How Not To Be Wrong: The Power of Mathematical Thinking. B*y Jordan Ellenberg, Penguin Press, New York, New York, 2014, 468 pages, $27.95.*

BOOK REVIEW

By Ernest Davis

In *How Not To Be Wrong*, Jordan Ellenberg takes on the daunting task of explaining to a lay audience why they should care about mathematics. In many ways, he succeeds brilliantly. The book is entertaining, informative, wise, and extremely well written. Ellenberg's specific objective is to explore the interaction of mathematical reasoning and common sense, and to show how mathematics is a powerful extension of common sense. "Math is like an atomic-powered prosthesis that you attach to your common sense, vastly multiplying its reach and strength," he writes. "The problems that we think about every day—problems of politics, of medicine, of commerce, of theology—are shot through with mathematics."

The most visible interactions of mathematics and common sense in recent years, and the most common applications of mathematics to problems of politics, commerce, and medicine, have been in probability and statistics; accordingly, about two-thirds of Ellenberg's book deals with the basic issues in those areas. He also considers the dangers of mindless linear extrapolation, the pitfalls in reporting a percentage of a sum formed of both positive and negative terms, the difficulties of finding a good voting scheme, and formalism in mathematical philosphy. A wide range of mathematical topics, including projective geometry, finite geometry, Lobachevskian geometry, error-correcting codes, the distribution of primes, the theory of the reals, and the non-standard theory of the reals, make cameo appearances.

Ellenberg's arguments require no mathematical background beyond arithmetic and very basic geometry. The many diagrams are almost all crudely hand-drawn, a very wise decision. The book includes only one nontrivial proof, but that one—Barbier's solution of Buffon's needle problem—is a beaut. At his best, Ellenberg is about as good a science writer as any I've come across. He achieves his best in the first chapter, with a story about Abraham Wald.

During World War II, Wald had the task of recommending to the Army Air Force sites on fighter planes where additional armor should be placed. The key data was the relative frequency of bullet holes in different parts of the planes after they returned from their missions. On average, the numbers of bullet holes per square foot were 1.11 in the engine, 1.73 in the fuselage, 1.55 in the fuel system, and 1.85 in the rest of the plane. What the data showed, Wald realized, was not that the engine was hit less often, but rather that planes hit in the engine were less likely to come back; the engine was thus the most important place to add armor.

In terms of pure writing technique, Ellenberg's pacing here is particularly admirable; he pulls off the trick of moving the story along, while making it seem as if he had all the time in the world. The writing is equally impressive—both crystal clear and captivating—in many other sections as well.

In describing the book as "wise," what I mean is that it has many of the intellectual virtues that I most value. Ellenberg is strikingly fair-minded on contentious topics; for instance, his account of the debate between frequentist and Bayesian statistics is as balanced as I have seen. (I think he could say more about the difficulties, in the Bayesian approach, of choosing a hypothesis space and assigning priors, but that's just a hobbyhorse of my own.) He has a strong sense of history, and a deep knowledge of it. He deals fairly with historical figures, making a serious effort to understand why they took the approaches and reached the conclusions they did, in a manner that brings to mind Stephen Jay Gould's essays. Ellenberg is very aware of the limitations of mathematical approaches, the meaninglessness of overly precise numbers, the futility of assigning a number when all that exists is a partial ordering. He decries the cult of the genius in mathematical mythology. His taste in deciding when to put himself into the story is very good.

The book does have flaws. The most conspicuous arises from Ellenberg's tendency to go on too long, sometimes much too long. He would have benefited from a more hard-hearted editor. An interminable story about MIT students and other groups who regularly made money from a Massachusetts lottery goes on for forty pages, and another about finding predictions from letter patterns in the Hebrew Bible gets twenty pages; two pages each would have been more than enough. The reader cannot safely skip these two chapters, though: The former contains, as a digression, the beautiful proof of Buffon's needle problem, the latter an important discussion of a Baltimore stockbroker.

In a similar way, the book sometimes goes beyond its mandate, in ways that I find unhelpful. I do not think that there is anything to be learned from comparing Antonin Scalia's formalist view of law with Hilbert's formalist view of mathematics; learning from the comparison, at the least, would require analysis much deeper than possible in a book like this one. In fact, I would have cut the entire chapter on mathematical foundations, which covers well-worn ground and does not contribute to the question of how not to be wrong.

A deeper problem is that the book does not exactly do what it sets out to do. As pointed out earlier, the ostensible aim is to show how math extends common sense. Most of the examples in the book, however, center on cases in which multiple mathematical arguments can be made; often, adjudication is via common sense rather than more mathematics.

To a large extent, therefore, the subject is as much "How invalid use of mathematics can confuse you" as "How correct use of mathematics can empower you." (In some respects, the book is an update of Darrell Huff's sixty-year-old classic *How to Lie with Statistics*.) Of course, this itself is an enormously important subject; but the discrepancy between the stated aim and the actual contents leaves the reader a little confused as to what has been accomplished.

Many of the debates necessarily end up unresolved. The book's last chapter is a defense of the wisdom of being in doubt. Ellenberg contrasts quotes from two public figures. First, Teddy Roosevelt:

> "It is not the critic who counts; not the man who points out how the strong man stumbles, or where the doer of deeds could have done them better. The credit belongs to the man who is actually in the arena . . . who errs, who comes short again and again because there is no effort without error and shortcoming."

The second quote is from John Ashbery: "For this is action, this not being sure." Ellenberg sides with Ashbery.

I have mixed feelings. Skepticism and intellectual caution are valuable qualities, particularly in these days of ubiquitous hype, but it is important to recognize their limitations. It is, after all, very easy not to be wrong by opting for the expedient of being too wise to commit to any answer. If one relies on this too much, one can end up like Gattling in Stephen Potter's *Lifemanship*:

> "[Gattling] was one of the most ignorant and ill-educated men I have ever met, and it was therefore always a particular pleasure to hear him say, to a perfectly ordinary question, `I don't know' slowly, kindly, and distinctly. He was able to indicate, by the tone of his voice, that although he know KNEW, RIGHT? practically everything about practically everything, and almost everything about this really, yet the mere fact that he knew such a tremendous lot about it made him realise, as we couldn't possibly, that the question was so inextricably two-sided that only a smart-Alec would ever dream of trying to pass judgement either way."

These complaints aside, the book is a splendid accomplishment, and very well worth reading, whatever your level of expertise. I learned a lot, and got a clearer perspective on all kinds of things—writing and teaching technique, history and biographical anecdotes, fallacies, social science, even a little math—and had a very enjoyable time in the process.I MEDDLED A LITTLE IN THE LAST PARAGRAPH; AS ALWAYS, FEEL FREE TO OBJECT

Ernest Davis is a professor of computer science at the Courant Institute of Mathematical Sciences, NYU.