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On the Computational Aspects of Gibbs–Markov Random Field Modeling of Missing-Data in Image Sequences

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***Abstract*— Gibbs-Markov random field (GMRF) modeling has been shown to be a robust method in the detection of missing-data in image sequences for a video restoration application. However, the maximum *a posteriori* probability (MAP) estimation of the GMRF model requires computationally expensive optimization algorithms in order to achieve an optimal solution. The continuous relaxation labeling (RL) is explored in this paper as an efficient approach for solving the optimization problem. The conversion of the original combinatorial optimization into a continuous RL formulation is presented. The performance of the RL formulation is analyzed and compared with that of other optimization methods such as stochastic simulated annealing, iterated conditional modes, and mean field annealing. The results show that RL holds out promise as an optimization algorithm for problems in image sequence processing.**

***Index Terms*— Gibbs–Markov random field, missing-data detection, relaxation labeling, simulated annealing.**

I. Introduction

This paper discusses the development of an efficient Gibbs–Markov random field (GMRF) model for the detection of missing data in image sequences [1]. The main application of the proposed model is the restoration of old degraded movies without causing distortions to nondegraded areas of image frames. The general procedure in the solution of a problem using GMRF modeling is to specify an objective function, $U(x)$, that quantifies the *a posteriori* assumptions of the problem at hand. A Gibbs distribution can then be specified as $P(x) = e^{-U(x)/T}$ where T is the global control parameter called temperature and therefore $P(x)$ is the *a posteriori* distribution.

The MAP estimation is carried out on $P(x)$ to determine the mode of the distribution. This mode represents the highest probability (lowest energy) solution of the problem. The MAP estimation is exceedingly tedious, especially as the number of unknowns in the objective function increases. The attempt at finding an optimal MAP solution is done via an appropriate optimization algorithm. There exist a number of optimization algorithms that are well studied in the literature. The commonly used algorithms [2] are stochastic simulated annealing (SSA), iterated conditional modes (ICM), and mean field annealing (MFA) [3]. In this paper, we investigate the use of relaxation labeling (RL) [4], [5]. RL was originally introduced for context and constraint modeling in labeling problems, and it has seldom been investigated for solving image-processing problems modeled by GMRF. Section II presents the GMRF model used for missing-data detection in image sequences. Section III outlines the formulation of the optimization algorithm using relaxation labeling. The performance of the proposed RL optimization algorithm is compared with other optimization algorithms and presented in Section IV with respect to quality and computational efficiency. Section V, summarizes the work presented in this paper.

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II. GMRF Model for the Detection of Missing-Data in Image Sequences

The detection of missing-data in image sequences is an important problem in image sequence restoration and the GMRF model has been used for the mentioned problem [1], [6]. Blotches in degraded movie frames exhibit the temporal discontinuity characteristic, which is commonly used for the detection of blotches. The detection model developed by Chong *et al.* [6] addresses the problem of incorrect detection due to poor motion compensation at the moving edges. It is found that the degree of incorrect detection (at the moving edges) in image sequences is reduced significantly by incorporating a moving-edge detector into the MRF model. This edge preserving MRF model [6] is adopted here for the detection of missing-data (blotches):

$$\begin{aligned} P(D=d|X=x) &= P(D=d)P(X=x|D=d) \\ &= \frac{1}{Z}\,\exp\Bigg(\frac{-1}{T}\sum_{i\in S}[-\beta_1+\phi(x_i,v_i))f(d_i) \\ &\qquad + (\beta_2+\phi(x_i,v_i))\delta(1-d_i) \\ &\qquad + \alpha(1-d_i)(x_i-x_{i+v_i})^2]\Bigg) \end{aligned} \tag{1}$$

where S denotes the pixel lattice of two adjacent frames from a sequence; x_i is the observed intensity at site i of the lattice S; D denotes the blotch detection frame which is to be estimated using MAP formulation; $d_i = 1$ indicates the presence of a blotch at site i and $d_i = 0$ denotes no blotch at site i; X denotes the observed image frame with the intensity of each pixel, x_i; x_{i+v_i} denotes the single motion-compensated neighbor pixel from the neighbor frame; v_i denotes the motion vector at site i which is computed using a multiresolution full-search block matching algorithm; $f(d_i)$ is the number of the four neighbors (first-order clique) of d_i with the same value as d_i; $\delta(\cdot)$ is the delta function, and function $\phi(x_i, v_i)$ is a moving edge detector. The moving edge detector $\phi(x_i, v_i)$ can be seen as a weighting function to alleviate the false detection of the temporal discontinuity that is due to poor motion estimate; the prior pdf will be weighted more than the likelihood function when a moving-edge is found. α, β_1, β_2 are the parameters used in the estimation.

An optimization algorithm will be required to solve (1). Nondeterministic (stochastic) optimization techniques such as SSA seek to maximize the *a posteriori* probability based on the controlling parameter (temperature) T. Deterministic optimization techniques such as relaxation labeling, on the other hand, would deal with the minimization objective (energy) function formulated as

$$\begin{aligned} U(D=d|X=x) &= \sum_{i\in S}\Big[-\beta_1+\phi(x_i,v_i))f(d_i) \\ &\quad + (\beta_2+\phi(x_i,v_i))\delta(1-d_i) \\ &\quad + \alpha(1-d_i)(x_i-x_{i+v_i})^2\Big]. \end{aligned} \tag{2}$$

III. Relaxation Labeling Formulation

The development of an MRF model for missing-data detection involves a discrete label set, where an optimization procedure is required to find the mode of the MAP configuration from this label set. Therefore the optimization procedure is combinatorial and it has been traditionally performed by using an algorithm such as SSA, ICM and MFA [2], [3], on the corresponding discrete solution space. The continuous relaxation labeling (RL) method is considered here for combinatorial optimization of the problem. Relaxation labeling can be considered a *constrained optimization* algorithm. The idea behind RL is the conversion of a discrete label set into a continuous one. However, RL is not readily applicable to combinatorial optimization and a proper conversion is necessary. This section shows how the original discrete-space combinatorial optimization problem can be converted into a problem of constrained real-optimization over a continuous-space. The continuous form is then solved by employing a continuous RL method such as in [4] or [5].

In RL, there exists a *certainty* for all possible labels at each site, where the certainty is a weight accorded to a particular label at that site. An initial labeling assignment is provided to all sites in the lattice, depending on the properties of the problem at hand. An iterative updating procedure is then employed to reach an *unambiguous* labeling at each site. This updating rule may take many forms, the classical one being that proposed in [7]. In the interests of computational efficiency, the updating rule proposed by Parent *et al.* in [4], will be employed here. Let $S=\{0,1,\cdots,m\}$ be a set of $(m+1)$ sites and $L=\{0,1,\cdots,M\}$ be a set of $(M+1)$ labels to be assigned to these sites. In RL the labeling state for each site $i\in S$ is represented by a $(M+1)$ position vector

$$p_i=[p_i(d_i)|d_i\in L] \tag{3}$$

subject to the following *feasibility* constraints:

$$\sum_{d_i\in L} p_i(d_i)=1 \quad \forall i \tag{4}$$

$$p_i(d_i)\geq 0 \quad \forall i,d_i. \tag{5}$$

The real value $p_i(d_i)$ reflects the strength or certainty with which i is assigned label d_i. The feasible labeling space is therefore a hyperplane in the nonnegative portion of the quadrant, defined by

$$\mathsf{P}=\{p|p_i(d_i)\geq 0, \sum_{d_i\in L} p_i(d_i)=1, i\in S, d_i\in L\}. \tag{6}$$

After updating, the final solution, $\breve{p}_i(d_i)$ should be *unambiguous*

$$\breve{p}_i(d_i)=0 \text{ or } 1 \quad \forall i,d_i. \tag{7}$$

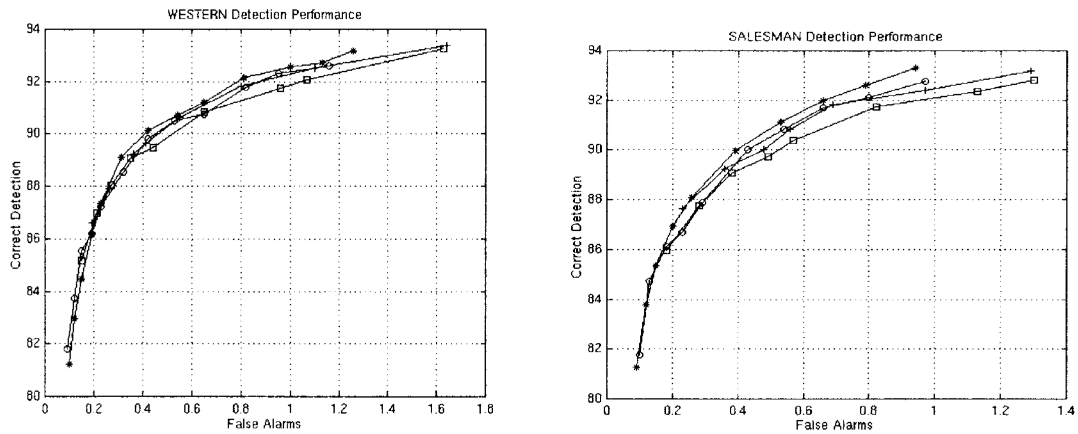
The space for the unambiguous labeling assignments can be considered to be the ‘corner’ points of the hyperplane P. Another interpretation of the labeling space is that it forms the convex hull of the unambiguous labeling assignment space P. The continuous RL can be regarded as an iterative process that maximizes the following gain function:

Fig. 1. Typical synthetically blotched image frames for the western sequence.

$$\begin{aligned} G(d_i) &= \sum_{i\in S}\sum_{d_i\in L} r_i(d_i)p_i(d_i) \\ &\quad +\sum_{i\in S}\sum_{d_i\in L}\sum_{(j\in S, j\neq i)}\sum_{d_j\in L} r_{i,j}(d_i,d_j)p_i(d_i)p_j(d_j) \end{aligned} \tag{8}$$

where $r_i(d_i)$ and $r_{i,j}(d_i,d_j)$ represent unary and binary *compatibility* functions. Higher order compatibility functions may be added. However, for most image processing problems, such as motion estimation and segmentation, second-order compatibilities suffice, as they can take care of up to second-order cliques. This is also suited to GMRF modeling. Once the gain function has been appropriately specified, an iterative updating process then maximizes this gain function based on the gradient $\boldsymbol{s}=\nabla G(\boldsymbol{d})$, where the gradient is with respect to $p_i(d_i)$. For the missing-data detection problem, the labeling set L is $\{0,1\}$, where 0 implies ‘nonblotch’ and 1 implies ‘blotch’ at each site i. Hence, the vector $p_i(d_i)$ consists of two components, $p_i(0)$ and $p_i(1)$. Though this is a simple application, the same approach can be easily extended to larger label sets for, say, motion estimation or restoration.

The RL optimization technique was compared with three other optimization schemes, namely, SSA, MFA and ICM. The SSA algorithm was developed using the Gibbs sampler [2]. For our experiments, the temperature-dropping schedule was set as $T\leftarrow 0.95T$; the initial temperature was set to ten. The number of temperature dropping iterations is set to 300. ICM is a deterministic algorithm based on maximizing local conditional probabilities. The algorithm is terminated when the number of changed labels in a particular iteration are less than 0.01% of the lattice, which corresponds to about six pixels for a 256 × 256 image. MFA is a deterministic optimization technique, which still retains many of the features of SSA. The initial assignment of the detection field D for the SSA, ICM, and MFA schemes is simply given by setting the labels at all sites to zero, i.e., $d_i=0$, $\forall i$. For the detection of missing-data, it is better to have a pessimistic initial labeling by setting all points



(a) (b)

Fig. 2. Detection performance for the four algorithms for (a) western and (b) salesman sequences. '*': RL. '+': ICM. '□': MFA. 'o': SSA.

to nonblotched. This prevents the accumulation of false alarms in neighborhoods. The equivalent is done for RL by setting $\vec{p}_i^0$ with components [1, 0], i.e., $p_i^0(0) = 1$ and $p_i^0(1) = 0$, $\forall i$.

IV. Results and Discussion

Experiments have been carried out on both real and synthetic image sequences that exhibit natural and simulated artifacts, respectively. The performance of the RL optimization algorithm is compared with that of the optimization algorithms outlined above (SSA, ICM, and MFA), in terms of solution quality and computational load. Solution quality for detection of missing data is determined by the *correct detection* and *false alarm* rates [1], which are known for the case of artificially blotched images.

Two sequences with synthetically added noise, were used to investigate the performance of the proposed RL algorithm. The two sequences used were a 16-frame 256 × 256 pixel resolution, WESTERN sequence and a 12-frame, 256 × 256-pixel resolution salesman sequence. The image sequences were artificially corrupted based on the Ising model and Gibbs sampler outlined in [1]. To make the problem more realistic, each pixel in an isolated blotch was randomly colored with gray level of variance 5. Fig. 1 shows the sample of the noise-corrupted western sequence.

Bidirectional image sequence processing is adopted here [8]. The motion vector fields are estimated using a three-level multiresolution full-search algorithm [9] on the degraded sequences. Boyce's checks [10] are incorporated to obtain robust vectors in the presence of noise. A search space of ± 4 pixels was employed at each level. Fig. 2 shows the detection performance of the four optimization algorithms, as a plot of correct detection rates versus false alarm rates. The results presented in Fig. 2(a) and (b) are obtained from an average of 16 and 12 frames for the western and salesman sequences, respectively.

The GMRF detector's characteristics were found by setting $\beta_1 = 50$, $\beta_2 = 15$ and $0.25 \le \alpha \le 5.0$. It can be seen that the proposed GMRF model with the RL optimization algorithm performs very well in general, and outperforms the other optimization algorithms. In fact the solution quality of the proposed RL optimization algorithm is slightly better or comparable to the performance of the SSA optimization algorithm. The temperature-cooling schedule of the SSA algorithm is conjectured to be the reason for the unexpected lowering of performance of the SSA optimization algorithm. ICM and MFA generally have performance lower than the RL and SSA counterparts. An important point to note is that at higher correct-detection rates, the difference in the false alarms becomes significant. ICM and MFA exhibit high false alarm rates for correct detection rates exceeding 90%, whereas RL and SSA stay relatively stable.

TABLE I
Time Taken on a Texas Instruments TMS320C40-50 MHz for the Various Optimization Algorithms to Converge

Algorithm	Ave. Time (s)	Ave. No. of Iterations Per Site	Approx. Floating Point Operations per Iteration
ICM	2.7	5	12
RL	2.8	5	15
MFA	7.7	5	30
SSA	100.0	9.1	120

The computational efficiency of an optimization algorithm and its solution quality are the key considerations for practical implementation. Table I (based on the western sequence, 16 frames) compares the computational costs of the various optimization algorithms considered, in terms of number of iterations and the actual processing time taken for the algorithms to converge. The exponential operation was taken as costing six multiplication operations. The ICM, MFA, and RL algorithms converge very fast, after just five iterations. In contrast, the number of iterations for SSA with the same specified set of parameters is approximately two times that of the ICM, MFA or RL algorithms. It can be concluded that RL is a promising solution to the GMRF problem as the computational efficiency of RL is comparable to ICM while the RL solution has the quality comparable to a SSA algorithm.

V. Conclusions

RL is essentially *parallel* in nature, as updates can be performed locally, with communication required only to update label values at the end of each iteration. This is very suited to a parallel SIMD implementation. One of the important factors that needs to be taken into account is the need for tuning the annealing schedule in the SSA and MFA algorithms, which is crucial to the performance of these algorithms. In contrast, RL and ICM do not require any kind of heuristic tuning. This is a major advantage. RL holds out promise as a cost-effective solution for motion estimation and other more complex image processing algorithms employing GMRF modeling.

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Adaptive Approximation Bounds for Vertex Based Contour Encoding

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***Abstract*—When approximating the shape of a region, a fixed bound on the tolerable distortion is set for approximating its contour points. In this work, an adaptive approximation bound for lossy coding of the contour points is proposed. A function representing the relative significance of the contour points is defined to adjust the distortion bound along the region contour allowing an adaptive approximation of the region shape. The effectiveness of the adaptive contour coding approach for a region-based coding system is verified through experiments.**

***Index Terms*— Contour approximation, region-based coding, vertex coding.**

I. INTRODUCTION

To circumvent the shortcomings of block-based image coding algorithms, the region-based image coding approach was introduced [1]. A major drawback of the region-based coding, however, is that the shape of each region (called the *contour*) has to be explicitly encoded. The amount of contour information occupies a significant portion of the coding rate, and the proportion becomes even larger as the required rates decrease. In this respect, the coding of contour information has attracted much attention with greater emphasis on the lossy coding of contour points for a higher coding gain [1]–[4].

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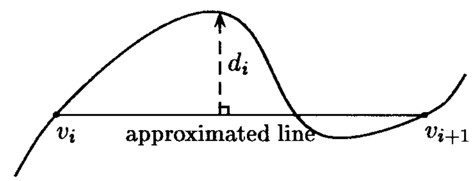

Fig. 1. Geometric distortion.

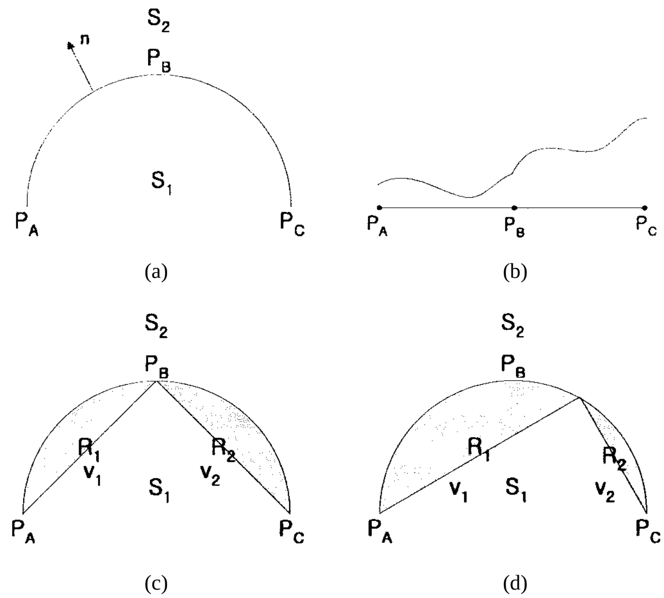

Fig. 2. Approximation error.

There are many approaches to approximately represent the shape of objects [5]: Contour based approaches such as polygonal approximation, and bitmap-based approaches such as context-base arithmetic encoding (CAE) [6]. Polygon approximation is one of the most widely used methods in region based coding due to its scalability with natural looking approximation and the fact that the bit-rate can be controlled by adjusting the degree of approximation. In this method, a contour is approximated by straight lines, and only those points corresponding to the endpoints of the lines (called *vertices*) need to be coded. There are many methods available for selecting the vertices, but regardless of the method employed, an upper bound on the tolerable error has to be set to control the degree of approximation accuracy. A distortion criterion is needed to set a fixed bound on the tolerable error between the original and the reconstructed region. The most popular criterion is the geometric distortion. Using the geometric distortion, the distance between a point on the original contour and the nearest point on the approximated contour must be shorter than a predefined threshold. It is very simple to implement, but fails to take into account the spatio-temporal properties of the region. Thus, the geometric distortion criterion does not reflect the actual error of the reconstructed region. Another distortion measure one can use is the mean square error (MSE) between the original region and the reconstructed region. Though this criterion is more adequate