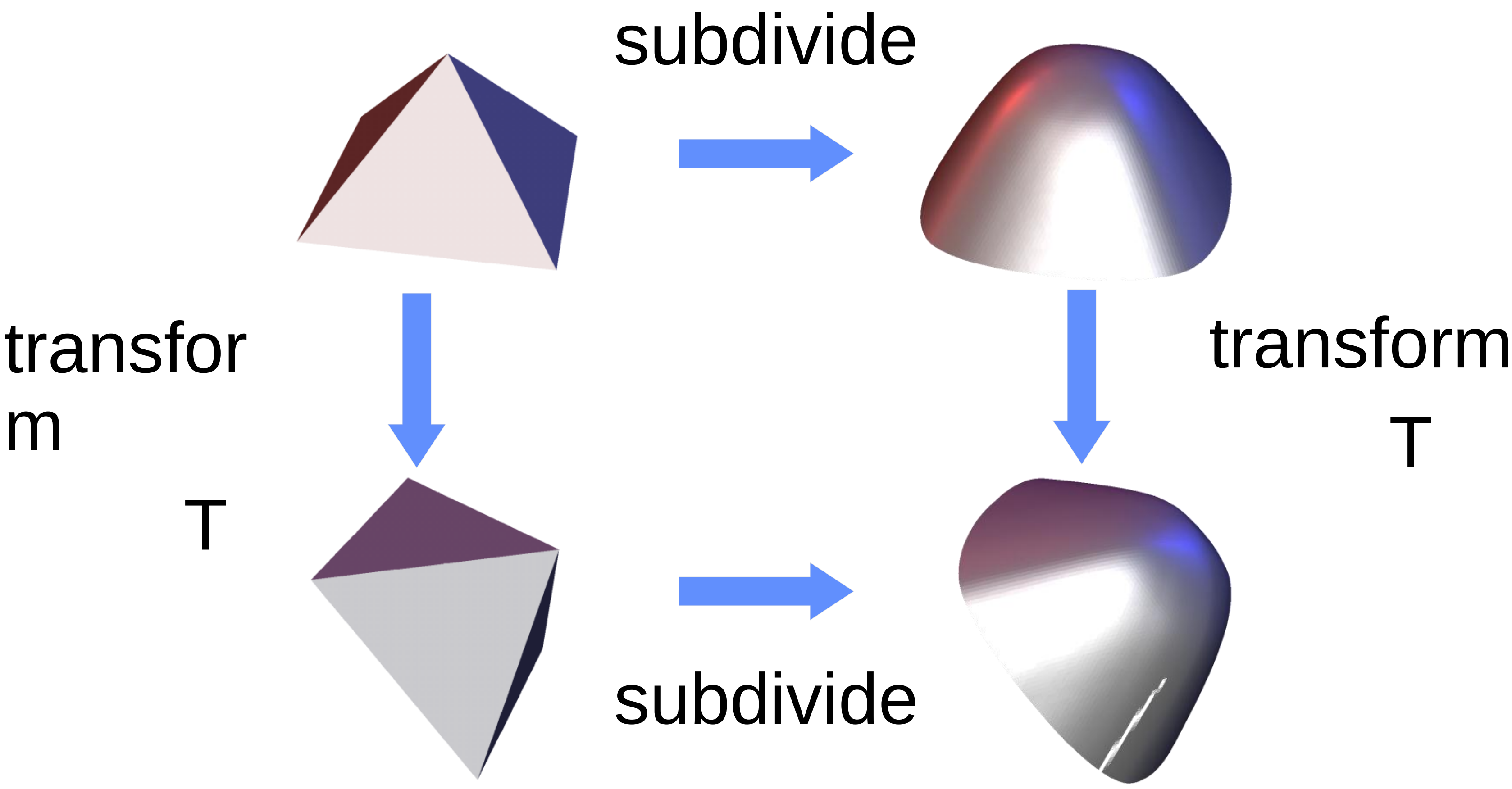
- small support



- smoothness and Fairness

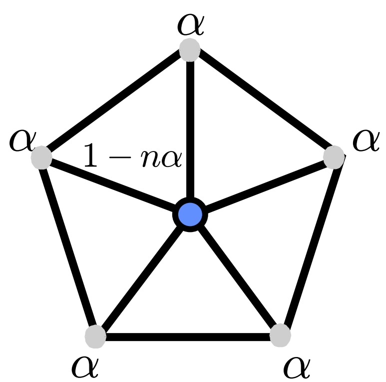


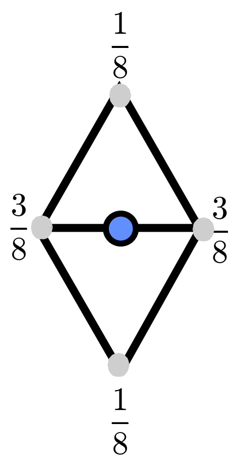
Invariance w.r.t rigid transforms



Invariance

Coefficients of masks must sum to 1



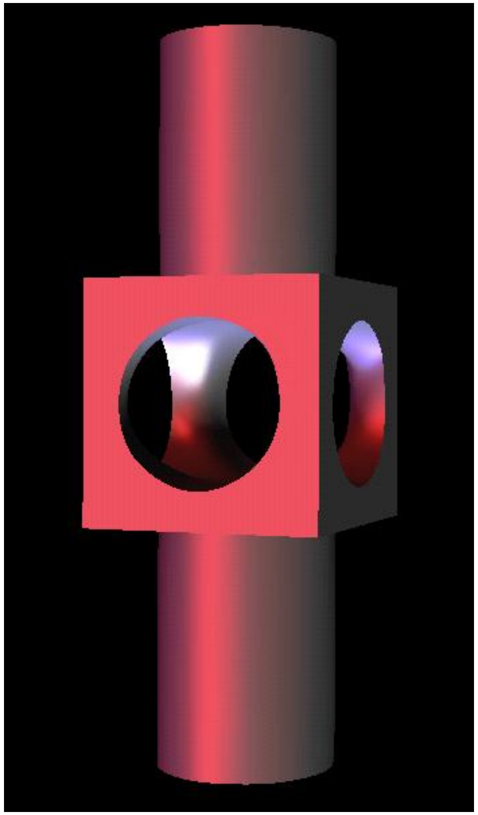


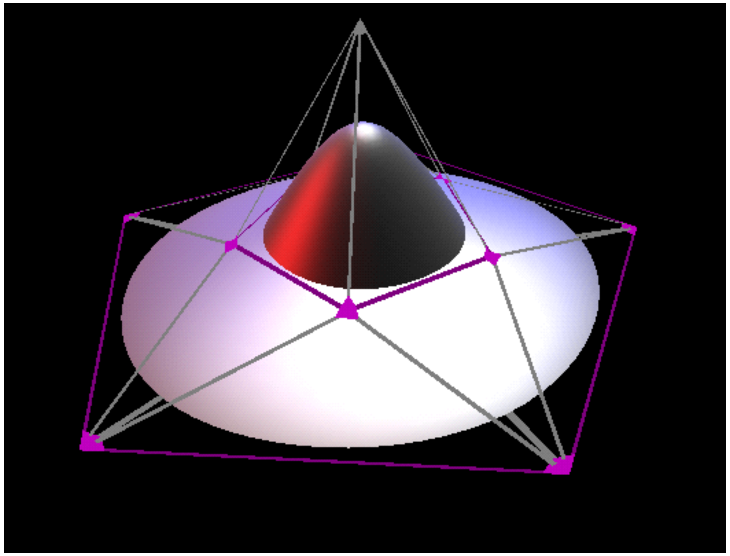
$$p = \sum a_i p_i$$

displacement

$$\sum a_i(p_i + t) = \underbrace{\left(\sum a_i\right)}_{1} t + p$$

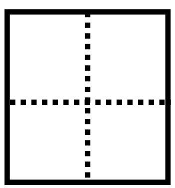
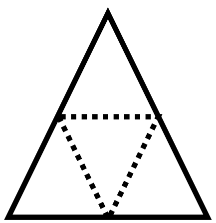
Crease Examples





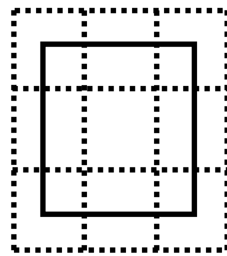
Subdivision Schemes

Primal

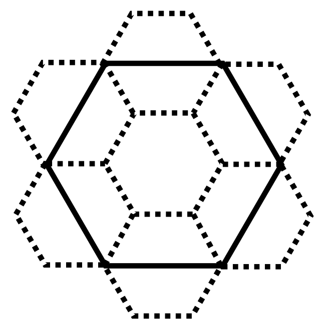
	Approx.	Interp.
	Catmull-Clark	Kobbelt
	Loop	Butterfly

Dual

- **(no interpolation)**



Doo-Sabin, Midedge

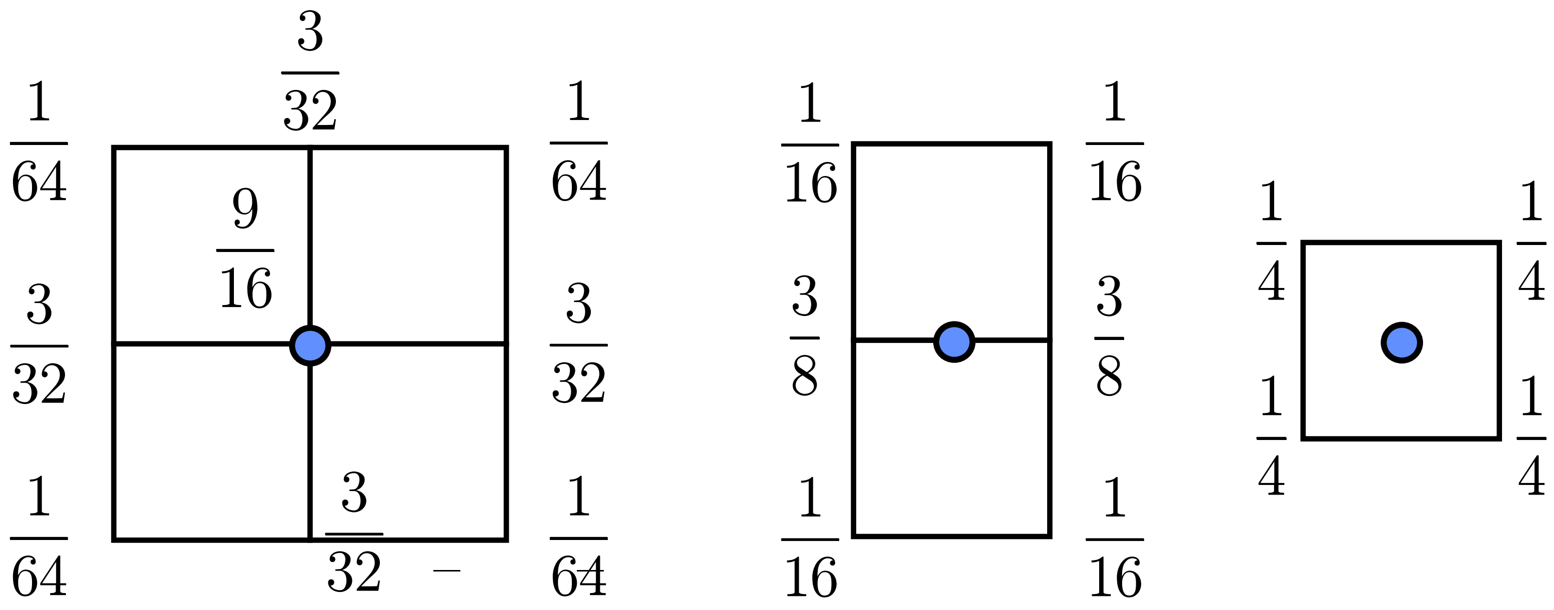


Dyn-Levin-Liu (non-linear)

Catmull-Clark Scheme

Primal, quadrilateral, approximating

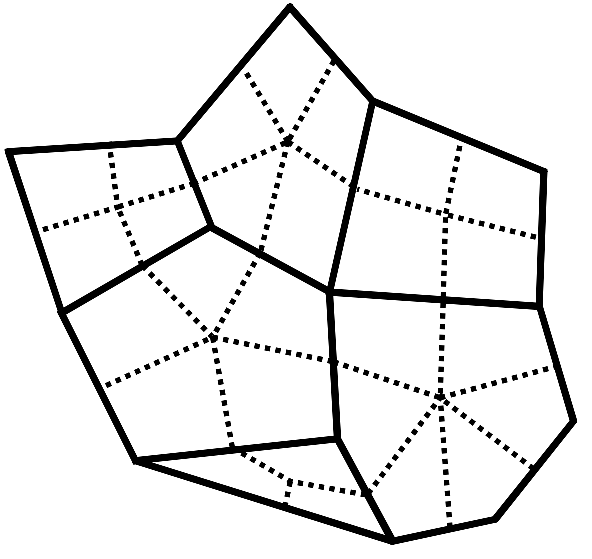
- tensor-product bicubic splines



Catmull-Clark Scheme

Reduction to a quadrilateral mesh

- do one step of subdivision with special rules: only quads remain

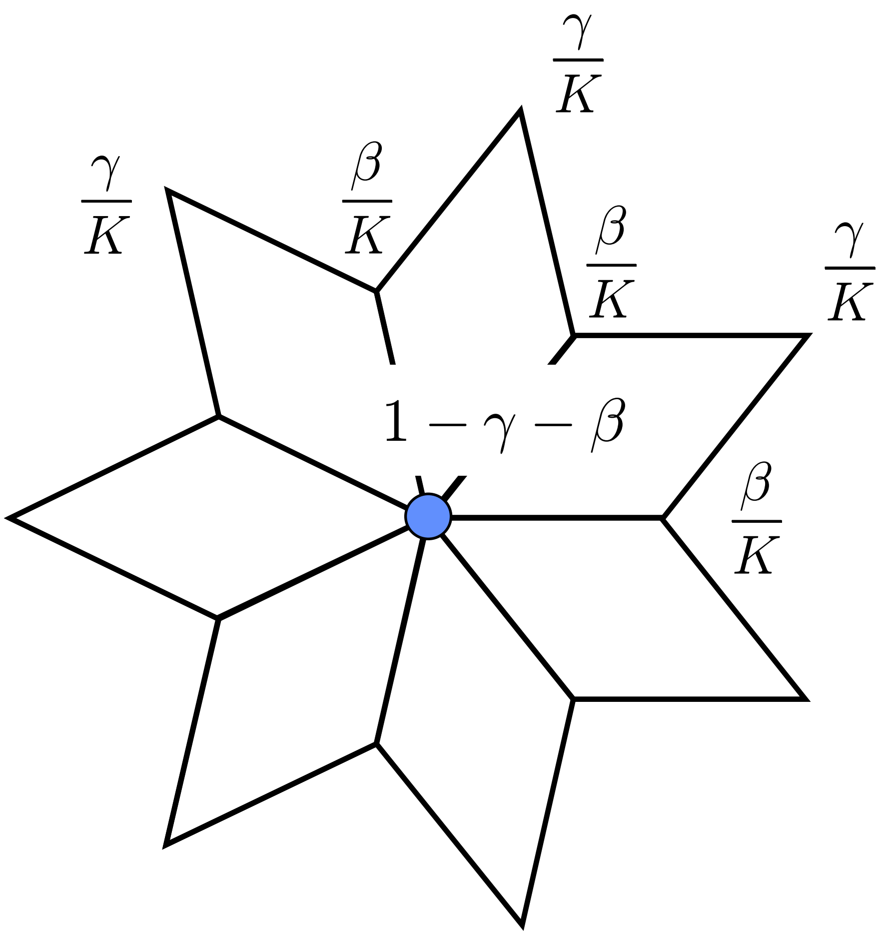


Catmull-Clark Scheme

Extraordinary vertices

$$\gamma = \frac{1}{4K}$$

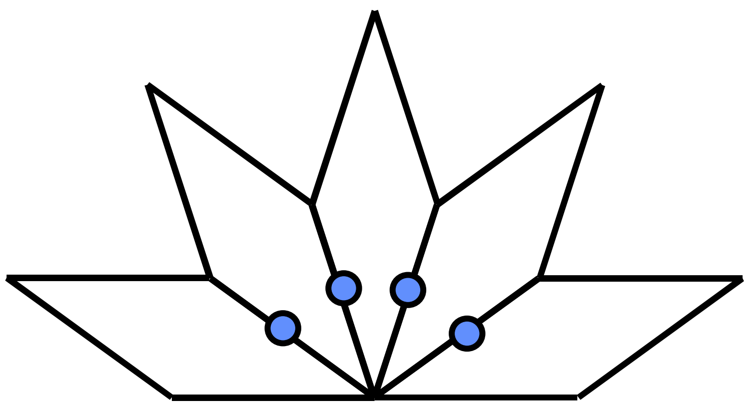
$$\beta = \frac{3}{2K}$$

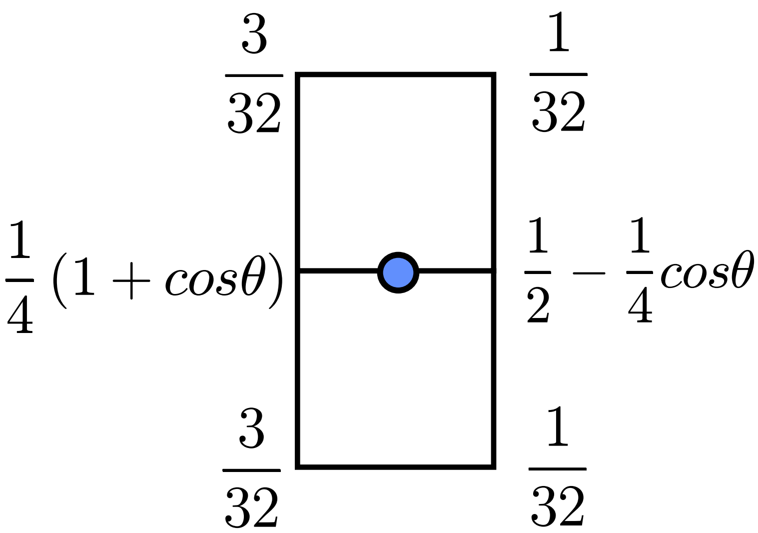


Catmull-Clark Scheme

Boundaries, creases, corners

- cubic spline (same as Loop!)
- need to fix rules for C^1-continuity





Implementing subdivision

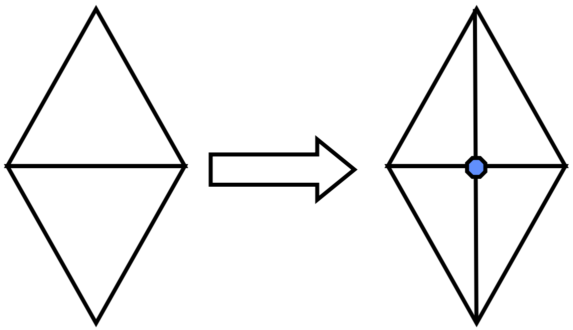
Operations needed:

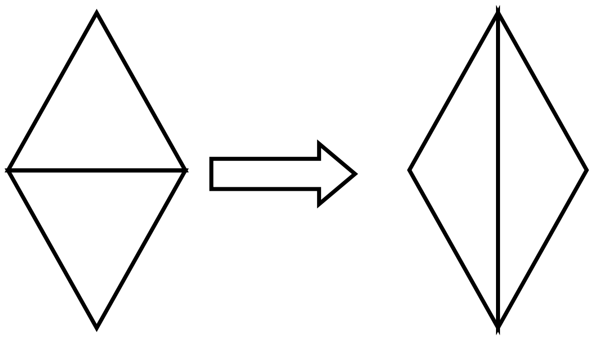
- **create a copy of the mesh maintaining vertex correspondence with the old mesh**
- **refine a mesh**
- **collect all neighbors of a vertex (for updating positions of old vertices, discussed at the last lecture)**
- **find vertices of two triangles sharing an edge (for computing positions of new vertices)**

Implementing subdivision

Uniform refinement

- can be achieved using two simple operations



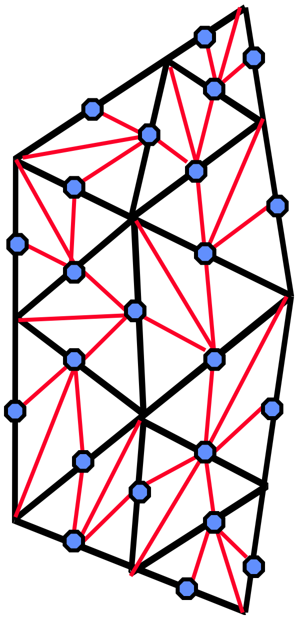


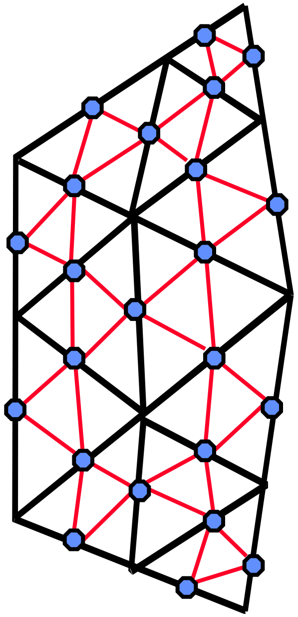
split two triangles adding a vertex

edge flip

Implementing subdivision

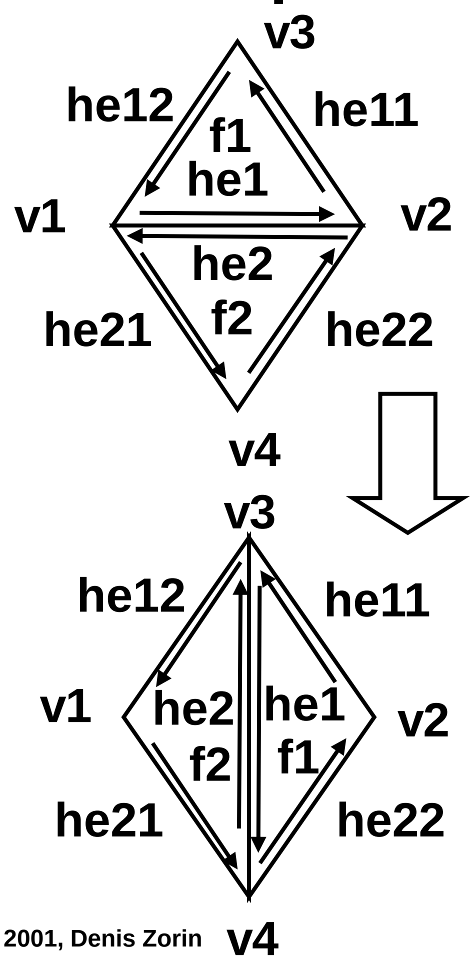
Step 1 (left): split all edges in any order, adding vertices for every edge and spit adjacent triangles in to two
Step 2 (right): flip all edges connecting an old vertex with a newly inserted one





Implementing an edge flip

Example: given a pair of half-edges he1,he2 flip the corresponding edge




```
he1.next = he22; he1.vertex = v4;
he2.next = he12; he2.vertex = v3;
he11.next = he1;
h12.next =  he21; h12.face = f2;
h21.next = he2;
he22.next = he11; he22.face = f1;
if (f2.halfedge == he22)
   f2.halfedge = he12;
if (f1.halfedge == he12)
   f1. halfedge = he22;
if(v1.halfedge == he1)
   v1.halfedge = he21;
if(v2.halfedge == he2)
   v2.halfedge = he11;
```

Building a half-edge data structure

Similar to building face-based triangular mesh

Input: a list of vertices, a list of faces, each face is a list of vertex indices enumerated CCW

1. Create arrays of vertices, faces and halfedges, one half-edge for every seq. pair of vertices of every face; initialize all pointers to zero.

2. For each face `f`**, with n vertices**

assign `f.halfedge` **to its first half-edge;**
for each vertex `v` **of a face, assign** `v->halfedge` **to the halfedge starting at it if nothing is assigned to it yet;**
for each half-fedge `he` **of a face, assign**
`he.face =f, he->next =`**next half-edge in the face,**
`he->vertex =` **next vertex in the face;**
record half-edge pointer he in the edge map:

```
edgemap(v[i],v[i+1]) = he
```

3. Go over all entries of the edge map, assign for half-edges `edgemap(i,j) edgemap(j,i)` **links to each other, if both exist**

Dealing with boundaries

To minimize implementation effort it is useful to create two halfedges for boundary edges, one of which has zero face pointer;

A boundary vertex `v` should always have `v.halfedge`

pointing to a boundary halfedge.

Then it is easy e.g. to find two boundary neighbors of a vertex.