Exploring the Rich Lore of the Pendulum

The Pendulum Paradigm: Variations on a Theme and the Measure of Heaven and Earth. *By Martin Beech. Brown Walker Press, Boca Raton, FL, February 2014. 290 pages, $25.95.*

BOOK REVIEW
By Ernest Davis

"In [the Pisa Baptistry] hangs the lamp whose measured swing suggested to Galileo the pendulum. It looked an insignificant thing to have conferred upon the world of science and mechanics such a mighty extension of their dominions as it has. Pondering, in its suggestive presence, I seemed to see a crazy universe of swinging disks, the toiling children of this sedate parent. He appeared to have an intelligent expression about him of knowing that he was not a lamp at all; that he was a Pendulum; a pendulum disguised, for prodigious and inscrutable purposes of his own deep devising, and not a common pendulum either, but the old original patriarchal Pendulum — the Abraham Pendulum of the world."
– Mark Twain, *The Innocents Abroad*

The pendulum—a bob connected to a fixed frame by a cord—is one of the simplest tools of experimental physics. Many of us likely recall working through the simple analysis in an early physics class and deriving the equation

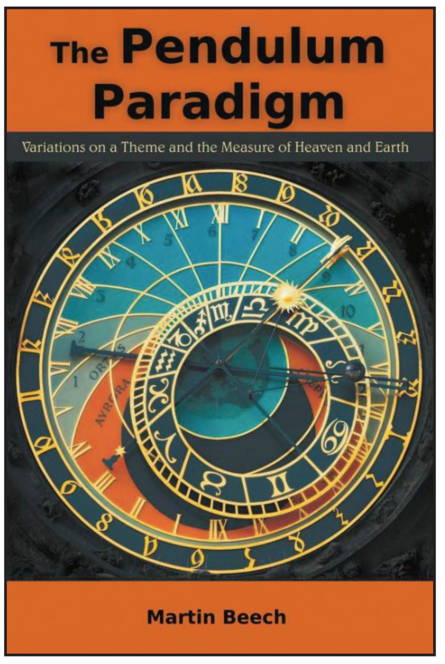
$$l\ddot{\theta} + g\sin(\theta) = 0.$$

Under the small angle approximation $\sin(\theta) \approx \theta$, we can find the solution $\theta(t) = C\sin(\omega t + \delta)$ with $\omega = \sqrt{g/l}$. With that, we are done with the pendulum; what more is left to say?

A lot more, actually, as Canadian astronomer Martin Beech demonstrates in his fascinating book, *The Pendulum Paradigm*. The lore of pendulums is deep, their history rich, their applications in science and technology multifarious, and their design and analysis surprisingly complex.

Firstly, the straightforward analysis that yields the aforementioned simple differential equation hides a multitude of sins. Dissipative forces spoil the equation's elegant purity with additional terms of more arbitrary mathematical form. The length l—from the pivot point to the center of mass—may be hard to measure with precision. The cord length likely varies with time on account of its elasticity or due to thermal expansion or contraction. And the position of the bob's center of mass might change; for instance, a wooden bob may absorb humidity.

The acceleration of gravity g can depend on one's height or geographical location. As Pierre Bouguer discovered in 1735, gravity might not even pull straight downward; camped at the base of Mount Chimborazo in Ecuador, he realized that all of the stars were several arc seconds away from their expected location relative to his plumb line. Although the frame that supports a pendulum is supposed to remain still, a seismograph—in its simplest form, just a pen attached to a pendulum over a scroll of paper—indicates that this assumption does not always hold. And as Léon Foucault's pendulum dramatically demonstrates, the observer is also not standing still; he and the pendulum are both rotating around Earth.



The Pendulum Paradigm: Variations on a Theme and the Measure of Heaven and Earth. By Martin Beech. Courtesy of Brown Walker Press.

As a result, the complications associated with designing and using pendulums tend to be twofold. On the one hand, engineers may design mechanisms to counteract these distracting issues. For example, an escarpment restores energy lost to friction. Henry Kater's pendulum makes a distance between two knife edges—which one can accurately measure—the critical length. George Graham's mercury compensation pendulum and John Harrison's gridiron pendulum both ingeniously keep the pendulum at a fixed length regardless of temperature.

On the other hand, one can use the perturbed responses of the pendulum to detect and measure the sources of perturbation. A seismograph measures small Earth movements from far-off earthquakes. Foucault's pendulum demonstrates Earth's rotation. Henry Cavendish's experiment—essentially a horizontal pendulum—calculates the gravitational constant (or, in Cavendish's terms, Earth's weight). Deviations in plumb lines can detect underground oil reserves. Of course, accomplishing these tasks effectively often involves solving the first problem as well, since one has to isolate the perturbations of interest.

The associated mathematics—summed up by the book's subtitle—consist mostly of variations on a single theme, namely the preceding second-order differential equation. By my count, *The Pendulum Paradigm* includes 21 variants of this equation, with assorted corrections for different circumstances. Of course, only a handful of these have an analytic solution.

Taking the pendulum as the point of departure, Beech's account explores a wide range of topics in the history of horology, gravitational physics, planetary science, and astronomy, mostly (though not entirely) confined to the 18th and 19th centuries. Readers learn about clock design, from the mechanical clock of Richard of Wallingford (c. 1330) to the Wishing Fish clock built by the artist Kit Williams; Pierre Louis Maupertuis's 1736 expedition to Lapland, Finland to adjudicate between Isaac Newton's theory that Earth is oblate and Jacques Cassini's theory that it is prolate; irregularities in Earth's gravitational field and shifts in its rotational axis; the ergodic and chaotic behaviors of pendulum systems with multiple degrees of freedom; and much more. The book is richly illustrated with diagrams and photographs.

There is plenty in *The Pendulum Paradigm* to engage a casual reader with an interest in science and its history, but Beech also provides careful descriptions and analyses of each issue he discusses. These do require some work on the part of the readers if they hope to fully appreciate his examinations. I cannot claim that I always followed my own advice, but the three or four times when I carefully worked through Beech's argument proved extremely worthwhile. In particular, it took me four readings and some concentrated thought to grasp the explanation of why the rotation period of Foucault's pendulum amounts to one day divided by the sine of the latitude. But in the end I was able to truly understand it. That in itself was worth the price of the book.

Ernest Davis is a professor of computer science at New York University's Courant Institute of Mathematical Sciences. His book Rebooting AI: Building Artificial Intelligence We Can Trust, with psychologist Gary Marcus, will appear this fall.