Automating Commonsense Reasoning for Elementary Physical Science

Proposal Submitted to the NSF, “Robust Intelligence.”

Ernest Davis, Courant Institute, New York University, New York, New York

Project Description

1 Introduction

A fundamental part of human experience is the interaction with physical materials of many different kinds: rigid solids, non-rigid solids of many different kinds (e.g. cloth, string, rubber bands, springs, gels, and so on) liquids and gasses. The rich pre-scientific understanding of these materials acquired by every child at an early age is critically important, both because it enables a person to deal effectively, in daily life and in more specialized activities, with a world full of these materials, and because this fundamental physical understanding serves as a grounding for more sophisticated knowledge of many kinds.

The implementation of this body of knowledge in an intelligent knowledge base and the analysis of the knowledge that would be entailed in this implementation would have immense value for many different purposes:

Robotics: An autonomous robot which needs to deal flexibly and sensibly with an uncontrolled environment containing these materials needs to understand them. This would include robots that work in a household; robots that work in complex and unpredictable industrial setting; robots that work in unusual and distant environments (undersea, Mars, etc.); and robots that work on medical applications (in order to understand the materials of the body and of the medical tools.)

Automated instruction: Interactive instructional computer programs could be made much more powerful and effective if they could draw on this body of commonsense physical knowledge. These could potentially be instructional programs for consumers (e.g. for using a sewing machine or for applying emergency first aid); for students, especially in physical science courses; or for workers.

Product and process design: The knowledge base could be used to design or validate tools constructed out of these materials, tools that operate on these materials, and techniques for using such tools.

Science knowledge base: An ambitious long-term project, called Project Halo, is underway to encode scientific knowledge in a knowledge base, the Digital Aristotle (Friedland et al. 2004a, 2004b). The first stage of this project encoded the knowledge in about a chapter’s worth of an introductory college chemistry textbook (Brown, LeMay, and Bursten 2000) and achieved a fair degree of success; the three competing systems developed achieved about the mean human score on questions in the area from the high school AP chemistry test. However, the subject matter in this first stage was carefully chosen to avoid the issues of spatial reasoning and of commonsense reasoning (Friedland et al. 2004a). But these issues obviously cannot be evaded forever if the project is to attain any reasonable coverage.

Grounding for science and mathematics: Experiences of simple physical interactions are a major epistemological grounding for the understanding of basic physical science and mathematics, directly (concepts of physical science correspond to elements of experience); as explananda (scientific theories explain commonsense experience); metaphorically (e.g. imagining atoms as hard round balls); and as contrasts (e.g. understanding the differences between atoms and hard round balls.) A first attempt at studying the grounding of mathematics in physical experience is in (Lakoff and Nuñez, 2001); however, their account is flawed and inadequate (see my review in (Davis, 2005b)).

Testbed for knowledge representation research: The domain of elementary physical reasoning is a testbed for research in knowledge representation and automated reasoning that is appealing, easily understood, and remarkably rich. It involves many of the central issues of knowledge representation, including:

- Taking knowledge that is "intuitive" and hard to verbalize, and representing it explicitly.
- Integrating commonsense and expert knowledge; spatial and symbolic knowledge; deterministic and probabilistic knowledge.
- Reasoning at multiple level of abstraction.

Thus, a fully-developed theory of commonsense physical reasoning and its relation to spatial reasoning would be a central conceptual infrastructure in the achievement of robust intelligence in programs for many different kinds of applications and in the development of a cognitive theory of the understanding of science and mathematics; and a important source of insight and experience in the development of automated reasoners for other domains.

2 Proposed Project

Our proposed project (named SOPHY) is to develop a knowledge base encoding basic knowledge about the physics of solids, liquids, and gasses. Our specific target is to express most of the macroscopic-level knowledge and reasoning (as opposed to the knowledge and reasoning at the molecular/atomic level) in chapter 10 of the freshman chemistry textbook *Chemistry: The Central Science* (Brown, LeMay, and Bursten 2000).[1]

The examples in this chapter illustrate seven different devices and experimental set ups: mercury barometers, closed end manometers, open end manometers, Boyle's U-tube experiment, gas collection over water, gas in a piston, and containers of gas joined with a stopcock. To these, we plan to add some further simple devices: boxes, shelves, doors, locks, jars, pipes, valves, and simple pumps.

The adequacy of our representation and our inference techniques will be demonstrated by the ability to carry out different types of reasoning, using more and less complete physical specifications and at higher and lower levels of abstraction, over these examples and variants of these examples.

3 Variants

As we have argued in (Davis 1998), the degree to which a reasoner understands a physical system is well tested by its ability to understand *variants* of the system: greater or lesser modifications of the system which may make no change in its behavior, a small change, or completely alter its behavior. (Similarly McCarthy (1997) has proposed the idea of *elaboration tolerance* as a measure of the adequacy of a representational system.) For example, in example 6 of a gas in a piston, one might ask how the behavior of the system would change if:

1. The piston had a small dent or bump in its top or bottom surface.
2. The piston had a small hole drilled through it from top to bottom.
3. The piston had a small hole drilled through it with both outlets on the bottom.
4. The cylinder were conical (a) narrowing inward; or (b) widening inward.

---

[1] This particular textbook was chosen because it was the one used in the Halo project.

5. The cylinder were open at the end opposite the piston.
6. The cylinder had a hole in the side.
7. The gas were replaced by a liquid.
8. The gas were replaced by a solid.
9. The cylinder contained some solids, of specified shapes and initial configuration, in addition to gas.
10. The system was placed horizontally rather than vertically.
11. The cylinder had a piston at each end.

Similarly, at the outset of the project, we will define several hundred variants of our sample scenarios (Davis 2005c). If our system can reason about all of them, it will have attained a substantial measure of flexibility.

4 Types of reasoning

We plan to support four fundamental types of reasoning over these physical systems:

1. **Plan projection.** Given the starting state of a physical system and containing an agent (or agents) and a plan of action for that agent, determine whether the plan is executable and what its effect will be. An important special case is **autonomous projection**, in which an inanimate physical system evolves on its own, with no agent.
2. **Comparative projection**. How a change in the conditions of a problem affects the outcome. For example: the denser the liquid in a barometer, the lower the column of liquid will be. The smaller the hole in a container, the more slowly liquid will leak through.
3. **Planning**. Given the starting state of a physical system containing an agent, and a task to be carried out, find a plan of action that will accomplish the task.
4. **Safety (unattainability)**. The purpose of many physical system is to ensure that specific states cannot be attained. An object in a closed box cannot come into contact with objects outside the box. An agent outside a locked door cannot go through the door. This is an issue that is rarely addressed in AI planning systems; most "complete" planners, when given a planning problem with no solution, generally go into an infinite loop, looking for longer and longer plans. (They are "complete" in the sense that if a plan exists, they will eventually find one.) It is, however, an important objective in the verification of software systems and computer security, so there may be techniques developed for those applications that can be used here.

We do not at all intend to come up with complete algorithms for any of these reasoning tasks; rather, we plan to develop methods that work over some class of interesting simple cases.

5 Material properties

We will use a simplified physical theory, just rich enough to support reasoning about the examples listed in section 2 — it will be complex and challenging enough! (The sketch below should be taken as approximate; it may prove to change the details.)

Material exists in three states: Solid, liquid, and gas. Solid objects are modelled as strictly rigid in shape. Different objects and different parts of a single object may be made of different materials; different materials differ in their density (perhaps also in other properties). The coefficient of friction between any two solid objects is taken to be 1/3 in all cases.

Liquids are taken to be incompressible (or perhaps infinitesimally compressible). We ignore such issues as viscosity, surface tension, adhesion, cohesion and mixtures. Different types of liquids are assumed to differ in their density; we assume that in an equilibrium state these will strictly separate out.

Gasses are taken to be ideal gasses. In an equilibrium state, a gas fills a topologically bounded region whose boundaries are all surfaces of solid objects, liquids, and the outer limits of the atmosphere. Different gasses may have different densities; however, it is assumed that, in an equilibrium state, any two gasses within a closed region mix uniformly. The pressure within a gas is a continuous function of space; in an equilibrium state, pressure is constant within a closed region.

Temperature changes continuously in space throughout a material; it may change discontinuously where two objects come into contact. Each material has a fixed thermal conductivity and thermal capacity; thus, temperature takes some finite time to propagate. Solid objects do not expand under heating (since they are rigid); liquids have a fixed coefficient of thermal expansion; gasses obey the ideal gas law. We shall not be much concerned with any kind of accurate modelling of thermal disequilibrium. We assume that there are objects (e.g. hot plates) whose temperature can be changed as a result of a mechanical motion; this is the hook an agent can use to change the thermal state of a system.

All experiments are assumed to be carried out in a small-scale terrestrial setting with a constant gravitational field, with the earth below and the atmosphere above.

Nature, in our theory, does not abhor a vacuum; e.g. the top of a barometer, above the level of the mercury, is assumed to be a true vacuum.

We omit many phenomena that are well-known and understood even within a commonsense understanding such as phase change; mixtures and solutions; and non-rigidity of solid objects. The above will be more than enough to deal with.

6 Partial Geometric Specification

The most important difference between the SOPHY project and all but a handful of the vast numbers of existing programs that do physical projection is that we focus on cases where the problem characteristics, especially the geometry, are specified only partially or abstractly. That is, almost all physical reasoning programs that deal with geometry at all assume that precise geometric and material characteristics are given in the problem specification. Given such precise boundary conditions, the programs can then make precise predictions. In many important applications, however, geometrical specifications are not given, either because

- Perception gives only partial information (e.g. significant parts of the objects involved are occluded.)
- The geometrical information has been inferred (e.g. from perceived physical behavior.)
- The geometrical information derives from natural language text, which rarely gives complete geometrical specifications; or from diagrams, which are often (deliberately) topologically correct but not otherwise geometrically precise.
- One wishes to make an inference about all objects in a class, not just about individual objects. E.g. one wants to infer that standard air pressure cannot support more than 39 inches of mercury, regardless of the shape of the column.

- One wishes to reason about the behavior of a physical system while one still in the middle of designing that system, and before one has fully specified the geometry of the objects.

Our aim is to develop reasoning engines that can use geometrical specifications across a wide range of detail and precision:

1. Exact. E.g. "The piston is a right circular cylinder 3 cm in radius and 2 cm thick. The interior of the cylinder is a right circular cylinder, 3 cm in radius and 6 cm long."
2. Within tolerance. E.g. "The piston is as in (1) but may have dents or bumps on the top or bottom surface up to 0.2 cm in depth."
3. Constrained in dimension or shape. E.g. "The interior of the physical cylinder is a geometric generalized cylinder. The horizontal cross section CH is constant and roughly circular; specifically, it is convex, it contains a circle of radius 2.9 cm and is contained in a circle of radius 3.1 cm. The vertical axis is between 2 cm and 3 cm long. The piston is a generalized cylinder that fits tightly around the piston; the axis of the piston is between 6 cm and 8 cm."
4. Purely qualitative. E.g. "The piston fits tightly inside the cylinder. The piston can move uniformly, with one degree of freedom, relative to the cylinder, within a fixed range. In each attainable position the union of the cylinder and piston is a shell containing a closed cavity. The volume of the cavity changes monotonically with the position of the piston."

Moreover, different parts of an object can be described at different levels of detail or abstraction. The same levels of description apply to representing the relative positions of different objects.

Since calculating with exact geometrical descriptions is very well understood, and reasoning with purely topological description has been fairly extensively studied (Randell, Cui, and Cohn, 1992; Kim, 1993) and generally supports only rather weak physical conclusions, our focus will be to work with the middle levels of representation: spatial representations that are not precise, but give some metric and shape information beyond the purely topological. Again, we are not looking for complete algorithms (reasoning with partial geometric specifications is almost always badly intractable or undecidable in the worst case) but rather looking for partial methods that give useful results in simple examples.

The representation of an action can generally be at the same level of abstraction as the given shape description or at a higher level but, generally, not at a lower level. For example, it makes sense to specify that the hand goes into a box whose shape is given precisely, but it is not useful to specify an exact trajectory for a hand among objects that are specified only in terms of their topological relations.

7 Abstracting transitory states

The behavior of physical systems often goes through highly complex transition states before setting into a state of equilibrium. For instance, when liquid is poured into a container, its shape and motion while being poured is extremely complicated and unstable with respect to small perturbations, whereas its final state – sitting at rest at the bottom of the container under a horizontal top surface – is extremely simple. There are three standard methods of projection for this problem: first, to ignore these transitional states altogether, and posit atomic transitions from one equilibrium state to the next; second, to view the pouring as a process that continuously reduces the volume in one container and increases it in the other, with no account of how this is mediated; and third, to use partial differential equations or simulation to trace the way in full detail through all the intermediate stages. The first two, though often useful abstractions, are too abstract for many kinds of commonsense reasoning (e.g. predicting what happens if an object is placed into the stream of liquid); the third is unusable in the context of commonsense reasoning, since it requires precise information of geometry, timing, and material characteristics, and it gives output that is too detailed, unreliable, and unstable. What we need is a representation and a mode of reasoning somewhere between the second and the third; that constrain the behavior during transitions, so that one can be sure that nothing too strange is happening, while not requiring that this behavior be worked out in full detail

8 Manipulation and Action

Our base level model of action is that one or more solid objects are directly controlled by the agent, and that the agent can either move them along a desired trajectory, or use them to exert a force against an object that is in contact. The objects being controlled may either be the robot's own manipulators or, more abstractly, the tools he is working with; for example, a robot can either think of moving its hand so as to pick up a beaker, or, more abstractly, it can think simply of moving the beaker, without envisioning its hand.

9 Abstraction

There are many different kinds of abstraction that can be applied to these kinds of physical scenarios and physical problems (Davis, 2000d). Several kinds of abstraction have been mentioned above. Some further examples include:

- A jar with a tight lid can be abstracted as a single object with an internal cavity as long as nothing occurs that will take off the lid. Or the jar and its contents can be further abstracted as a single solid mass with no internal cavity if one is not concerned with the behavior of the contents within the jar or with features like the moment of inertia.
- A simple hinged valve can represented in full detail, giving the shapes of the object parts involved in constructing the hinge. Or the hinge can be abstracted as a constraint that the valve is connected to the frame along a fixed axis. Or the entire valve can be further abstracted as an abstract condition of either being closed or open.
- Dimension reduction: The behavior over time of a three dimensional systems can often be adequately characterized in terms of a fixed three dimensional structure plus a collection of one-dimensional time-varying parameters; or, in many cases, still more abstractly, as a fixed two- or one-dimensional structure plus a collection of one-dimensional time-varying parameters.

These abstractions can be applied independently. An effective reasoner needs to be able to choose the appropriate abstractions, to reason at any level of abstraction, to understand high-level reasoning in terms of the low-level theory and vice versa, and to combine high-level and low-level reasoning together.

10 Difficult issues avoided

By focussing on scenarios from simple physics experiments, the SOPHY project will be able to avoid several of the most difficult issues in commonsense physical reasoning including reasoning with uncertainty and reasoning about ensembles of solid objects, such as chains or heaps. Controlled experiments in physics tend to be set up, when possible, to avoid situations with uncertain outcomes. Ensembles don't happen to arise in the experimental set ups we are considering.

11 Research plan

Our plan of research is two-pronged; working on algorithms and implementation for the simpler and better understood parts of the problem, and working on theory, particularly representation and inference, for the more abstract and less well-understood parts of the problem. The key parts of our theory and system are as follows:

Theoretical:

1. A representation language capable of expressing the knowledge and the problems in this domain. Since, in this project, we are avoiding probabilistic reasoning and reasoning about ensembles, and intensional operators don't arise much in physical domains, first-order logic without set theory should suffice. Ontologically, we will take time to be real-valued and space to be Euclidean space. A major aspect of the design of this language (indeed, of the entire SOPHY project) is to design the spatial sub-language so that it is expressive enough to support the problems we need but restrictive enough to permit efficient inference for simple problems.
2. A set of rules, expressed in the representation language (1), sufficient to capture the knowledge we want to express and to justify the inferences we want to carry out.
3. A meta-level theory of abstraction that characterizes the use and relation of multiple levels of abstraction. There is an extensive AI literature on abstraction (Ellman and Giunchiglia, 2005), both domain-independent (e.g. Nayak and Levy, 1995; Giunchiglia et al., 1997) and specific to physical reasoning (e.g. Choueiry, Iwasaki, and McIlraith, 2005; Davis, 1995a) that we can draw upon; but the new domain will undoubtedly raise new issues and demand new techniques, at both levels.
4. Algorithms for the tasks of projection, comparative projection, planning, and safety evaluation.

Implementational:

5. A knowledge base implementing the rules in (2). We will probably use an existing open-source knowledge base architecture, such as Otter (Kalman, 2001), supplemented with special purpose modules.
6. Meta-structures over the knowledge base (5) that express the meta-theory of abstraction (3).
7. Implementation of the algorithms (4).
8. Explanation generation. Since the algorithms are operating over a knowledge base, it should be possible to generate a crude explanation in the form of a proof structure.
9. User interface.

It is difficult to project how fast the various parts of this will progress, but an approximate (probably overly optimistic and overly systematic) time line for the next seven years might be as follows:

6/05-5/06: Develop domain axiomatization.
Develop knowledge-base projection algorithm for base level of abstraction.
6/06-5/08: Encode the knowledge base, and implement projection.
Develop theory of abstraction.

6/08-5/10: Implement levels and techniques of abstraction.
Develop algorithms for planning, safety.

6/10-5/12: Implement planning, safety.
Extension, evaluation, and tuning of implementation.

12 Evaluation

The SOPHY project will be evaluated in two ways. First, to what extent can it solve problems associated with the physical systems discussed in section 2, and the variants enumerated in (Davis, 2005c)? Second, to what extent does it represent the knowledge in the relevant parts of the texbook (Brown, LeMay, and Bursten, 2000)? Both of these can be given roughly numerical measures, by enumerating a set of problems to be solved and of individuated facts to be expressed. Obviously, though, these numerical measures cannot be taken very seriously, or treated as experimentally reproducible results, given the inherent vagueness of what it means to "solve a problem" and to "express knowledge" and of the individuation of "pieces of knowledge" in the textbook; the arbitrary nature of the collection of variants; the fact that no wall has been set up between the training set and the test set on either of these measures; and the fact that the evaluator will be the same as the system designer (myself). An additional limitation of the second measure is that merely representing a domain fact in a knowledge base does not by any means guarantee that that fact can be used effectively in inference. On the other hand, since there is (as far as I know) no one else working on any task remotely comparable, there is no great need for a precise numerical method to compare the quality of the result.

Why not, one might ask, use the kind of evaluation measure used for the Halo (Friedland et al. 2004a) project? The Halo project has developed a very systematic method of evaluation — one of the most systematic and careful that has ever been applied to work in knowledge-based reasoning — which has received deservedly much favorable attention. The measure is the success rate on answering questions from the relevant section of the advanced placement high school chemistry exam, both in finding the correct answer, and in explaining the answer. The three competing KR teams were presented with a training set of problems, and then their systems were tested on a separate, previously unseen, test set drawn from the same corpus. The grading of the answers was done by an independent set of domain experts. The translation of the English language AP questions into the input formalism was done by the system designers, but overseen by the administrators of Halo.

However, this kind of measure is not really suitable to the subject matter and objectives of the SOPHY project; and gearing the project to address this kind of measure will divert it from the central problems that need to be solved. The AP exam was designed, after all, to test human high school students' knowledge of chemistry. Leaving aside the difficult and much debated question of how effectively the AP test does that, or of how effectively any test of this kind can do that, obviously this is a quite different undertaking than evaluating a computer system's knowledge of chemistry. The starting points are different. For the human test-takers, one can assume linguistic competence, the ability to read a diagram, and in general, the ability to apply commonsense reasoning to problems and to connect the course teachings to a commonsense understanding of the world. For a computer system, one cannot assume any of this; one can assume, however, that unlike humans, they do not make errors in calculations or in following fully specified procedures and that they can easily master arbitrary quantities of rote memorization. Since the AP test is designed to test humans, it focuses on those aspects of understanding chemistry that are particularly *difficult* for humans and not the aspects that are particularly difficult for computers. Therefore, using the AP test as measure for computer systems tends to exaggerate how well they understand chemistry.

Even in the Halo project, whose subject matter, as mentioned above, was specifically chosen to avoid the issues of qualitative reasoning, spatial reasoning, and commonsense reasoning, there is a substantial difference between what is needed to solve the AP problems and what would be needed to support a general purpose reasoner over the same domain. It is notable, for instance that the Ontoprise knowledge base (I have not looked carefully at the others) managed to avoid using any kind of temporal representation or temporal reasoning; they represent a chemical reaction, such as $O_2 + 2H_2 \rightarrow 2H_2O$, just as an uninterpreted binary relation between the left and right side, with no notion that the right side is later in time than the left side. Thus, it would not be possible, using this representation, for a reasoner to reason that after the reaction, the products of a reaction are available and can be used for something else, but the reagents are gone, and can no longer be used. Apparently, the AP exam never tests this; it seems safe to say that even quite poor human chemistry students understand that much about reactions. Clearly, though, it is a central aspect of temporal reactions and when one wishes to reason about chemical reactions in a setting larger than answering AP questions, it will not be possible to avoid it. The limitations of using AP questions in the SOPHY domain are much more severe, as the SOPHY domain was specifically chosen to involve the issues of qualitative, spatial, and commonsense reasoning.

As for the protocol with training set, test set, and external evaluators, we have nothing like the resources to do such a thing, and there would be little point anyway. The Halo project was large and very well funded with three competing knowledge representation teams. Careful evaluation was of critical importance so that the comparative strengths and weaknesses of the different teams could be compared. SOPHY will be just me and maybe a graduate student; we cannot hire the external experts, and there is no comparison to be carried out in any case.

13 Related Work

The previous research relevant to this proposal falls mostly into two categories, which have not previously interacted much: work on rule-based approaches to commonsense physical reasoning; and work on algorithmic approaches to physical reasoning.

13.1 Rule-based physical reasoning

This is a comparatively small body of research. It was initiated by Pat Hayes' "Naive Physics Manifesto" (1979), which proposed a large-scale project of encoding naive physics in an axiomatic system, deferring the problems of effective implementation. Hayes provided an extensive instance of his project in the "Ontology for Liquids" (1985), which constructed an ontology and an axiomatic system for reasoning about liquids.

After this exciting start, the field languished. Schmolze (1986) presented a rather shallow set of axioms for a number of commonsense physical domains.

In (Davis 1988) I developed a large axiomatic system for the solution of a single problem in solid object dynamics; showing that a small marble dropped inside a large funnel will come out the bottom. This work was justly criticized as requiring a large number of axioms to produce a single conclusion, with little to suggest that the axioms can be applied or that the technique can be generalized beyond the particular example or that the long proof could be constructed automatically.

I continued work on axiomatizing a variety of physical domains. (Davis 1990, chap 7) gave a complete axiomatization of the reasoning in the ENVISION program (de Kleer and Brown 1985) and partial axiomatizations of QP theory (Forbus 1985), of solid object kinematics, and dynamics and of liquids. (Davis 1992a) fixed and completed the axiomatization of QP theory. (Davis 1992b) discusses infinitary problems that arise in axiomatizing a variety of domains, chiefly physical ones. (Davis 1993) presented two ontologies, with axiomatizations, for reasoning about cutting solid objects. (Davis 1994) gives a theory of continuous branching time, which integrates a standard branching time ontology of action choice with the continuous model of time needed for physical reasoning. (Davis 1998) reviews work in this field, with a discussion of the pros and cons of various approaches and an analysis of the major difficulties that are encountered. It also contains, incidentally, a partial axiomatization of the behavior of string.

Sandewall (1989) developed a logical description of a microworld of point objects moving along surfaces. The chief focus of this work was integrating non-monotonic logic with temporal logic.

Three researchers — Lifschitz (1997), Morgenstern (1997, 2001), and Shanahan (1997, 2004) — published work on the problem, posed by me, (Miller and Morgenstern 1998) of axiomatizing reasoning about an egg being cracked into a bowl. The papers are independent; each presents a separate axiomatization.

The problem encompasses so many different types of physical interactions that an complete treatment cannot feasibly be attained, so these papers are necessarily only preliminary studies. They do, however, make many interesting observations about the domains and suggestions for directions of axiomatization.

The Halo project (Friedland et al. 2004) is discussed extensively above.

13.2 Reasoning about Fluids

The classic study of reasoning about liquids in the AI literature is Hayes (1985) "Ontology for Liquids", discussed above. This constructs a language for representing liquids and presents a taxonomy of simple kinds of behavior. Projection is carried out using constraints that describe how histories of different kinds can be fit together. However, the language, particularly the spatial language, is not clearly defined, and the physical analysis does not come close to handling all cases.

The other major study[2] is that of (Kim 1993). The system described here does qualitative prediction ("envisionment") for physical systems of liquids and gasses in solid containers; it can handle basic instances of many of the systems described here in section 2. The spatial language combines topological predicates with order relations on vertical height. The envisionment algorithm begins by dividing a given situation into separate physically significant regions (a "place vocabulary") and then using a form of qualitative reasoning (see section 13.4) to carry out a qualitative prediction. Kim's work is by far the closest to SOPHY in the literature. We hope in SOPHY to go beyond it chiefly (a) by using a much richer spatial vocabulary. (As far as I can tell, Kim's system is unable to deduce that one can pour out of a beaker by tilting it or even to express the notion of tilting a beaker); (b) by incorporating multiple levels of abstraction; (c) by addressing tasks other than projection.

13.3 Solid Object Physics

There are many systems that carry out computations of one kind or another about the physical interactions of solid objects. A complete survey would be far beyond the scope of this proposal; we can here only sketch a few major threads.

Most of the work on solid objects physics has worked entirely within the kinematic theory, which consists of the three laws that objects are rigid, that they move continuously, and that two objects do not overlap. It turns out that these three constraints suffice for the analysis of many interesting systems, particularly mechanical systems with few degrees of freedom. In almost kinematic calculations, the program starts with exact geometric specifications of the objects involved. It next computes the *configuration space*; that is, the space of positions in which the objects do not overlap. The particular problem to be solved, such as prediction or motion planning, is then computed from the configuration space. This basic outline was first put forward in (Faltings 1987) and (Joskowicz 1987). Subsequent work has improved the algorithms and found a wide range of applications (e.g. Sacks and Joskowicz 1993; Joskowicz and Taylor 1996). Of particular interest to the issue of evaluation is the survey carried out in (Joskowicz and Sacks 1991) which determined that a large fraction of the mechanisms enumerated in a standard encyclopedia of mechanisms can be explained purely in kinematic terms.

Simulators for the behavior of solid objects using a full dynamic theory have been developed in the context of computer-aided engineering in projects such as (Wehage and Haug 1982) and in the context of AI in (Gelsey 1995). These carry out a exact simulation of behavior given exact geometric specifications of the objects involved. (Sacks and Joskowicz 1998) presents an algorithm that efficiently carries out dynamic simulation for two-dimensional assemblies using configuration spaces to expedite the problem of collision

---

[2] I thank Ken Forbus for bringing this to my attention. This excellent work, which was apparently never published other than as a doctoral thesis, deserves to be much more broadly known.

detection. WHISPER (Funt 1980) simulated dynamic behavior of two-dimensional systems of solid objects in a occupancy array representation.

The works in solid object physics most relevant to SOPHY are the studies of qualitative reasoning about the behavior of physical systems. The first projects of this kinds were NEWTON (de Kleer 1977), which gave a qualitative analysis of the behavior of a point object on a roller-coaster track, and FROB (Forbus 1980), which analyzed the behavior of point objects in among two-dimensional obstacles.

Programs such as (Faltings 1987), (Nielsen, 1988), and (Sacks and Joskowicz 1993) took a configuration space that had been computed exactly from exact shape descriptions, divided the configuration space into significant regions, and thus were able to compute qualitative properties of the systems from the relations between these regions. (Forbus, Nielsen, and Faltings 1991) extended this approach with a qualitative representation of forces and motions, and thus producing a system for qualitative dynamic prediction.

The work of Stahovich, Davis, and Shrobe (2000) is similar in spirit to (Forbus, Nielsen, and Faltings 1991), but much more elegant and systematic. This program does qualitative simulation for planar systems of objects, each of which moves with one degree of freedom under the "quasi-static" assumption that the inertia of objects is negligible as compared to the driving forces and frictive (dissipative) forces, and that collisions are inelastic. The input to the program is a representation of the "qc-space", which gives, for each pair of interacting objects, a qualitative description of the configuration space of the feasible (non-overlapping) positions and the contact positions of the two objects. (The paper vaguely states that the qc-space can be computed from an informal sketch of the mechanism, but it is not at all clear how this is to be done.) The possible qualitative behaviors of the mechanism is then predicted in terms of trajectories through qc-space, using rules for balancing forces.

Other directions for the study of qualitative reasoning include (1) studies of kinematic reasoning in cases where the shapes of the objects involved are known only to within a given tolerance (Joskowicz, Sacks, and Srinivasan 1997; Davis 1995b); (2) studies of abstraction techniques that can be applied to solid object kinematics (Nielsen 1988; Davis 1995a).

13.4 Qualitative Reasoning

Qualitative Process (QP) theory (Forbus 1985) defined a language for characterizing the behavior of physical systems in terms of the interaction of processes and gave an algorithm for qualitative prediction ("envisionment") based on the sign of the derivatives of key quantities. The QSIM (Kuipers, 1985, 1994) and ENVISION (de Kleer and Brown 1985) systems defined similar calculi, but with a less specific physical framework. A great deal of work was done in the late 80's and early 90's extending and applying these theories; for example, Weld (1988a, 1988b) developed two methods for comparative projection in a QSIM setting. However, the inherent restriction to systems whose state is characterized by one-dimensional parameters and whose dynamics is characterized by the sign of derivatives soon proved to be a serious limitation.

13.5 Other related work

Extensive work on qualitative geometric languages — particularly topological relations, but also other kinds of geometric relations — has been been within the qualitative spatial reasoning research community (see the survey in Cohn and Hazarika 2001). However, these languages are mostly either too weak to support much significant physical inference; too restrictive to be applied to the range of problems we wish to consider; or too unrestricted to give any guidance for practical spatial representations (e.g. the first-order language over regions with topological relations and a measure of distance). In (Davis, 1997) I proposed a constraint language with a rich collection of spatial primitives; I still think this general direction is correct, but it needs a lot of special purpose inference rules to make it run. Also of interest is the large collection of spatial primitives in OpenCyc (OpenCyc); I have not been able to get any information about the power of the associated inference engine.

14 Strengths of the Proposal

I have been working in the areas of commonsense physical reasoning and qualitative spatial reasoning for twenty years, and have published extensively in these areas. I have written a textbook on commonsense reasoning (Davis 1990) with an extensive discussion of spatial and physical reasoning. I have also supervised one doctoral thesis (Joskowicz 1988) in the area of physical reasoning.

The project will be much enriched by ties and communication with other ongoing projects at NYU and elsewhere. The large research endeavors at Courant in modelling and simulation and in computational geometry, which involve both the computer science and the mathematics departments, are sources of much inspiration and information as to more mainstream approaches to physical and spatial reasoning. I can also draw on the expertise of the NYU research group in program verification — Clark Barrett, Ben Goldberg, and Amir Pnueli — in dealing with automated logical inference of all kinds.

15 Broader Impact

As discussed in the introduction, the chief impact of this work is in the long term. The project is a first step toward a general knowledge base that could support automonous robotics; automated science instruction; tools for planning, design, and verification of laboratory experiments and tools; and a knowledge base of scientific knowledge.

The project will also have impact in the short term:

- The training of doctoral students. At the moment, one doctoral student is working with me, on an unrelated topic (web search); but over my career I have graduated roughly one doctoral student every two and one half years, and I am hopeful of finding a student or two to work on SOPHY.
- The production of a public electronic resource. The knowledge base and the reasoning engines will be published on the web when they attain reasonably stable states.
- Dissemination of knowledge beyond the immediate research community. I am working on a book, addressed to a semi-popular audience, on the subject of how commonsense physical and spatial knowledge interacts with formal mathematical calculations. This will draw on many of issues that I have studied in my research.

16 Results from Recent NSF-supported Research

Grants

“Physical and Spatial Reasoning across Multiple Scales,” NSF IRI-9625859, $236,000, 8/96-8/99.

“Qualitative Reasoning about Loosely Constrained System of Solid Objects” NSF IIS-0097537, $289,000, 6/01-5/04.

During the last five years of NSF support, we have carried out in-depth studies in three general areas of commonsense reasoning: physical reasoning; qualitative spatial reasoning; and reasoning about multi-agent planning and communication. Our research group has also carried out research in automated planning in domains with continuous change; automated planning for the placement of resources in a network; and a number of techniques for for improved retrieval of web documents.

Physical Reasoning

Our studies of physical reasoning have led to the following results:

1. A detailed methodological study (Davis 1998) of the strengths and weaknesses of the microworlds approach to the conceptualization of commonsense physical domains, and its relation to the "cluster" approach proposed by Hayes (1979).
2. A taxonomy and analysis of the types of abstraction used in commonsense reasoning about solid objects (Davis 2005d).

Spatial Reasoning

3. A study, carried out with N.M. Gotts and A.G. Cohn, of the constraint language over the geometric primitives RCC8 + Conv. RCC8 contains the binary topological predicates, "$A = B$", "A is a non-tangential partial part of B", "A is a tangential partial part of B," "A overlaps B," "A is in external contact with B," and "A is disconnected from B" and their inverses. "Conv(A)" is the predicate "A is convex." We derived tight bounds on expressivity — two regions are distinguishable in the language if and only if they are not related by an affine transformation — and tight bounds on computability — the decision problem is exactly as hard as the decision problem for a set of algebraic constraints of comparable size over the reals. (Davis, Gotts, and Cohn 1999)
4. An algorithmic analysis of sets of constraints of the form "The distance from A to B is much less than the distance from C to D." We show that such constraint sets, and a number of natural extensions, are solvable in polynomial time. We also show that the first-order language over such constraints is decidable. (Davis 1999a)
5. An analysis of the qualitative properties of continuous shape transformations, where "continuous" is interpreted relative to four different metrics over regions. (Davis 2001a)
6. An analysis of the expressivity of the first-order language allowing quantification over regions, and containing the two predicates, "C(x,y)" (regions x and y are connected) and "sphere(x)" (region x is a sphere). We have proven that this theory is extraordinarily expressive and contains, roughly speaking, most of the properties of regions that have been defined in the literature of geometry. (Davis 2005e).

Multi-agent plans and communication

7. A new, highly expressive, language of informative communications, that allows an agent to express almost any property of the current situation, defined in terms of logical combinations of conditions that hold currently, held in the past, or will hold in the future, including facts about agents' knowledge and facts about other communicative acts. (Davis, 2004; 2005a). We have proven that this theory is consistent with a large class of physical theories, and that it avoids the "unexpected hanging" problem.
8. Leora Morgenstern (IBM Watson Labs) and I have developed a new, highly expressive, theory of multi-agent planning. This subsumes the very general notion of a plan and of plan correctness developed in (Davis, 1994) and the theory of informative communications described in (7) above and extends these to include a similarly expressive theory of requests. We have shown that this theory is strong enough to support validation of a simple sample multi-agent plan, and that it is consistent (Davis and Morgenstern, 2004).

Planning with Continuous Time

9. Ji-Ae Shin (Shin, 2004; Shin and Davis, 2004 and 2005) implemented a planner that solves problems in domains that involve continuously changing numerical fluents. Her technique is to compile these problems into Boolean combinations of linear constraints and propositional atoms; to use LPSAT (Wolfman and Weld, 1999), an existing constraint engine, to find a solution to the constraints; and then to interpret the solution as a plan. We have proven that the planner is sound and complete.

Planning for Component Placement in a Network.

10. Tatiana Kichkaylo (2005) developed two planners for placing software components on a network to achieve a specified goal, subject to constraints on CPU power and network bandwidth. The Sekitei system (Kichkaylo, 2003; Kichkaylo, Ivan, and Karamchti, 2003) uses a combination of constrained forward propagation and regression. It can efficiently find plans over networks with hundreds of nodes. The GPPS system (Kichkaylo 2004) plans deployment of software components and file transfers for grid architectures.

Web Search Engines

11. Ziyang Wang is a doctoral student in the area of web mining and the analysis of link structures. He has proposed a modification to the PageRank algorithms that gives a higher value to an in-link from a distant page as compared to an in-link from a closely related pair, and shown in a number of test cases that this gives better results (Wang, 2003). His doctoral thesis deals with the problem of monitoring a local web site for new information, and presenting this to the user.

Development of human resources

Four doctoral students have been associated with this research project within the last five years.

Tamir Klinger, “Adversarial Reasoning: A Logical Approach for Computer GO.” January 2001.

Ji-Ae Shin, “TM-LPSAT: Encoding Temporal Metric Planning in Continuous Time.” May 2004.

Tatiana Kichkaylo, “Creation of component-based applications by planning.” December 2004

Ziyang Wang, Web mining, expected to complete May 2006.

It should be noted that Shin and Kichkaylo are women, an underrepresented category in computer science.

Publications:

E. Davis. “The Naive Physics Perplex.” *AI Magazine*, Winter 1998, Vol. 19. No. 4. pp. 51-79.

E. Davis. “Order of Magnitude Comparisons of Distance.” *Journal of AI Research,* vol. 10, 1999, pp. 1-38.

E. Davis. “Guide to Axiomatizing Domains in First-Order Logic.” In *Electronic Newsletter on Reasoning about Actions and Change,* Issue 99002. 1999.

E. Davis. “Metrics on Regions and Continuous Shape Deformation.” *Fundamenta Informaticae,* 2001, vol. 46, pp. 31-54.

E. Davis. “Knowledge Representation.” In *The International Encyclopedia of the Social and Behavioral Sciences* Neil J. Smelser and Paul B. Baltes (eds.) Elsevier Science Pubs., Oxford, 2001.

E. Davis. “Two Machine Learning Textbooks: An Instructor’s Perspective,” *Artificial Intelligence,* 2001, vol. 131, pp. 191-198.

E. Davis. “A First-Order Theory of Communicating First-Order Formulas.” *Principles of Knowledge Representation and Reasoning,* 2004.

E. Davis. “Knowledge and Communication: A First-Order Theory,” *Artificial Intelligence,* to appear.

E. Davis. “Mathematics as Metaphor: A review of *Where Mathematics Comes From,* by G. Lakoff and R. Nuñez,” *Journal of Experimental and Theoretical Artificial Intelligence,* to appear.

E. Davis. “The Expressivity of Quantifying over Regions.” In preparation.

E. Davis. “Abstraction in Commonsense Physical Reasoning: A Taxonomy.” In preparation.

E. Davis, N.M. Gotts, and A.G. Cohn “Constraint Networks of Topological Relations and Convexity.” *CONSTRAINTS,* 1999, Vol. 4 No. 3, pp. 241-280.

E. Davis and L. Morgenstern, “Progress in Formal Commonsense Reasoning,” *Artificial Intelligence,* 2004, vol. 153, pp. 1-12.

E. Davis and L. Morgenstern. “A First-Order Theory of Communication and Multi-Agent Plans.” *Journal of Logic and Computation,* to appear.

T. Kichkaylo. “Planning with Arbitrary Monotonic Resource Functions,” *ICAPS-2003.*

T. Kichkaylo. “Planning for Grid Applications with Explicit Reservations,” *ICAPS-2004.*

T. Kichkaylo. *Construction of Component-Based Applications by Planning,* Ph.D. Thesis, NYU Comp. Sci. Dept., 2005.

T. Kichkaylo, A. Ivan, and V. Karamcheti. “Constrained Component Deployment in Wide-Area Networks using AI Planning Techniques.” *IDPDS-2003.*

T. Klinger. “Adversarial Reasoning: A Logical Approach to Computer Go.” Ph.D. Thesis, NYU Comp. Sci. Dept, 2000.

J. Shin. “TM-LPSAT: Encoding Temporal Metric Planning in Continuous Time.” Ph.D. Thesis, NYU Comp. Sci. Dept., 2004.

J. Shin and E. Davis. “Continuous Time in a SAT-Based Planner.” *AAAI-04.*

J. Shin and E. Davis, “Processes and Continuous Change in a SAT-based Planner,” *Artificial Intelligence,* to appear.

Z. Wang. “Improved Link-Based Algorithms for Ranking Web Pages.” NYU Tech. Report #TR2003-846, 2003.

References Cited

T.L. Brown, H.E. LeMay, and B. Bursten. 2003. *Chemistry: The Central Science,* Prentice Hall.

B. Choueiry, Y. Iwasaki, and S. McIlraith. 2005. Towards a practical theory of reformulation for reasoning about physical systems, *Artificial Intelligence,* vol. 162, pp. 145-204.

A.G. Cohn and S. Hazarika. 2001. Qualitative Spatial Representation and Reasoning: An Overview. *Fundamenta Informaticae,* **46** pp. 1-21.

E. Davis. 1988. A Logical Framework for Commonsense Predictions of Solid Object Behavior. *AI in Engineering,* vol. 3 no. 3, pp. 125-140.

E. Davis. 1990. *Representations of Commonsense Knowledge.* Menlo Park, Calif.: Morgan Kaufmann.

E. Davis. 1992. Axiomatizing Qualitative Process Theory. *Third International Conference on Knowledge Representation and Reasoning.*

E. Davis. 1992. Infinite Loops in Finite Time: Some Observations. *Third International Conference on Knowledge Representation and Reasoning.*

E. Davis. 1993. The Kinematics of Cutting Solid Objects. *Annals of Mathematics and Artificial Intelligence* 9(3,4): 253-305.

E. Davis. 1994a. Knowledge Preconditions for Plans. *Journal of Logic and Computation,* vol. 4, no. 5, Oct. 1994, pp. 721-766.

E. Davis. 1994b. Branching Continuous Time and the Semantics of Continuous Action. *Second Intl. Conference on AI Planning Systems.*

E. Davis. 1995a. Approximation and Abstraction in Solid Object Kinematics. NYU Computer Science Tech. Report #706, September 1995

E. Davis. 1995b. Approximations of Shape and Configuration Space. NYU Computer Science Tech. Report #703, September 1995.

E. Davis. 1997. A Highly Expressive Language of Spatial Constraints. NYU Computer Science Tech. Report #714, September 1995.

E. Davis. 1998. The Naive Physics Perplex. *AI Magazine*, Winter 1998, Vol. 19. No. 4. pp. 51-79.

E. Davis. 1999a Order of Magnitude Comparisons of Distance. *Journal of AI Research,* vol. 10, 1999, pp. 1-38.

E. Davis. 2001a. Metrics on Regions and Continuous Shape Deformation. *Fundamenta Informaticae,* Vol. 46 Nos. 1-2, pp. 31-54.

E. Davis. 2004. A First-Order Theory of Communicating First-Order Formulas. *KR-2004.*

E. Davis. 2005a. Knowledge and Communication: A First-Order Theory. *Artificial Intelligence,* to appear.

E. Davis. 2005b. “Mathematics as Metaphor: Review of *Where Mathematics Comes From,* by G. Lakoff and R. Nuñez,” *Journal of Experimental and Theoretical Artificial Intelligence,* to appear.

E. Davis. 2005c. Variants of simple physical systems as a testbed for a knowledge-based reasoner. In preparation.

E. Davis. 2005d. Abstraction in simple physical reasoning: A taxonomy. In preparation.

E. Davis. 2005e. The expressivity of quantifying over regions. In preparation.

E. Davis, N.M. Gotts, and A.G. Cohn. 1999. Constraint Networks of Topological Relations and Convexity. *CONSTRAINTS,* Vol. 4 No. 3, pp. 241-280.

E. Davis and L. Morgenstern (2005). A First-Order Theory of Communication and Multi-Agent Plans. *Journal of Logic and Computation,* to appear.

J. de Kleer. 1977. Multiple Representations of Knowledge in a Mechanics Problem Solver. *Proc. IJCAI-77* pp. 299-304.

J. de Kleer and J.S. Brown. 1985. A Qualitative Physics Based on Confluences. In D. Bobrow (ed.) *Qualitative Reasoning about Physical Systems.* Cambridge, Mass.: MIT Press.

T. Ellman and F. Giunchiglia (eds.) 2005. *Artificial Intelligence,* vol. 162, Special Issue on Abstraction.

B. Faltings. 1987. Qualitative Kinematics in Mechanisms. *IJCAI-87,* pp. 436-442.

K. Forbus. 1980. Spatial and Qualitative Aspects of Reasoning about Motion. *AAAI-80.*

K. Forbus. 1985. Qualitative Process Theory. In Bobrow, D. (ed.) *Qualitative Reasoning about Physical Systems,* Cambridge, Mass.: MIT Press.

K. Forbus, P. Nielsen, and B. Faltings. 1991. Qualitative Spatial Reasoning: The CLOCK project. Technical Report 9, Northwestern University, Institute for the Learning Sciences.

N. Friedland et al. 2004a. Toward a quantitative platform-independent quantitative analysis of knowledge systems, *KR-2004.*

N. Friedland et al. 2004b. Project Halo: Toward a Digital Aristotle, *AI Magazine,* Winter 2004, vol. 25 no. 4, pp. 29-47.

B. Funt. 1980. Problem-Solving with Diagrammatic Representations. *Artificial Intelligence.* Vol. 13, No. 3, pp. 201-230.

A. Gelsey. 1995. Automated Reasoning about Machines. *Artificial Intelligence,* vol 74, pp. 1-53.

F. Giunchiglia, A. Villafiorita, and T. Walsh. 1997. Theories of abstraction. *AI Communications,* vol. 10, pp. 167-176.

P. Hayes. 1979. The Naive Physics Manifesto. In Michie, D. (ed.) *Expert Systems in the Microelectronic Age.* Edinburgh: Edinburgh University Press.

P. Hayes. 1985. Naive Physics 1: Ontology for Liquids. In Hobbs, J. and Moore, R. (eds.) *Formal Theories of the Commonsense World.* Norwood, N.J.: Ablex Pubs.

L. Joskowicz. 1987. Shape and Function in Mechanical Devices. *IJCAI-87,* pp. 611-615,

L. Joskowicz. 1988. Reasoning about Shape and Kinematic Function in Mechanical Devices. NYU Ph.D. Thesis.

L. Joskowicz and E. Sacks. 1991. Computational Kinematics. *Artificial Intelligence,* 51: 381-416.

L. Joskowicz, E. Sacks, and V. Srinivasan. 1997. Kinematic Tolerance Analysis, *Computer-Aided Design,* Vol. 29, No. 2.

L. Joskowicz and R. Taylor. 1996. Interference-free insertion of a solid body into a cavity: an algorithm and a medical application. *International Journal of Robotics Research.* Vol. 15, No. 3.

J.A. Kalman, 2001. *Automated Reasoning with Otter,* Rinton Press.

T. Kichkaylo. 2003. "Planning with Arbitrary Monotonic Resource Functions," *ICAPS-2003.*

T. Kichkaylo. 2004. Planning for Grid Applications with Explicit Reservations, *ICAPS-2004.*

T. Kichkaylo. 2005. *Construction of Component-Based Applications by Planning,* Ph.D. Thesis, NYU Comp. Sci. Dept.

T. Kichkaylo, A. Ivan, and V. Karamcheti. 2003. Constrained Component Deployment in Wide-Area Networks using AI Planning Techniques. *IDPDS-2003.*

H. Kim. 1993. *Qualitative Reasoning about Fluids and Mechanics.* Ph.D. thesis, Institute of Learning Sciences, Northwestern University.

B. Kuipers. 1985. Qualitative simulation. In Bobrow, D. (ed.) *Qualitative Reasoning about Physical Systems,* Cambridge, Mass.: MIT Press.

B. Kuipers. 1994. *Qualitative Reasoning: Modeling and Simulation with Incomplete Knowledge.* MIT Press.

T. Klinger 2000. *Adversarial Reasoning: A Logical Approach to Computer Go.* Ph.D. thesis, NYU Comp. Sci. Dept.

G. Lakoff and R. Nuñez. 2001. *Where Mathematics Comes From.* Basic Books.

V. Lifschitz. 1997. Cracking an Egg: An Exercise in Formalizing Commonsense Reasoning. Fourth Symposium on Logical Formalizations of Commonsense Reasoning.

J. McCarthy. 1997. Elaboration tolerance. Fourth Symposium on Logical Formalizations of Commonsense Reasoning.

R. Miller and L. Morgenstern. 1998. Commonsense Problem Page. http://www-formal.stanford.edu/leora/cs/

L. Morgenstern. 1987. Knowledge preconditions for actions and plans. *Proc. Tenth Intl. Joint Conf. on Artificial Intelligence.* pp. 867-864.

L. Morgenstern. 1997. Formulating a Problem in Commonsense Physical Reasoning: The Egg-Cracking Problem. Fourth Symposium on Logical Formalizations of Commonsense Reasoning.

L. Morgenstern. 2001. Mid-Sized Axiomatizations of Commonsense Problems: A Case Study in Egg Cracking. *Studia Logica,* vol. 67 pp. 333-384.

L. Morgenstern and E. Davis. 2005. A First-Order Theory of Communication and Multi-Agent Plans. *Journal of Logic and Computation,* to appear.

R. Moore. 1985. A formal theory of knowledge and action. In J. Hobbs and R. Moore (eds.) *Formal Theories of the Commonsense World* pp. 319-358, ABLEX Publishing, Norwood, NJ

P. Nielsen. 1988. A Qualitative Approach to Mechanical Constraint. *Proc. AAAI-88*, pp. 270-274.

OpenCyc. Spatial Relations. http://www.cyc.com/cycdoc/vocab/spatial-vocab.html

D.A. Randell, Z. Cui, and A.G. Cohn. 1992. A Spatial Logic Based on Regions and Connection. *KR-92.*

E. Sacks and L. Joskowicz. 1993. Automated modelling and kinematic simulation of mechanisms. *Computer-Aided Design.* Vol. 25, No. 2. pp. 106-118.

E. Sacks and L. Joskowicz. 1998. Dynamical Simulation of Planar Assemblies with Changing Contacts Using Configuration Spaces. *Journal of Mechanical Design,* Vol. 120.

E. Sandewall. 1989. Combining Logic and Differential Equations for Describing Real-World Systems. In Proceedings of the First International Conference on Knowledge Representation and Reasoning, 412-420. Menlo Park, Calif: Morgan Kaufmann.

E. Sandewall. 1994. *Features and Fluents: representation of knowledge about dynamical systems.* Oxford University Press.

J. Schmolze. 1986. Physics for Robots. In Proceedings of the Fifth National Conference on Artificial Intelligence, 44-50. Menlo Park, Calif: American Association for Artificial Intelligence.

M. Shanahan. 1997. A Logical Formalisation of Ernie Davis's Egg Cracking Problem. Fourth Symposium on Logical Formalizations of Commonsense Reasoning.

M. Shanahan. 2004. An Attempt to Formalise a Non-Trivial Problem in Common Sense Reasoning. *Artificial Intelligence,* to appear.

J. Shin. 2004. TM-LPSAT: Encoding Temporal Metric Planning in Continuous Time. Ph.D. Thesis, NYU Comp. Sci. Dept.

J. Shin and E. Davis. 2004. Continuous Time in a SAT-Based Planner. *AAAI-04.*

J. Shin and E. Davis. 2005."Processes and Continuous Change in a SAT-based Planner, *Artificial Intelligence,* to appear.

T. Stahovich, R. Davis, and H. Shrobe. 2000. Qualitative rigid-body mechanics. *Artificial Intelligence* Vol. 119, pp. 19-06.

Z. Wang. 2003. Improved Link-Based Algorithms for Ranking Web Pages. NYU Tech. Report #TR2003-846

R.A. Wehage and E.J. Haug. 1982. Dynamic analysis of mechanical systems with intermittent motion. J. Mechanical Design. Vol. 104, pp. 784-788.

D. Weld. 1988a. Comparative Analysis. *Artificial Intelligence,* vol. 36, pp. 333-374.o

D. Weld. 1988b. Exaggeration. *AAAI-88*, pp. 291-295.

S. Wolfman and D. Weld. 1999. The LPSAT Engine and its application to Resource Planning. *IJCAI-99.*