

“Arbitrary Numerical” Science Problems: Problems, Solutions, Output from GPT4 + Wolfram Alpha and GPT4 + Code Interpreter

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This document includes 32 problems in high school and college-level physics and mathematics with numerical or vector answers.

For each problem, we give here:

1. The problem.
2. The correct solution, with an explanation of how it was derived. In some cases, we have included the Matlab code used to compute the correct solution.
3. The answer produced by GPT4+WA.
4. An edited form of the output of GPT4+WA. Editorial comments in this output are in italics.
5. A comment, including whether the answer was right, wrong, or got partial credit.
6. The answer produced by GPT4+CI.
7. In some cases, where it has features of interest, an edited form of the output of GPT4+CI. Editorial comments in this output are in italics.
8. A comment, as in (5).
9. Occasionally, we have also included the output of GPT-3.5 or GPT-4 without plug-ins, for comparison.

The excerpts from the outputs have been extracted unsystematically to show features of the systems reasoning that seemed to us interesting. The excerpts from GPT4+WA are mostly fairly complete; those from GPT4+CI are more brief, except where that had features of particular interest.

The authors carried out the tests of GPT4+WA between June 28 and July 10, 2023. They carried out the tests of GPT-4+CI between July 10 and July 12, 2023.

Unless otherwise noted, all questions are marked correct if the relative error is less than 1%. If the answer is a vector, then the question will be marked correct if $|\vec{u} - \vec{v}|/|\vec{v}| < 0.1$, where \vec{v} is the correct value and \vec{u} is the proposed answer.

1. Object A is a spherical shell of gold, 1 meter in radius, 10 cm thick. Object B is a spherical shell of copper, 10 cm in radius, 5 cm thick. They are placed so that there is an 8 cm gap between them. What is the net gravitational force they exert on each other?

Answer: 3.4486 dyne

Matlab

```
% By a theorem of Newton's, the gravitational force exerted by spherical shell
```

```

% is the same as if all the mass were concentrated at the center.
GoldDensity = 19.3; % gm/cm^3 -- Wikipedia.
GoldVolume = (4/3) * pi * (100^3-90^3); %cm^3
GoldMass = GoldDensity * GoldVolume; % gm
CuDensity = 8.96 % gm/cm^3 --- Wikipedia
CuVolume = (4/3) * pi * (10^3-5^3); % cm^3
CuMass = CuDensity * CuVolume;
Distance = 100+10+8; %cm
GravConst = 6.674 * 10^(-8); % dyne cm^2/gm^2 -- Wikipedia
Force = GravConst * GoldMass * CuMass / Distance^2

```

GPT4+WA Answer: 0.0000345N (Newtons)

GPT4+WA Calculation:

Given:

$$\text{Density of gold}(\rho_{\text{gold}}) = 19300 \text{ kg/m}^3$$

$$\text{Density of copper}(\rho_{\text{copper}}) = 8960 \text{ kg/m}^3$$

$$\text{Radius of the gold sphere}(R_1) = 1 \text{ m}$$

$$\text{Thickness of the gold sphere}(T_1) = 0.1 \text{ m}$$

$$\text{Radius of the copper sphere}(R_2) = 0.1 \text{ m}$$

$$\text{Thickness of the copper sphere}(T_2) = 0.05 \text{ m}$$

$$\text{Gravitational constant}(G) = 6.67430 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$$

$$\text{Distance between the centers of the shells}(D_{\text{center}}) = R_1 + R_2 + 0.08 \text{ m}$$

Volume of the gold and copper shells:

$$V_1 = \frac{4}{3}\pi(R_1^3 - (R_1 - T_1)^3)$$

$$V_2 = \frac{4}{3}\pi(R_2^3 - (R_2 - T_2)^3)$$

Mass of the gold and copper shells:

$$M_1 = \rho_{\text{gold}} \times V_1$$

$$M_2 = \rho_{\text{copper}} \times V_2$$

Gravitational force:

$$F = \frac{G \times M_1 \times M_2}{D_{\text{center}}^2}$$

$$F \approx 0.0000345 \text{ N}$$

Comment: Correct.

GPT4+CI: Correct.

GPT4 default: $4.26 \cdot 10^{-9}$ Newtons.

Comment: Correct formulas, errors in calculation.

- Let b, c, d be points at the center of Buenos Aires, Cincinnati, and Delhi, respectively. Consider the plane P that contains b, c, and d (it cuts through the earth). What is the area in square kilometers of the triangle b,c,d lying in P?

Answer: 41,111,600 km².

Matlab

```
% Convert latitude and longitude to a unit 3D vector
function v=LatAndLongToUnit(LatDeg, LatMin, LatS, LongDeg, LongMin, LongS)
Theta = LatS*(LatDeg+(LatMin/60))*(pi/180);
Phi = LongS*(LongDeg+(LongMin/60))*(pi/180);
v = [cos(Theta)*cos(Phi), cos(Theta)*sin(Phi), sin(Theta)];
end

b=LatAndLongToUnit(34, 36, -1, 58, 22, -1)
c=LatAndLongToUnit(39, 6, 1, 84, 30, -1)
d=LatAndLongToUnit(28, 37, 1, 77, 14, 1)
bc=c-b
bd=d-b
cd=d-c
EarthRadius=6371; % The magnitude of the cross product
answer = norm(cross(bc,bd))*EarthRadius^2/2 % is the area of the parallelogram
x=norm(bc)
y=norm(bd)
z=norm(cd)
s=(x+y+z)/2
check = sqrt(s*(s-x)*(s-y)*(s-z))*EarthRadius^2 % Heron's formula
```

GPT4+WA Answer: The area of the triangle formed by the cities Buenos Aires, Cincinnati, and Delhi on the plane that cuts through the Earth is approximately $1.9745222932407066 \cdot 10^7$ square kilometers.

$$\begin{aligned} b &= \text{GeoPosition} \left[\left\{ 34 + \frac{36}{60} + \frac{36}{3600}, -\left(58 + \frac{22}{60} + \frac{12}{3600}\right) \right\} \right] \\ c &= \text{GeoPosition} \left[\left\{ 39 + \frac{8}{60} + \frac{24}{3600}, -\left(84 + \frac{30}{60} + \frac{23}{3600}\right) \right\} \right] \\ d &= \text{GeoPosition} \left[\left\{ 28 + \frac{40}{60} + \frac{12}{3600}, 77 + \frac{12}{60} + \frac{36}{3600} \right\} \right] \\ \text{triangle} &= \text{GeoPolygon} [\{b, c, d\}] \\ \text{area} &= \text{GeoArea} [\text{triangle}, \text{UnitSystem} \rightarrow \text{"Metric"}] \\ &\approx 1.9745222932407066 \times 10^7 \text{ km}^2 \end{aligned}$$

Comment: Wrong. At least part of the problem is that it takes the latitude of Buenos Aires to be 28° N rather than 23° S. Also, it is computing the area of the spherical triangle on the sphere rather than the triangle in the plane.

GPT4+CI Answer: The area of the triangle formed by Buenos Aires, Cincinnati, and Delhi in the plane that cuts through the Earth is approximately $1.16 \cdot 10^{14}$ square kilometers.

Comment: Off by a factor of $2.8 \cdot 10^6$. GPT computes the area of the spherical triangle (not the triangle in the plane) in square kilometers and then makes the following mistake:

```
area_sq_km = area * 10**6
```

GPT4 default

Calculating the area of a triangle on a plane that spans thousands of kilometers across the Earth is complex, and will require some approximations. The Earth's surface is not flat, but nearly spherical (technically, an oblate spheroid), so a plane triangle doesn't perfectly represent the actual distances and areas. However, for simplicity, we can make a rough approximation using the spherical model of the Earth.

We first need to find the great circle distances between Buenos Aires, Cincinnati, and Delhi. These are the shortest distances between two points on the surface of a sphere, which we can calculate using the Haversine formula. However, we also need the specific latitudes and longitudes for these cities. As of my knowledge cutoff in September 2021, the coordinates are:

Buenos Aires, Argentina: -34.6037° S, 58.38° W

Cincinnati, Ohio, USA: 39.1031° N, 84.5120° W

Delhi, India: 28.7041° N, 77.1025° E

Using the Haversine formula, the great circle distances between these cities (given the Earth's average radius of approximately 6371 km) can be calculated.

Let's denote the distances as follows:

Buenos Aires to Cincinnati = BC

Buenos Aires to Delhi = BD

Cincinnati to Delhi = CD

Next, to calculate the area of a spherical triangle (the triangle that lies on the surface of the sphere, as opposed to the planar triangle that lies on the plane passing through the three points), we can use the formula derived from the spherical excess theorem:

$$\text{Area} = E \cdot R^2$$

Where: $E = BC + BD + CD - \pi$ (spherical excess, where BC, BD, CD are the angles of the triangle in radians) R = radius of the Earth (approximately 6371 km)

Please note that the distances calculated above are the sides of the spherical triangle (in km), and these sides should be converted to radians by dividing by the radius of the Earth before being used in the spherical excess calculation. We use these angles (in radians), not the distances, in the formula for the spherical excess.

The output will be the area of the spherical triangle. However, you are asking for the area of the triangle on the plane passing through the three points. This can be quite different from the area of the spherical triangle, especially for large triangles spanning a significant fraction of the globe. Such a plane triangle is generally smaller than the corresponding spherical triangle because the sphere's surface curves away from the plane.

Unfortunately, calculating the area of this plane triangle accurately is more complicated because the Earth's surface isn't flat and the triangle's vertices are not at the same altitude (they're at the Earth's surface, which curves). As such, a purely planar calculation would not be appropriate.

But, if we assume the Earth is a perfect sphere and we approximate the plane by a "great circle plane" that passes through the Earth's center and the three cities, then the triangle would be a spherical triangle and we could use the above-mentioned area formula for a spherical triangle.

Keep in mind that these are only approximations and that the actual distances and areas on the Earth's surface may be slightly different due to its oblate spheroidal shape and variations in topography. Furthermore, the "plane" triangle you are asking about is not a standard geometrical figure and cannot be accurately calculated with simple formulae.

Comment: Declined to answer. What it says is true, as far as that goes.

3. X is a regular tetrahedron of zinc with a side length of 2 moving at 100 m/sec. Y is a molecule of NaCl that is moving with speed v . X and Y have equal kinetic energy. What is $1 - (v/c)$, where c is the speed of light?

Answer: $8.41 \cdot 10^{-21}$

Matlab:

```
ZincVolume=2^3/(6*sqrt(2)) % Wolfram Alpha
ZincDensity = 7.14 % gm/cm^3. Wikipedia
ZincMass = ZincVolume*ZincDensity
VZinc = 100/(3*10^8) % Speed of Zinc in units where C=1
ZincKE = ZincMass*VZinc^2
NaClMass = 9.7*10^(-23) % gm. Wolfram Alpha
LorenzFactor = ZincKE/NaClMass
Q = 1/LorenzFactor^2 % 1-(v/c)^2
Answer = Q/2
```

GPT4+WA Answer: The value of $1 - v/c$ is approximately -87665.67

GPT4+WA Explanation: *GPT makes 6 calls to WA, to retrieve some of the physical constants and to carry out the computations.*

- (a) Calculate the mass of the regular tetrahedron of zinc:

The volume V of a regular tetrahedron with side length a is given by the formula:

$$V = \frac{a^3}{6\sqrt{2}}$$

The density ρ of zinc is 7.14 g/cm^3 . So, the mass m of the tetrahedron is given by $m = \rho V$. The mass of the tetrahedron in kg is approximately 0.00673 kg .

- (b) Use the kinetic energy formula to find the kinetic energy of the tetrahedron:

The kinetic energy formula is $KE = mv^2/2$ where m is the mass and v is the velocity. The kinetic energy of the tetrahedron is approximately 33.65 J .

- (c) Set the kinetic energy of the tetrahedron equal to the kinetic energy of the NaCl molecule and solve for v :

The mass m of a NaCl molecule is approximately $9.704 \cdot 10^{26}$.

Using the kinetic energy formula, the velocity v of the NaCl molecule is approximately $2.62 \cdot 10^{13} \text{ m/s}$.

- (d) Calculate $1 - v/c$ where c is the speed of light:

The speed of light c is approximately $3 \cdot 10^8 \text{ m/s}$. The value of $1 - v/c$ is approximately -87665.67 .

Comment: Wrong. GPT has failed to consider the relativistic correction on kinetic energy in computing the speed of the molecule.

GPT4+CI Answer: The value of $1 - v/c$ is approximately -87665.67

GPT4+CI's discussion after generating the answer:

Upon reviewing the calculations, it seems that the mass of the zinc tetrahedron is much larger than the mass of the NaCl molecule, which leads to an extraordinarily high speed for the NaCl molecule when we set the kinetic energies of the two objects equal.

This result appears to be correct given the input parameters: a large zinc tetrahedron and a single NaCl molecule. However, the result implies that the NaCl molecule would need to travel

much faster than the speed of light to have the same kinetic energy as the zinc tetrahedron, which is not physically possible according to the theory of relativity.

This suggests that there may be a misunderstanding or mistake in the problem as it is currently stated. Could you please check the problem statement or provide additional information?

Comment: GPT does not seem to have a reliable thorough grasp of the theory of special relativity.

GPT4 with no plug-ins: *Described a similar computation and found all the correct physical values and formulas, but did not attempt the calculation.*

4. Viewed from Vega, what is the angle between Sirius and the Sun?

Answer: 0.0972 radians

Matlab:

```
AngleSEV = 2.755 % radians. Wolfram Alpha
DistES = 8.597 % light years. Wolfram Alpha
DistSV = 33.41 % light years. Wolfram Alpha
AngleEVS = asin(sin(AngleSEV)*DistES/DistSV)
```

GPT4+WA Answer: The angle between Sirius and the Sun, as viewed from Vega, is approximately 0.005060 degrees.

GPT4+WA Calculation:

The coordinates of the three celestial bodies are as follows:

Sirius: Right Ascension (RA) = 6h 45m 8.92s, Declination (Dec) = $-16^{\circ} 42' 58''$

Vega: RA = 18h 36m 56.336s, Dec = $38^{\circ} 47' 1.3''$

Sun: RA = 6h 29m 58.95s, Dec = $23^{\circ} 35' 40.02''$

We can calculate the angle between two points on the celestial sphere using the spherical law of cosines. The formula is: $\cos(\delta) = \sin(\delta_1) \sin(\delta_2) + \cos(\delta_1) \cos(\delta_2) \cos(\alpha_1 - \alpha_2)$ where:

δ is the angle between the two points,

δ_1 and δ_2 are the declinations of the two points,

α_1 and α_2 are the right ascensions of the two points.

We need to calculate two angles: between Sirius and the Sun, and between Sirius and Vega. Then, we subtract the first from the second to get the angle between the Sun and Vega as viewed from Sirius.

$$\begin{aligned}
\delta_1 &= -16 - \frac{42}{60} - \frac{58}{3600} \\
\alpha_1 &= 6 + \frac{45}{60} + \frac{8.92}{3600} \\
\delta_2 &= 23 + \frac{15}{60} + \frac{40.02}{3600} \\
\alpha_2 &= 6 + \frac{29}{60} + \frac{48.95}{3600} \\
\delta_3 &= 38 + \frac{47}{60} + \frac{1.3}{3600} \\
\alpha_3 &= 18 + \frac{36}{60} + \frac{56.336}{3600} \\
\text{angle}_1 &= \arccos(\sin(\delta_1)\sin(\delta_2) + \cos(\delta_1)\cos(\delta_2)\cos(\alpha_1 - \alpha_2)) \\
\text{angle}_2 &= \arccos(\sin(\delta_1)\sin(\delta_3) + \cos(\delta_1)\cos(\delta_3)\cos(\alpha_1 - \alpha_3)) \\
\text{angle} &= \text{angle}_2 - \text{angle}_1 \\
\text{angle} &\approx 0.00506 \text{ degrees}
\end{aligned}$$

Comment: GPT has entirely misunderstood the geometric situation. What it has done is to compute the direction of the three stars as viewed from earth (as other parts of the output make clear, the angle to the sun is computed as viewed from San Antonio, presumably at the time the experiment was run.) Thus, the three stars are viewed as being on the surface of a sphere centered at earth, and then the spherical angle between the line from Vega to Sun and the line from Vega to Sirius is computed. We have not found the source for the trigonometric formula, converting ascensions and declination angles into an angle on the unit sphere, so we do not know whether this is correct for the problem being solved. There is no indication that GPT or WA has converted degrees to radians in doing the calculation. Finally, the calculation itself does not seem to be done correctly, whether the angles are interpreted as degrees or radians.

GPT4+CI Answer: Didn't answer the question. Didn't call CI.

GPT4+CI Output:

The angle between Sirius and the Sun as viewed from Vega depends on the time of year, as well as the specific time of day. This is because the Earth and Vega both orbit the Sun, but at different rates, and because the Earth also rotates on its axis.

There were two more paragraphs, equally far from the mark.

Comment: No answer.

5. You have an empty cylindrical open container whose inside has a diameter of 8 centimeters and a height of 20 centimeters. and a pitcher with 200 ccs of water. You first empty the pitcher into the cylinder, then put a solid rock cube, 4 cm on a side, into the container so that it is sitting flush against the bottom of the container. What is the height of the water in the container?

Answer: 5.2521 cm

Matlab:

```
AreaOfCyl = pi*4^2
AreaAtBottom = AreaOfCyl-4^2
VolumeAtBottom = 4*AreaAtBottom
VolumeAtTop = 200 - VolumeAtBottom
```

HeightAtTop = VolumeAtTop/AreaOfCyl
Height = 4+HeightAtTop

GPT4+WA Answer: The height of the water in the cylinder after the cube is placed in it is approximately 2.72 cm.

$$\begin{aligned}V_{\text{cylinder}} &= \pi r^2 h \\&= \pi (4 \text{ cm})^2 (20 \text{ cm}) \\&= 320\pi \text{ cm}^3 \\&\approx 1005.31 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}V_{\text{cube}} &= a^3 \\&= (4 \text{ cm})^3 \\&= 64 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}V_{\text{water}} &= V_{\text{initial}} - V_{\text{cube}} \\&= 200 \text{ cm}^3 - 64 \text{ cm}^3 \\&= 136 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}h_{\text{water}} &= \frac{V_{\text{water}}}{\pi r^2} \\&= \frac{136 \text{ cm}^3}{\pi (4 \text{ cm})^2} \\&\approx 2.72 \text{ cm}\end{aligned}$$

Comment: GPT has misunderstood the situation. It computes the volume of the water minus the volume of the rock, and then computes the height of that amount of water poured into an empty cylinder.

GPT + CI Answer: 5.25 cm high.

Comment: Exactly right.

6. There are six small spherical masses placed at various positions in space. Measured in a coordinate system with a 1 meter unit length:

A has mass 2 kg and is at $\langle 2, 0, 0 \rangle$;

B has mass 3 kg and is at $\langle 0, 2, 0 \rangle$;

C has mass 1 kg and is at $\langle 0, 0, 2 \rangle$;

D has mass 4 kg and is at $\langle 1, 1, 1 \rangle$;

E has mass 5 kg and is at $\langle 1, 0, 1 \rangle$;

F has mass 2 kg and is at $\langle 0, 1, 1 \rangle$.

What is the instantaneous acceleration of F under the gravitational attraction of A,B,C,D,E? Express your answer as a vector.

Answer: $\langle 0.403, -0.080, -0.056 \rangle * 10^{-9} \text{ m/sec}^2$

Matlab:


```
% Gravitational force exerted on mass ma at point pa by mass
% mb at point pb, taking the gravitational constant to be 1.
```

```
function f = GravForce(ma,pa,mb,pb)
    v = pb-pa;
    r = norm(v);
    u = v/norm(v);
    f = ma*mb*u/(r^2)
end

g1 = GravForce(2,[0,1,1],2,[2,0,0]) + ...
GravForce(2,[0,1,1],3,[0,2,0]) + ...
GravForce(2,[0,1,1],1,[0,0,2]) + ...
GravForce(2,[0,1,1],4,[1,1,1]) + ...
GravForce(2,[0,1,1],5,[1,0,1])
GravConst = 6.674*10^(-11) % N*kg^2/m^2 -- Wikipedia
acc = g1*GravConst/2
```

GPT4+WA Answer: $\langle 1.6690710^{10}, 1.6690710^{10}, 1.6690710^{10} \rangle \text{m/s}^2$ This means that mass F is accelerating towards the origin of the coordinate system, with each component of its acceleration vector having approximately the same magnitude.

$$\begin{aligned}
 G &= 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \\
 \vec{r}_A &= (2, 0, 0), \quad m_A = 2 \text{ kg} \\
 \vec{r}_B &= (0, 2, 0), \quad m_B = 3 \text{ kg} \\
 \vec{r}_C &= (0, 0, 2), \quad m_C = 1 \text{ kg} \\
 \vec{r}_D &= (1, 1, 1), \quad m_D = 4 \text{ kg} \\
 \vec{r}_E &= (1, 0, 1), \quad m_E = 5 \text{ kg} \\
 \vec{r}_F &= (0, 1, 1), \quad m_F = 2 \text{ kg} \\
 \vec{a}_F &= G \left(\frac{m_A}{|\vec{r}_F - \vec{r}_A|^2} \frac{\vec{r}_A - \vec{r}_F}{|\vec{r}_F - \vec{r}_A|} + \frac{m_B}{|\vec{r}_F - \vec{r}_B|^2} \frac{\vec{r}_B - \vec{r}_F}{|\vec{r}_F - \vec{r}_B|} + \frac{m_C}{|\vec{r}_F - \vec{r}_C|^2} \frac{\vec{r}_C - \vec{r}_F}{|\vec{r}_F - \vec{r}_C|} + \right. \\
 &\quad \left. \frac{m_D}{|\vec{r}_F - \vec{r}_D|^2} \frac{\vec{r}_D - \vec{r}_F}{|\vec{r}_F - \vec{r}_D|} + \frac{m_E}{|\vec{r}_F - \vec{r}_E|^2} \frac{\vec{r}_E - \vec{r}_F}{|\vec{r}_F - \vec{r}_E|} \right) \\
 \vec{a}_F &= (-1.66907 \times 10^{-10}, -1.66907 \times 10^{-10}, -1.66907 \times 10^{-10}) \text{ m/s}^2
 \end{aligned}$$

Comment: It is not clear what is going wrong here. The formula given in Latex is correct. GPT comment that mass F is accelerating toward the origin is incorrect, since F is at $\langle 0, 1, 1 \rangle$. The raw output from Wolfram Alpha is very long and unformatted; we have not attempted to analyze it.

GPT + CI Answer: $\langle 4.03 \cdot 10^{-10}, -7.99 \cdot 10^{-11}, -5.63 \cdot 10^{-11} \rangle \text{ m/s}^2$.

Comment: Exactly right.

- How many total eclipses of the moon were there between Jules Verne's death and Neil Armstrong's moon landing? An exact integer value is required.

Answer: 52.

Jules Verne died March 24, 1905.

Neil Armstrong landed on the moon July 20, 1969

There were 52 total lunar eclipses in between:
02-09-06, 08-04-06, 06-04-09, 11-27-09, 05-24-10, 11-17-10,
03-22-13, 09-15-13, 01-08-17, 07-04-17, 12-28-17, 05-03-20,
10-27-20, 04-22-21, 02-20-24, 08-14-24, 06-15-27, 12-08-27,
06-03-28, 11-27-28, 04-02-31, 09-26-31, 01-19-35, 07-16-35,
01-08-36, 05-14-38, 11-07-38, 05-03-39, 03-03-42, 08-26-42,
12-19-45, 06-14-46, 12-08-46, 04-13-49, 10-07-49, 04-02-50
09-26-50, 01-29-53, 07-25-53, 01-19-54, 11-18-56, 05-13-57,
11-07-57, 03-13-60, 09-05-60, 12-30-63, 06-25-64, 12-19-64,
04-24-67, 10-18-67, 04-13-68, 10-06-68

Source: Wikipedia article, "List of lunar eclipses in the twentieth century."

GPT4+WA Answer: 1.

GPT4+WA Computation: This is the complete explanation generated by GPT.

```
Total Lunar Eclipses=  
Length(TotalLunarEclipse(DateRange(DateObject[{1905,3,24}],  
DateObject[{1969,7,20}]))))  
=1
```

Comment: It is hard to guess why Wolfram Alpha got this so wrong.

GPT4+CI Answer:

Jules Verne died on March 24, 1905, and Neil Armstrong landed on the moon on July 20, 1969.

A lunar eclipse occurs when the Earth comes between the Sun and the Moon, blocking the sunlight that usually reflects off the Moon's surface, and causing the Moon to appear dark from Earth.

Lunar eclipses can be total (the entire Moon is covered by the Earth's shadow), partial (only part of the Moon is covered), or penumbral (the Moon passes through the outer part of the Earth's shadow, causing a much more subtle darkening).

For this question, we're only interested in total lunar eclipses, which occur roughly twice a year.

As an AI model developed by OpenAI, my ability to access real-time data or databases is restricted as of my last update in September 2021. However, data about lunar eclipses is based on predictable astronomical phenomena, so we can use data available up to 2021 to extrapolate for this period.

Unfortunately, I don't have the capability to compute this directly because it involves complex astronomical calculations. You would need to access a specific database or astronomical software to get an exact count of the total lunar eclipses that occurred between March 24, 1905, and July 20, 1969.

NASA maintains a comprehensive catalog of lunar eclipses dating back to -1999 (2000 BCE) and projected forward to +3000 (3000 CE). This catalog would be ideal for answering your question. You can find it on NASA's website and use the search or sort functions to count the total lunar eclipses between the two dates you're interested in.

I apologize for any inconvenience this may cause. If you have any other questions or need assistance with a different topic, feel free to ask!

8. A quantity of chlorine gas is in a right prism whose base is a triangle with sides 5 cm, 7 cm, and 4 cm and whose altitude is 8 cm. The temperature is the freezing point of mercury, and the pressure is 2 atmospheres. What is the mass of the chlorine?

Answer: 0.5781 gm

Matlab:

```
s=(5+7+4)/2
Area = sqrt(s*(s-5)*(s-7)*(s-4)) % Heron's formula
VolumeCC = Area * 8
Liters = VolumeCC / 1000
Temp = 273.15 -38.83 % Wikipedia
IdealGasConst = 0.082057 % L*atm*K^{-1}*mol^{-1}. Wikipedia
Moles = Liters*2/(Temp*IdealGasConst)
MassPerMoleCl2 = 70.9 % gm. Wolfram Alpha.
Mass = Moles * MassPerMoleCl2
```

GPT4+WA Answer: The mass of the chlorine gas in the prism is approximately 0.0005780932129660225 kg.

GPT4+WA Output:

We use the ideal gas law: $PV = nRT$

Where:

P = pressure = 2 atm

V = volume = $78.3837 \times 10^{-6} \text{ m}^3$

R = ideal gas constant = $0.08206 \text{ L atm K}^{-1}\text{mol}^{-1}$

T = temperature = $273.15 - 38.83 \text{ K}$

n = number of moles = $\frac{PV}{RT}$

m = mass = $n \times$ molar mass of chlorine = $n \times 70.906 \text{ g/mol}$

So,

$m \approx 0.0005780932129660225 \text{ kg}$

Comment: Exactly right.

GPT4 default: Sets up the equations correctly but make mistakes doing the computation. E.g. when computing the volume it correctly computes the area of the base as $\sqrt{98}$ but then writes for the volume $V = \sqrt{96} \cdot 8 = \sqrt{769} = 27.7$

GPT4+CI Answer: The mass of the chlorine gas in the prism is approximately 0.578g

Comment: Exactly right.

9. A train has two whistles, one at middle C and one at the F above middle C. The train is driving past a station without stopping or changing speed. On approaching the station, it blows the low whistle; when it passes the station, it switches to the high whistle. However, to the people standing at the station, it sounds like the whistle dropped by a whole tone. How fast is the train moving?

Answer: 68.4 m/s

Explanation: A half-tone in music is a factor of $2^{1/12}$. The interval from C to F is 5 half-tones. So the effect is that a note as the train approaches is dropped by 7 half tones or a factor of $2^{7/12}$. Let s be the speed of sound, then $(s + v)/(s - v) = 2^{7/12}$. Solving for v we get

$$v = s \cdot \frac{2^{7/12} - 1}{2^{7/12} + 1}$$

The speed of sound is 343 m/s so the train is moving at 68.4 m/s. (One can work this out by hand to an accuracy of 1% using the fact that a fifth in music – 7 half tones – is 1.5 in the natural musical scale.)

GPT4+WA Answer: The speed of the train is approximately 86.94 m/s.

GPT4+WA Calculation: The problem you're describing is a classic example of the Doppler effect, which is the change in frequency or wavelength of a wave in relation to an observer who is moving relative to the wave source. In this case, the wave source is the train's whistle and the observer is a person standing at the station.

The frequency of middle C is approximately 261.63 Hz and the frequency of the F above middle C is approximately 349.23 Hz. A whole tone lower than F is E, which has a frequency of approximately 329.63 Hz.

GPT then attempts to set up an equation for Wolfram Alpha to solve. Wolfram Alpha is unable to solve the first two equations that are presented to it — perhaps a formatting mistake in the Wolfram Alpha input language. On GPT's third attempt to formulate the equation, Wolfram Alpha finds the solution $v=86.04$. Unfortunately, the large number of unreadable symbols and other format issues makes it impossible to determine what any of these equations are.

Comment: GPT has misunderstood the problem. The problem that it is solving is to determine, if the train is sounding the C whistle as it approaches and it sounds as a whole tone under F, how fast is the train moving? GPT makes a second mistake in thinking that E is a whole tone under F; a whole tone under F is actually E flat. Finally, the final solution returned does not correspond to GPT's formulation of the problem described above; it must have made an error in setting up the equation.

GPT4+CI Answer: $64.87 \text{ /s} = 145 \text{ mph}$

GPT4+CI Output: *GPT interpreted the problem correctly but made a mistake on the equation for the Doppler shift in an approaching object. GPT wrote*

$$f' = f \left(\frac{c + v}{c} \right)$$

whereas the equation should be

$$f' = f \left(\frac{c}{c - v} \right)$$

Comment: Wrong.

10. A physical process generates photons whose energies follow a random distribution of the following form: For positive energy e , the probability density at e is proportional to the value of e in a Gaussian distribution with mean 2 Ev and standard deviation 0.01 Ev . The probability of a negative value is zero. What is the expected value of the wavelength of a photon produced by this process? (Give the mathematical answer, assuming that the above description is exact, and assuming the standard relation between energy and wavelength in a photo. The answer is not physically plausible.)

Answer: Infinite.

Explanation: The wavelength of a photon with energy e is hc/e . The mean wavelength of a photon in the beam is therefore

$$\int_{e=0}^{\infty} N_{2,0.01}(e) \cdot \frac{hc}{e} de$$

where $N_{2,0.01}$ is the Gaussian distribution with mean 2.0 and standard deviation 0.1. In the limit as $e \rightarrow 0^+$, the quantity $N_{2,0.01}(e)$, though extremely tiny, is still a positive quantity, so the integral diverges at 0.

GPT4+WA Answer: Unable to answer.

GPT4+WA Output:

GPT4 begins by using the physical principles involved to correctly set up the above integral. It calls WA to compute the integral. This call fails, because GPT4 has used the symbols E as the variable of integration, and E is a reserved symbol in WA. GPT4 rewrites the integral using variable x and tries WA again:

REQUEST TO WOLFRAM

```
{ "input": "mu = 2; sigma = 0.01; h = 6.62607015*10^-34; c =
299792458; eV = 1.602176634*10^-19; integral =
NIntegrate[(1/(x*sqrt[2*pi*sigma^2]))*Exp[-(x-mu)^2/(2*sigma^2)], {x,
0, Infinity}]; wavelength = h*c/(integral*eV); wavelength" }
```

RESPONSE FROM WOLFRAM

"ComplexInfinity

Errors generated during evaluation:

Integrate::izero : Integral and error estimates are 0 on all integration subregions. Try increasing the value of the MinRecursion option. If value of integral may be 0, specify a finite value for the AccuracyGoal option.

Power::infy : Infinite expression -- encountered.
0."

Note: As far as we can judge, the response from WA does not mean that the integral diverges; it means that the integral seems to be 0, but Wolfram Alpha is not confident of that. GPT then writes:

I apologize for the confusion. It seems that the integral is not converging, which is causing issues with the calculation. This might be due to the singularity at $E = 0$ and the rapid decay of the Gaussian function for large E .

A possible workaround is to use a change of variables to avoid the singularity. Let's try using $u = 1/E$.

GPT's attempt to change the variable of integration leads it to an incorrect integral form. It calls WA and gets the same error message as above. GPT then concludes:

I apologize for the confusion. It seems that the integral is not converging, which is causing issues with the calculation. This might be due to the singularity at $E = 0$ and the rapid decay of the Gaussian function for large E .

In practice, this kind of problem might be approached using numerical methods or approximations. However, it's important to note that the physical interpretation of this problem is somewhat unusual: the energy of a photon cannot be negative, and a Gaussian distribution of energies would not typically be cut off at $E = 0$. This could be part of the reason why the calculation is not yielding a physically meaningful result.

I'm sorry for any confusion. If you have any other questions or need further clarification, please let me know!

Comment: GPT4+WA correctly discerns that the integral is problematic but does not determine that it diverges. Score: 0.75

GPT4+CI Answer: GPT4+CI sets up the integral correctly and calls CI to evaluate it. However, that returned an error. GPT continued:

I apologize for the confusion. The integral computation seems to be failing due to the complexity of the integrand, which involves a combination of exponential and reciprocal functions. This is a known limitation of symbolic computation systems, which sometimes struggle with certain types of integrals.

However, it's worth noting that in this particular case, the expected value of the wavelength of a photon produced by this process doesn't have a straightforward physical interpretation. The energy of a photon is quantized, meaning it can only take on certain discrete values, and the relationship between energy and wavelength is inversely proportional. So, even if we could calculate this expected value, it wouldn't necessarily correspond to a possible state of the system.

If you're interested in a more approximate or numerical approach to this problem, it could be possible to sample energies from the given Gaussian distribution, compute the corresponding wavelengths, and then calculate the average wavelength from the sample. This would give a Monte Carlo estimate of the expected value of the wavelength. However, keep in mind that this would still be a statistical estimate and not an exact calculation.

Comment: Wrong.

11. The wavelengths of the photons in a beam of light are uniformly distributed across the range of visible (to humans) light. What is the mean energy of a photon in electron volts?

Answer: 2.3688 eV.

Explanation: The range of visible light is around 3800 to 7000 Angstroms. A photon with wavelength λ has energy hc/λ . Thus, if λ is uniformly distributed between 3800 and 7000, the mean energy is

$$\int_{\lambda=3800}^{7000} (1/3200)(hc/\lambda) d\lambda = (hc/3200) \log_e \lambda \Big|_{3800}^{7000}$$

Matlab:

```
Planck = 4.136 * 10^(-15) % eV*sec
SpeedOfLight = 3*10^8 % m/sec
Meter2Ang = 10^(10)
SpeedOfLight * Meter2Ang * Planck / 3200 * log(7000/3800)
```

GPT4+WA Answer: 2.19 eV.

GPT4+WA Explanation: The range of wavelengths for visible light is from 380 nm (violet light) to 750 nm (red light). The energy of a photon can be calculated using the formula:

$$E = \frac{hv}{\lambda}$$

Since the wavelengths are uniformly distributed, the mean wavelength is the average of the minimum and maximum wavelengths. Therefore, we can calculate the mean energy of a photon in the visible light spectrum by substituting the mean wavelength into the formula.

Let's calculate the mean wavelength first.

The mean wavelength of the visible light spectrum is 565 nm. Now, let's calculate the energy of a photon with this wavelength. Note that we need to convert the wavelength from nanometers to meters (1 nm = 11091109 m) for the calculation.

The mean energy of a photon in the visible light spectrum is approximately 2.19 eV.

Calculate the mean energy of a photon using the formula

$$E_{mean} = \frac{hv}{\lambda_{mean}} = \frac{4.135667696 \cdot 10^{15} \text{eV}\cdot\text{s} \cdot 2.99792458 \cdot 10^8 \text{m/s}}{565 \cdot 10^9 \text{m}} = 2.19 \text{eV}$$

Comment: The method that GPT4+WA is using — calculate the mean wavelength, then apply the formula to compute the mean energy — would work if the energies were uniformly distributed, but not if the wavelengths are uniformly distributed.

GPT4+CI Answer: 2.31 eV.

Comment: Correct. The reason the answer is numerically different from ours above is that we took 380 nm as the lower bound on visible light whereas GPT took that to be 400 nm.

12. A pendulum is hanging on a 2 meter cord attached to the ceiling 3 meters above the floor. It is brought to a position 25 degrees from the vertical and released. It swings past the bottom and the cord is cut when it is 10 degrees from the vertical on the far side. Then the bob flies through the air and hits the ground. What is the distance from the point where the bob is released to the point where it hit the ground?

Answer: 2.3606 meters

```
% Take the point of attachment of the pendulum as the origin
g = 9.8 % Acceleration of terrestrial gravity in m/sec^2
d2r = pi/180 % Degrees to radians
s25 = sin(25*d2r) % Handy abbreviations
c25 = cos(25*d2r)
s10 = sin(10*d2r)
c10 = cos(10*d2r)
pr = [-2 * s25, -2*c25] % release point
pc = [2 * s10, -2*c10] % cut point
hd = 2 * (c10 - c25) % height difference between the cutpoint and the release point.
s = sqrt(2*g*hd) % speed of the bob at release time
vc = s * [c10,s10]; % velocity of bob when cut
tca = vc(2)/g % time between cut and apex of flight
hca = (g*tca^2)/2 % height difference from cut point to apex
ha = -2*c10 + hca % height of apex of flight
haf = 3 + ha % height difference between apex and floor
taf = sqrt(2*haf/g) % time from apex to floor
pf = [pc(1) + (tca+taf)*vc(1),-3]
answer = norm(pf-pr)
```

GPT4+WA Answer: 1.35m.

GPT4+WA Calculation:

$$\begin{aligned}
L &= 2 \text{ m} && \text{(length of the pendulum)} \\
g &= 9.81 \text{ m/s}^2 && \text{(acceleration due to gravity)} \\
\theta_{\text{initial}} &= 25^\circ && \text{(initial angle from the vertical)} \\
\theta_{\text{final}} &= 10^\circ && \text{(final angle from the vertical)} \\
h_{\text{total}} &= 3 \text{ m} && \text{(total height from the ground)} \\
\Delta h &= L - L \cos(\theta_{\text{initial}}) - (L - L \cos(\theta_{\text{final}})) && \text{(change in height)} \\
v &= \sqrt{2g\Delta h} && \text{(speed of the bob when the cord is cut)} \\
t &= \sqrt{\frac{2h_{\text{total}}}{g}} && \text{(time it takes for the bob to hit the ground)} \\
d &= v \cos(\theta_{\text{final}})t && \text{(horizontal distance the bob travels)} \\
d &\approx 1.35 \text{ m} && \text{(distance from the point where the bob is released} \\
&&& \text{to the point where it hits the ground)}
\end{aligned}$$

Comment: GPT has calculated some relevant quantities, but overall its calculation is off-track and the final value is not only wrong but meaningless. Specifically: Δh is correctly the distance in height between the release point and the cut point. v is correctly the speed at the cut point. t is the time it would take to fall from the ceiling to the floor, which is irrelevant. d is the horizontal distance travelled by the bob at the cut point if it could travel for time t , which would be relevant, but not the final answer, if t were the right value.

GPT4+CI Answer: 10.7 cm.

Comment: Wrong. Again, GPT invokes some correct equations and computes some relevant quantities, but again it misunderstands the situations and invokes an irrelevant equation.

13. An spherical asteroid 500 km in diameter travels in an essentially perfect circular orbit of radius 2.4 astronomical units. On January 4, 2023, the earth was at perihelion, and as it happens, the asteroid was in perfect opposition. What was the solid angle of the asteroid in the sky as seen by an earth observer?

Answer: $4.35 \cdot 10^{-12}$

Matlab:

```

perihelion = 1.471 * 10^8 % km. Wikipedia
au = 1.498 * 10^8 % km. Wikipedia
asteroid = 2.4*au
distance = asteroid - perihelion
radiusAngle = 250/distance
solidAngle = pi*radiusAngle^2

```

GPT4+WA Answer: $6.09 \cdot 10^{-12}$ steradians.

GPT4+WA Calculation:

$$\begin{aligned}
d &= 500 \text{ km} && \text{(diameter of the asteroid)} \\
D &= 2.4 \times 149597870.7 \text{ km} && \text{(distance to the asteroid)} \\
\theta &= \arctan\left(\frac{d}{D}\right) && \text{(angular radius in radians)} \\
\Omega &= 2\pi (1 - \cos(\theta)) && \text{(solid angle in steradians)} \\
\Omega &\approx 6.09 \times 10^{-12} \text{ steradians}
\end{aligned}$$

Comment: GPT is using the distance from the sun to the asteroid, not the distance from the earth to the asteroid.

GPT4+CI Answer: $4.27 \cdot 10^{-12}$.

Comment: Right. The small discrepancy between this and ours above is almost certainly due to the small-angle approximation that we made.

14. Two twin stars, one of 5 solar masses, the other of 10 solar masses, orbit each other in circular orbits in a period of 4 earth years. How far apart are they, in astronomical units?

Answer: 6.21 Au.

Explanation:

Let L be the distance between them in AUs. They each rotate around in a circle centered at their center of gravity, which is $2L/3$ from the small star and $L/3$ from the large star.

Let $M = 5$ be the mass of the smaller star in solar masses.

Let Q be the ratio of the centripetal acceleration of the large star to the centripetal acceleration of the earth in its orbit.

Comparing the gravitational acceleration, $Q = M/L^2 = 5/L^2$ Comparing the centripetal acceleration, the star is moving on a path of radius $L/3$ Au at an angular velocity $1/4$ of earth. Hence $Q = L/48$. So $L^3 = 5 \cdot 48$, so $L=6.21$ Au.

Matlab: Alternative solution using a generalized form of Kepler's third law:

$$a^3/T^2 = G(m_1 + m_2)/4\pi^2.$$

Year = $3.154 \cdot 10^7$ %sec

Grav = $6.6743 \cdot 10^{-11}$ % N*m²/kg²

SolarMass = $1.99 \cdot 10^{30}$ % kg

M1 = 10*SolarMass

M2 = 5*SolarMass

Period = 4*Year

DistInMeter = (Grav*(M1+M2)*Period²/(4*pi²))^(1/3)

DistInAU = DistInMeter/($1.496 \cdot 10^{11}$)

GPT4+WA Answer: GPT found the correct answer in meters $9.2 \cdot 10^{11}$ m. However, it was unable to convert that to AU, despite eight calls to Wolfram Alpha.

GPT4+WA Calculation:

Let's solve this problem step by step using Kepler's third law of planetary motion, which states that the square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

The formula for Kepler's third law, when applied to two bodies of masses m_1 and m_2 orbiting each other, is given by

$$T^2 = (G(m_1 + m_2)/4\pi^2) \cdot a^3.$$

where:

T is the period of the orbit,

G is the gravitational constant,

m_1 and m_2 are the masses of the two bodies, and

a is the semi-major axis of the orbit (which is the distance between the two bodies in the case of a circular orbit).

We can rearrange this to solve for a .

$$a = \left(\frac{(T^2 G (m_1 + m_2))}{4\pi^2} i \right)^{1/3}$$

Given that:

$T = 4$ years

$G = 6.67430 \cdot 10^{-11} \text{ m}^2\text{kg}^{-1}\text{s}^{-2}$

$m_1 = 5$ solar masses, and $m_2 = 10$ solar masses,

we can substitute these values into the fomrula to find a . Note that we need to convert the solar masses to kilograms and the years to seconds for the units to be consistent. The conversion factors are:

1 solar mass = 1.9885×10^{30} kg, and

1 year = 3.1536×10^7 seconds.

Let's calculate this.

... [Request and response from Wolfram]

The distance between the two stars, calculated in meters is approximatesly $9.292 \cdot 10^{11}$ m.

However, GPT was unable to convert this correct value in meters into AU, despite eight separate calls to Wolfram Alpha.

Comment: GPT4+WA set up the problem perfectly and got the correct answer in meters. It is extraordinary that the system was unable to convert that to astronomical units.

GPT4+CI Answer: 12.43 Au.

Comment: Off by a factor of 2, because it inserted a wholly unnecessary factor of 2 in its formulation of Kepler's third law:

$$d_m = 2 * ((T_seconds**2 * G * (M1_kg + M2_kg) / (4 * math.pi**2))**(1/3))$$

Also curiously, GPT had to call CI four times because it "kept forgetting" to load the **math** package.

Let's calculate these values now.

Finished working Show work \downarrow

I apologize for the interruption in the calculation. Let's restart the calculation for the distance between the two stars.

Finished working Show work \downarrow

I apologize for the oversight. It seems that I forgot to import the math module. Let me correct that and run the calculations again.

Finished working Show work \downarrow

I apologize for the error. I forgot to import the math module, which is necessary for performing mathematical operations such as the cube root and pi. Let's correct this and redo the calculation.

Finished working Show work \downarrow

I apologize for the confusion. I forgot to import the `math` module, which is necessary for the mathematical calculations. Let me correct that and try the calculations again.

Finished working Hide work \uparrow

15. Draw a circle, on the earth's surface, going through Cairo, Peking, and Moscow. Let S be the area of the part of the earth's surface inside the circle and let P be the area of the circle in the plane of the circle. What is S/P ?

Answer: 1.108.

Matlab:

```
% Since we are computing a ratio, we take the earth's radius to be 1
c = LatAndLongToUnit(30,2,1,31,14,1) % Unit vector corresponding to Cairo
p = LatAndLongToUnit(39,54,1, 116,24,1) % Peking
m = LatAndLongToUnit(55,45,1,37,37,1) % Moscow
a = zeros(3); % coefficient matrix
q = zeros(3,1); % constant terms
% Equation of the perpendicular bisector plane of line cp is dot(x,p-c) = 0
% (Note that the origin is always in the plane.)
a(1,:) = p-c
% Likewise perpendicular bisector of cm
a(2,:) = m-c
a(3,:) = cross(p-c,m-c) % normal to plane of three cities
q(3,1) = dot(c,a(3,:))
f = a\q % circumcenter is intersection of these 3 planes
f=f';
r = norm(f-c) % radius of circumcenter
norm(m-f) % check for correctness
```

```

norm(p-f)
PlanarArea = pi*r^2 % area of planar circle
SphereArea = 2*pi*(1-norm(f)) % Formula from Wikipedia "Spherical sector"
Answer = SphereArea/PlanarArea

```

GPT Answer: 0.578

GPT Explanation:

The spherical law of cosines states that for a spherical triangle with sides of lengths a , b , and c (measured as angular distances), and the angle C opposite side c , the following relationship holds:

$$\cos(c) = \cos(a)\cos(b) + \sin(a)\sin(b)\cos(C)$$

We can use this formula to find the angle C (which is the angle at the center of the Earth subtended by the circle). The sides a , b , and c are the distances between the cities, which we have already calculated.

Let's proceed with this calculation.

```

REQUEST TO WOLFRAM \
{ "input": "a = 4695.316320788477; b = 1800.806190323255; c =
3608.113384695386; angleC = ArcCos[(Cos[a]Cos[b] +
Sin[a]Sin[b]Cos[c])]; N[angleC]" }
RESPONSE FROM WOLFRAM
"1.4137334777561732"
Used Wolfram
REQUEST TO WOLFRAM
{ "input": "theta = Pi - 1.4137334777561732; ratio = 1/2 * (1 -
Cos[theta]); N[ratio]" }
RESPONSE FROM WOLFRAM
"0.5782089440059505"

```

The ratio of the area of the part of the Earth's surface inside the circle (S) to the area of the circle in the plane of the circle (P) is approximately 0.578.

Comment: The answer is obviously impossible — for any simple closed curve C in plane P , the region in P bounded by C is smaller in area than the region bounded by C in any other surface — and the calculation bears no relation that we can see to the question being asked.

GPT4+CI Answer: The ratio of the area of the part of the Earth's surface inside the circle to the area of the circle in the plane of the circle, S/P , is approximately 2.

This is an upper bound for the ratio, based on the assumption that the central angle of the spherical cap is 90 degrees. The actual value could be less than 2, depending on the exact positions of Cairo, Peking, and Moscow.

Comment: GPT4+CI didn't even bother retrieving the latitudes and longitudes of the three cities. All it did was observe that the maximum value of S/P is attained for a hemisphere, and it calculated that for a hemisphere, the value is 2.

16. Assume that the probability of having any particular isotope of a chemical follows their frequency on earth. What is the probability that a randomly constructed molecule of glucose will have 4 atoms of ^{12}C , 2 atoms of ^{13}C , 11 atoms of ^1H , 1 atom of ^2H , 3 atoms of ^{18}O , 3 atoms of ^{16}O .

Answer: $4.797 \cdot 10^{-13}$

Matlab:

```
nchoosek(6,4) * .0106^2 * 0.989^4 * ...
nchoosek(12,1) * .99985^11 * 0.000145 * ...
nchoosek(6,3) * 0.998^3 * 0.00205^3
Answer: 4.797 * 10^-13
```

GPT4+WA Answer: $7.60 \cdot 10^{-17}$

GPT4+WA Calculation

Here are the natural abundances we found ^{12}C : 98.9%

^{13}C : 1%
 ^1H : 99.99%
 ^2H : 0.01%
 ^{18}O : 0.2%
 ^{16}O : 99.8%

The probability P is given by

$$(0.989)^4(0.01)^2(0.9999)^{11}(0.0001)(0.002)^3(0.998)^3 = 7.60x10^{-17}.$$

Comment: GPT has omitted the combinatorial factors $C(6,2) \cdot C(12,1) \cdot C(6,3)$.

GPT4+CI Answer: $6.22 \cdot 10^{-13}$.

Comment: GPT4+CI sets up the calculation correctly, but gets an answer different from ours. GPT4+CI is using different values than we did for the frequency of the rare isotopes; those numbers are probably not known, possibly not even defined, with any huge precision in any case. But when we take GPT's numbers and do the computation in Matlab, we get an answer of $3.50 \cdot 10^{-13}$. We cannot explain the discrepancy.

17. Two ^{31}K phosphorus nuclei, with no electrons, are isolated in space, with coordinates $\langle 0,0,0 \rangle$ and $\langle 10,10,10 \rangle$ in a coordinate system whose unit length is 1 Angstrom. What is the instantaneous acceleration (a vector with unit length of Angstrom/sec²) of the nucleus at the origin due to the electrostatic force?

Answer: $\langle -1.9420 \cdot 10^{27}, -1.9420 \cdot 10^{27}, -1.9420 \cdot 10^{27} \rangle$

Matlab

```
Mass = 5.1433*10^(-26) % kg
Charge = 15*1.602*10^(-19) % coulomb
CoulombConst = 8.988 * 10^9 % Newton * m^2/C^2
Angstrom2Meter = 10^(-10)
Distance = 10*sqrt(3)*Angstrom2Meter
Force = CoulombConst*Charge^2/Distance^2 % Newtons
Acceleration = -Force/(Mass*Angstrom2Meter) * [1,1,1]/sqrt(3)
```

GPT4+QA Answer: $\langle 6.91 \cdot 10^{27}, 6.91 \cdot 10^{27}, 6.91 \cdot 10^{27} \rangle$ Angstrom/sec²

GPT Explanation:

The charge of a phosphorus nucleus is the charge of a proton times the atomic number of phosphorus. The atomic number of phosphorus is 15, so the charge of a phosphorus nucleus is $15 \cdot 1.602 \cdot 10^{19}$ Coulombs.

The electrostatic force between two charges is given by Coulomb's law:

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

where k is Coulomb's constant $8.99 \cdot 10^9 \text{Nm}^2/\text{C}^2$. q_1 and q_2 are the charges, and r is the distance between the charges.

The distance between the two nuclei is the Euclidean distance between their coordinates, which is

$$(10 - 0)^2 + (10 - 0)^2 + (10 - 0)^2 = 10^3 \text{ Angstroms or } 10^3 \cdot 10^{-10} \text{ meters.}$$

The acceleration of the nucleus at the origin due to this force is given by Newton's second law:

$$a = \frac{F}{m}$$

where m is the mass of the nucleus. The mass of a phosphorus nucleus is approximately the mass of a proton times the atomic number of phosphorus. The mass of a proton is approximately $1.67 \cdot 10^{-27}$ kg, so the mass of a phosphorus nucleus is approximately $15 \cdot 1.67 \cdot 10^{-27}$ kg.

Substituting the force and mass into Newton's second law gives the acceleration:

$$a = \frac{8.99 \cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \cdot (15 \cdot 1.602 \cdot 10^{19} \text{ C})^2}{(10^3 \cdot 10^{10} \text{ m})^2 \cdot 15 \cdot 1.67 \cdot 10^{27} \text{ kg}}$$

This acceleration is a vector pointing from the origin to the other nucleus, so its direction is the same as the direction of the vector from the origin to the other nucleus. The direction of this vector in Cartesian coordinates is $(10,10,10)$, so the direction of the acceleration is also $(10,10,10)$. The magnitude of this vector is $10^2 + 10^2 + 10^2 = 10^3(10^2 + 10^2 + 10^2) = 10^3$, so to get a unit vector in the same direction, we divide the vector by its magnitude, giving $((10/10\sqrt{3}, 10/10\sqrt{3}, 10/10\sqrt{3}) = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

Used Wolfram Alpha

The magnitude of the acceleration is approximately $6.91 \cdot 10^{17} \text{ m/s}^2$.

To convert this to Angstroms per second squared, we multiply by 10^{10} , since 1 meter is 10^{10} Angstroms. This gives an acceleration of approximately $6.91 \cdot 10^{27} \text{ \AA/s}^2$.

The direction of the acceleration is the same as the direction of the vector from the origin to the other nucleus, which is $(1/3, 1/3, 1/3)$.

Therefore, the acceleration of the nucleus at the origin due to the electrostatic force is approximately $6.91 \cdot 10^{27} \text{ \AA/s}^2$ in the direction $(1/3, 1/3, 1/3)$.

Comment: There are several separate mistakes here. The sign is wrong; GPT does not use the fact that like charges repel. The mass of the phosphorus nucleus is miscalculated; GPT is ignoring the neutrons in the nucleus. GPT computes the sum $10^2 + 10^2 + 10^2$ as 10^3 , though it is not clear whether that affects the final result. The unit directional vector, first correctly calculated as $\langle 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3} \rangle$ is then misremembered as $\langle 1/3, 1/3, 1/3 \rangle$.

GPT4+CI Answer: The instantaneous acceleration of the phosphorus nucleus at the origin due to the electrostatic force from the other phosphorus nucleus is approximately $1.94 \cdot 10^{27}$ Angstrom/sec² in the direction of the vector $\langle 1, 1, 1 \rangle$.

Comment: The sign is wrong, and the magnitude, as stated, is wrong by a factor of $\sqrt{3}$.

18. An irregular (house-shaped) pentagon has vertices numbered 1 through 5 in order. The pentagon has right angles at vertices 1, 3, and 4, and 135-degree angles at 2 and 5. Side 2-3 and 4-5 have length 1 and side 3-4 has length 2. The pentagon is placed on a planar coordinate system so that the numbering of the vertices is in clockwise order, vertex 3 is at the origin, and vertex 5 is on the positive y-axis. What are the coordinates of vertex 1?

Answer: $\langle 1.5269, 2.3446 \rangle$

Matlab:

```
% Problem 18
% Consider the pentagon in a standard orientation where vertex 3 is at the origin and
% vertex 4 is on the negative x-axis. Compute the rotation necessary to place 5 above
% 3, and then apply that rotation to 1.
```

```
theta = atan2(1,-2)-(pi/2)
standard1 = [-sqrt(2);1+sqrt(2)]
answer = [cos(theta), sin(theta); -sin(theta), cos(theta)]*standard1
standard5 = [-2;1]
check = [cos(theta), sin(theta); -sin(theta), cos(theta)]*standard5 % check
```

GPT4+WA Answer: $(-1/2, 1/2)$

GPT4+WA Calculation:

Given: Vertex 3 is at the origin, so its coordinates are $(0,0)$. Vertex 5 is on the y-axis, and since the length of side 4-5 is 1, vertex 5 is at $(0,1)$. The angle at vertex 4 is a right angle, so vertex 4 must be on the x-axis. Since the length of side 3-4 is 2, vertex 4 is at $(2,0)$. The angle at vertex 2 is 135 degrees, which means that the line from vertex 3 to vertex 2 is at a 45-degree angle to the x-axis. Since the length of side 2-3 is 1, vertex 2 is at $(-1,1)$. To find the coordinates of vertex 1: Since the angle at vertex 1 is a right angle, vertex 1 must lie on the line perpendicular to the line from vertex 2 to vertex 1. This line has a slope of -1, and it passes through vertex 2, so its equation is $y=1-(x+1)+1$. The line from vertex 1 to vertex 5 is also at a 45-degree angle to the x-axis, so it has a slope of 1. This line passes through vertex 5, so its equation is $y=1+(x-0)+1$. Solving these two equations for x and y, we get: $x = 1/2, u = 1/2$. Therefore, the coordinates of vertex 1 are $(1/2, 1/2)$.

Comment: These calculations make no sense and bear no relation to the geometry of the problem. GPT did not call WA.

GPT4+CI Answer: $\langle 2, 1 \rangle$

Comment: The answer is wrong. The explanation makes no sense. GPT did not call CI.

19. Consider a cube with unit length sides, where the vertices of one face are numbered A..D in counterclockwise order, as viewed from the center of the cube; the vertices of the opposite face are named E to H; and there are edges AE, BF, CG, and DH. Rotate the cube so that vertex A is at the origin, vertex G is on the positive z axis, and vertex B is in the x-z plane, with positive x coordinate. What are the coordinates of vertex E?

Answer: $\langle -0.4082, -0.7071, .5774 \rangle = \langle -\sqrt{1/6}, -\sqrt{1/2}, \sqrt{1/3} \rangle$

Explanation: Consider the cube in a standard position where $\vec{a} = \vec{0}$, $\vec{b} = \hat{i}$, $\vec{d} = \hat{j}$. $\vec{e} = \hat{k}$; thus $\vec{g} = \langle 1, 1, 1 \rangle$. Since \vec{a} is at the origin after the rotation, this is a pure rotation around the origin; it is therefore equivalent to multiplication by an orthogonal matrix M with determinant 1. The dot product $M(2, :)$ with any vector v is the y -coordinate of the rotated position of v . In particular since after the rotation both \vec{b} and \vec{g} have y -coordinate zero, $M(2, :)$ must be orthogonal to both; hence, it is their cross-product, normalized. Since the x coordinate of the rotated place of g is 0, and since M is orthogonal, $M(1, :)$ is orthogonal to both \vec{g} and $M(2, :)$; hence, it is the normalized form of their cross product. Since M is orthogonal, $M(3, :)$ is the cross product of $M(1, :)$ and $M(2, :)$. All that remains is to make sure the signs are OK, to check, and to compute $M \cdot \vec{e}$.

Matlab

```
% Problem 19
```

```

b = [1,0,0]
g = [1,1,1]
e = [0,0,1]
m = zeros(3);
u = cross(b,g) % both b and g get mapped to a vector with y component = 0
m(2,:) = u / norm(u)
v = cross(m(2,:),g)
m(1,:) = sign(dot(v,b))*v/norm(v)
m(3,:) = cross(m(1,:),m(2,:))
if (dot(m(3,:),g) < 0)
    m(2,:) = -m(2,:)
    m(3,:) = -m(3,:)
end
m*m' %Check: rotation matrix?
det(m)
m*b' %Check: b mapped into xz plane with positive x?
m*g' %Check: g mapped into positive z axis?
answer = m*e'

```

GPT4+WA answer: (1,1,1).

GPT4+WA explanation: Given the description, we can infer the following:

Vertex A is at the origin, so its coordinates are (0,0,0).

Vertex G is on the positive z axis, so its coordinates are (0,0,1).

Vertex B is in the x-z plane with a positive x coordinate, so its coordinates are (1,0,0).

Since E is directly opposite A in a cube with unit length sides, its coordinates will be (1,1,1).

Comment: GPT has entirely misunderstood the problem. In particular, it has missed the actual specification of the positions of the vertices in standard position and it has missed the fact that the cube is rotated into a new position.

GPT4+CI Answer: $\langle -1, 0, 0 \rangle$

Comment Again, the analysis bears no relation to the problem.

20. Joe and Jim each have a bank account which they started on January 1, 2000. Joe started his account with with \$1000; Jim started his with \$900. Joe's account pays 5% every December 31, and he adds an additional \$500 every January 1. Jim's account pays 10% every December 31. When will Jim's account have more money in it than Joe's?

Answer: 2053. (More precisely, December 31, 2052).

Matlab:

```

joe = 1000
jim = 900
count = 0
while (jim <= joe)
    count = count+1
    jim = jim*1.10
    joe = joe*1.05 + 500
end
2000+count

```


GPT answer: After 53 years.

GPT calculation:

Let $A_{\text{Joe}}(n)$ and $A_{\text{Jim}}(n)$ denote the amount of money in Joe's and Jim's accounts after n years.

$$A_{\text{Joe}}(0) = 1000, \quad A_{\text{Jim}}(0) = 900$$

$$A_{\text{Joe}}(n) = 1.05 \cdot A_{\text{Joe}}(n-1) + 500, \quad n \geq 1$$

$$A_{\text{Jim}}(n) = 1.1 \cdot A_{\text{Jim}}(n-1), \quad n \geq 1$$

We want to find the smallest n such that $A_{\text{Jim}}(n) > A_{\text{Joe}}(n)$.

By running the above recurrence relations, we find that the smallest such n is 53.

Comment: Exactly right.

GPT4+CI Answer: 2005.

GPT4+CI Calculation:

To determine when Jim's account will have more money than Joe's, let's calculate the balance of their accounts over time. We'll start from January 1, 2000, and track the balances at each December 31 until Jim's account surpasses Joe's.

Year 2000:

Joe's initial balance: \$1000

Jim's initial balance: \$900

Year 2001:

Joe's balance: $\$1000 + \$500 = \$1500$

(Joe adds \$500 on January 1)

Jim's balance: $\$900 + (\$900 * 0.10) = \$990$

(Jim's account earns 10% interest)

Year 2002:

Joe's balance: $\$1500 + (\$1500 * 0.05) = \$1575$

(Joe's account earns 5% interest)

Jim's balance: $\$990 + (\$990 * 0.10) = \$1089$

(Jim's account earns 10% interest)

Year 2003:

Joe's balance: $\$1575 + (\$1575 * 0.05) = \$1653.75$

(Joe's account earns 5% interest)

Jim's balance: $\$1089 + (\$1089 * 0.10) = \$1197.90$

(Jim's account earns 10% interest)

Year 2004:

Joe's balance: $\$1653.75 + (\$1653.75 * 0.05) = \$1736.44$

(Joe's account earns 5% interest)

Jim's balance: $\$1197.90 + (\$1197.90 * 0.10) = \$1317.69$

(Jim's account earns 10% interest)

22. A point \mathbf{p} is chosen at random within the 100-dimensional box $\mathbf{B} = [0, 100]^{100}$ following a uniform distribution. What is the probability that the Euclidean distance from \mathbf{p} to the boundary of \mathbf{B} is less than 1?

Answer: $1 - 0.98^{100} \approx 1 - 1/e^2 = 0.8647$

Explanation: The distance from \mathbf{p} to the boundary is $\min_{i=1..k} \min(p_k, 100 - p_k)$. That is, the distance is more than 1 if and only if $\mathbf{p} \in [1, 99]^{100}$. So the probability that the distance is less than one is $(100^{100} - 98^{100})/100^{100}$.

GPT4+WA Answer:

GPT4+WA Explanation: This is a high-dimensional geometry problem. The probability that a point chosen uniformly at random within a 100-dimensional box is within a distance of 1 from the boundary is equivalent to the ratio of the volumes of two 100-dimensional hypercubes.

Let's denote the side length of the larger hypercube (the box \mathbf{B}) as L , and the side length of the smaller hypercube (the box \mathbf{B} but with a layer of thickness 1 removed from all sides) as L' . In this case, $L = 100$ and $L' = 100 - 2 = 98$.

The volume V of a 100-dimensional hypercube with side length L is given by L^{100} .

So, the volume of the larger hypercube $V_B = L^{100} = 100^{100}$, and the volume of the smaller hypercube $V_{B'} = L'^{100} = 98^{100}$.

The probability P that a point is within a distance of 1 from the boundary is then given by

$$1 - \frac{V_{B'}}{V_B}$$

Comment: Right

GPT4+CI Answer: 0.867.

Comment: Right

23. A point \mathbf{p} is chosen at random within the 100-dimensional box $\mathbf{B} = [0, 1]^{100}$ following a uniform distribution. What is the mean value of the Euclidean distance from \mathbf{p} to the boundary of \mathbf{B} ?

Answer: $1/202$.

Explanation: Pick one particular face of the hypercube; for convenience the lower face in the x -dimension $\mathbf{F} = \{0\} \times [0, 1]^{99}$. Let $\mathbf{c} = \langle 0.5, 0.5, \dots, 0.5 \rangle$ be the center of \mathbf{B} . Let \mathbf{P} be the hyperpyramid consisting of the convex hull of $\mathbf{F} \cup \{\mathbf{c}\}$. It is obvious, and easy to show, that the points in \mathbf{B} whose closest boundary point is \mathbf{F} are exactly those in \mathbf{P} and that the distance of a point in \mathbf{P} to \mathbf{F} is just its x -coordinate. Moreover, since \mathbf{B} can be partitioned into such pyramids, one for each face of \mathbf{B} and they all have identical distributions of distances to the boundary, the mean distance to the boundary is the same in \mathbf{P} . Therefore the mean distance for a point in \mathbf{B} to the boundary of \mathbf{B} is the same as mean distance for the distance from a point in \mathbf{P} to \mathbf{F} which is just the mean x -coordinate of a point in \mathbf{P} .

The rest is just calculus. The cross-section of \mathbf{P} at x -coordinate x is the box $[x, 1 - x]^{99}$ of 99-d volume $(1 - 2x)^{99}$. We could integrate this to compute the volume of \mathbf{P} but it's simpler to note that there is one such pyramid for each face of \mathbf{B} ; there are 200 such faces; and their union is all of \mathbf{B} which has volume 1. So the volume of \mathbf{P} is $1/200$. So the probability density of a uniform choice in \mathbf{P} as a function of x is $200(1 - 2x)^{99}$.

The expected value of x in \mathbf{P} is therefore

$$\int_{x=0}^{0.5} 200(1 - 2x)^{99} x dx = 200 \int_{u=0}^1 u^{99} \cdot ((1 - u)/2) \cdot (du/2) = (200/4) \int_{u=0}^1 u^{99} - u^{100} du =$$

$$(200/4) \cdot (u^{100}/100 - u^{101}/101)|_0^1 = 1/202.$$

GPT4+WA Answer: 5

GPT4+WA Calculation: The mean Euclidean distance from a point chosen uniformly at random within a hypercube to the boundary can be calculated using the formula:

$$\frac{\sqrt{d}}{2}$$

Where d is the dimension of the hypercube. In this case $d = 100$. Let's calculate this.

[Call to Wolfram Alpha]

The mean Euclidean distance from a point chosen uniformly at random within a 100-dimensional hypercube to the boundary is 5 units.

Comment: Wrong. The formula is pulled out of thin air.

Answer from GPT4+CI: 0.01875.

GPT4+CI Calculation: *GPT4 computes the integral*

$$\int_0^{1/2} y \cdot 2y^2(1-y)dy$$

entirely ignoring the dimension of 100.

Comment: Wrong.

24. A pound of graphite is inside a closed cylindrical container with a radius of 50 cm and a height of 200 cm, filled with air at room temperature (22° C) and atmospheric pressure. The graphite reacts with the oxygen, producing equal masses of carbon monoxide and carbon dioxide until one of the reactant chemicals is exhausted. How many moles of each chemical is in the container at the end?

Answer: No oxygen, 11.86 moles of carbon monoxide, 7.61 moles of carbon dioxide, 18.2 moles of carbon.

Matlab:

```

CAAtomMass = 12.011
OAtomMass = 16
COMolecMass = CAAtomMass+OAtomMass
CO2MolecMass = CAAtomMass+2*OAtomMass
MolesOfCarbon = 453.6/CAAtomMass % Wikipedia
Temp = 273.15 + 22 % Wikipedia
Liters = pi*200*50^2/1000 % Volume in liters
IdealGasConst = 0.082057 % l*atm*K^{-1}*mol^{-1}. Wikipedia
MolesOfAir = Liters/(Temp*IdealGasConst)
MolesOfO2 = MolesOfAir * .2095
MoleFracOfCO = CO2MolecMass/(COMolecMass+CO2MolecMass)% Molar fraction of CO in product
A = 2*MolesOfO2/(2-MoleFracOfCO)
MolesOfCO = MoleFracOfCO * A
MolesOfCO2 = (1-MoleFracOfCO) * A
MolesOfCO*COMolecMass - MolesOfCO2*CO2MolecMass %check
MolesOfCO + 2*MolesOfCO2 - 2*MolesOfO2 %check
EndMolesOfCarbon = MolesOfCarbon - (MolesOfCO+MolesOfCO2)

```

GPT4+WA Answer: 10.541 moles of carbon (graphite) left; 13.614 moles of CO produced; 0 moles of CO₂ produced; 0 moles of oxygen left.

GPT4+WA Explanation: The number of moles of carbon (graphite) is approximately 37.769 moles.

The volume of the cylinder is approximately 1571 liters.

The number of moles of air in the cylinder is approximately 64.83 moles.

The number of moles of oxygen in the cylinder is approximately 13.614 moles.

The limiting reactant is the reactant that is completely consumed in the reaction and determines when the reaction will stop. In this case, the limiting reactant is oxygen because we have less moles of oxygen than carbon.

For reaction 1 (producing CO), we need 0.5 moles of oxygen for each mole of carbon. We have 13.614 moles of oxygen, so we can react all of this with carbon to produce CO. This will consume $2 * 13.614 = 27.228$ moles of carbon.

For reaction 2 (producing CO₂), we need 1 mole of oxygen for each mole of carbon. We have no oxygen left, so we can't produce any more CO₂.

Therefore, at the end of the reaction, we have:

$37.769 - 27.228 = 10.541$ moles of carbon (graphite) left. 13.614 moles of CO produced. 0 moles of CO₂ produced. 0 moles of oxygen left.

Answer: GPT misunderstood the question. It assumed that the carbon monoxide reaction runs until the oxygen is exhausted; then, of course, the carbon dioxide reaction can't run.

GPT4+CI Answer: In terms of moles, there are approximately 24.0 moles of CO and 15.3 moles of CO₂ in the container.

This calculation assumes that all the oxygen in the air reacts with the carbon.

Comment: Wrong. The calculation carried out by GPT4+CI first assumes that all of the oxygen is used up in the reaction $2C + O_2 \rightarrow 2CO$ and then sets the mass of CO₂ equal to the mass of CO.

25. Suppose that you have a gram of pure radium 223. What is the mass of the helium that that will generate via fission in a week?

Answer: 0.0248 gm

Explanation: Radium 223 undergoes a series of nuclear reactions:

Radium 223 \rightarrow Radon 219 + alpha

Radon 219 \rightarrow Polonium + alpha

Polonium 215 \rightarrow Lead 211 + alpha

Lead 211 \rightarrow Bismuth 211 + beta

Bismuth 211 \rightarrow Thallium 207 + alpha

Thallium 207 \rightarrow Lead 207 + beta

Thus each atom of radium generates 4 alpha particles.

The first reaction has a half-life of 11.43 days; the others have half-lives ranging from fractions of a second to a few minutes.

Matlab:

```
radiumMass = 223
heliumMass = 4
halfLife = 11.43 % Half life of radium 223 in days
f = (1/2)^(7/halfLife)
answer = (1-f)*4*heliumMass/radiumMass
```

GPT4+WA Answer:

GPT4+WA Explanation: The decay chain of radium-223 involves several alpha decays, which produce helium-4 nuclei. Each alpha decay reduces the mass number of the decaying nucleus by 4 and the atomic number by 2.

The decay chain of radium-223 is as follows:

Radium-223 (223 Ra) undergoes alpha decay to produce radon-219 (219 Rn).

Radon-219 undergoes alpha decay to produce polonium-215 (215 Po).

Polonium-215 undergoes alpha decay to produce lead-211 (211 Pb).

Lead-211 undergoes beta decay to produce bismuth-211 (211 Bi), which does not produce helium.

Bismuth-211 undergoes alpha decay to produce thallium-207 (207 Tl).

Thallium-207 undergoes beta decay to lead-207 (207 Pb), which does not produce helium.

From this decay chain, we can see that 4 alpha decays occur. Each alpha decay produces a helium-4 nucleus. Therefore, for each radium-223 atom that decays, 4 helium-4 atoms are produced.

The next step is to calculate the number of radium-223 atoms that will decay in a week. The half-life of radium-223 is 11.43 days. We can use the formula for radioactive decay:

$$N = N_0 \cdot 0.5^{(t/T)}$$

where:

N is the final quantity remaining after the time t ,

N_0 is the initial quantity,

T is the half-life of the decaying quantity

We can rearrange this formula to find the number of atoms that have decayed after a certain time:

$$N_{\text{decayed}} = N_0 - N = N_0 - N_0 \cdot 0.5^{(t/T)} = N_0 \cdot (1 - 0.5^{(t/T)})$$

We can calculate the number of radium-223 atoms in 1 gram using Avogadro's number and the molar mass of radium-223 (223 g/mol):

$$N_0 = \frac{1\text{g} \cdot 6.022 \cdot 10^{23}\text{atoms/mol}}{223\text{g/mol}}$$

Finally, we can calculate the mass of the helium produced. Each helium-4 atom has a mass of 4 atomic mass units (amu), and 1 amu is approximately 1.66×10^{-24} g. The total mass of helium produced is then:

$$N_{\text{decayed}} \cdot 4\text{He/decay} \cdot 1.661024\text{g/He}$$

The mass of helium that will be generated from 1 gram of radium-223 via fission in a week is approximately 0.0062 grams.

Comment: GPT lost track of the fact that a helium nucleus has mass 4 amu and therefore its answer is off by a factor of four. However, it did better than author Ernie Davis who, until he saw GPT's solution, did not realize that there were four alpha decays involved, and, in a second error, used the factor $0.5^{t/T}$ instead of $1 - 0.5^{t/T}$.

GPT4+CI: 0.0062 grams.

Comment: In this instance, GPT did not consider the whole decay chain, only the first reaction. However, since that is a reasonable reading of the question, we mark this as correct.

26. A small school has 100 students and three extramural activities: the basketball team, the chess club, and the drama society. There are 60 students in the band, 48 in the chess club, and 55 in the drama club. There are 18 students who do both band and chess, 28 who do both band and drama, 31 who do both chess and drama, and 8 students who are in all three groups. What is the probability that a student is not in the band given that they are either in the drama club or the chess club but not both?

Answer: 11/41

Explanation: It is easily determined from the above information that $BCD = 8$. $BC\bar{D} = 10$
 $\bar{B}CD = 20$. $\bar{B}\bar{C}D = 22$. $\bar{B}C\bar{D} = 23$. $B\bar{C}\bar{D} = 7$. $\bar{B}\bar{C}\bar{D} = 4$. $\bar{B}C\bar{D} = 6$. The above answer is immediate.

GPT4+WA Answer: Unable to answer.

GPT4+WA Explanation: First, let's denote:

Total number of students, $T=100$.

Number of students in the band, $B=60$

Number of students in the chess club, $C=48$

Number of students in the drama society, $D=55$

Number of students in both band and chess, $BC=18$

Number of students in both band and drama, $BD=28$

Number of students in both chess and drama, $CD=31$

We are interested in the probability that a student is not in the band given that they are either in the drama club or the chess club but not both. This can be denoted as $\neg B \cap (C \oplus D)$ where $\neg B$ means "not in the band", and $C \oplus D$ means "in the chess club or the drama society, but not both".

The number of students in either the chess club or the drama society, but not both, can be calculated as:

$$C \oplus D = C + D - 2 \cdot CD + BCD$$

The number of students not in the band but in either the chess club or the drama society, but not both, can be calculated as:

$$\neg B \cap (C \oplus D) = C \oplus D - BD + BCD$$

[GPT then tries seven times to get Wolfram Alpha to evaluate the two above expressions, but it fails every time, apparently due to variable name conflicts with the Wolfram Alpha language]

Comment: GPT starts off on the right track with the formula for probability. The two formulas it generates for the size of the sets $C \oplus D$ and $\neg B \cap (C \oplus D)$ have the right flavor but are not right. Then the process runs into some kind of bugs in the interface between GPT and WA.

GPT4+CI: 0.268

Comment: Right.

27. You draw 20 cards from a deck of cards labelled 1..60. To your surprise, the cards can be arranged as an arithmetic sequence. What is the probability that the successive difference is 3? Give an exact answer as a rational number.

Answer: $3/66 = 1/22$.

Explanation: There are 41 sequences of difference 1 (starting with every value from 1 to 41); 22 sequences with difference 2 (starting from 1 to 22); and 3 with difference 3 (starting points 1, 2, and 3).

GPT4+WA first answer: 1/1397281501935165

GPT4+WA second answer, after human intervention: 1/820.

GPT4+WA Explanation:

First, we need to find out how many arithmetic sequences with a common difference of 3 and length 20 exist within a sequence of length 60.

An arithmetic sequence is defined by its first term and its common difference. In this case, the common difference is 3. The first term of the sequence can be any number from 1 to 60, but we need to ensure that the entire sequence fits within this range.

Since the length of the sequence is 20 and the common difference is 3, the last term of the sequence will be $a + 19 \cdot 3 = a + 57$. To ensure that the last term does not exceed 60, the first term a must be in the range from 1 to $60 - 57 = 3$.

Therefore, there are 3 possible arithmetic sequences with a common difference of 3 and length 20 within a sequence of length 60.

Next, we need to find out how many ways we can draw 20 cards from a deck of 60. This is a combination problem, and the number of combinations of 60 items taken 20 at a time is given by the binomial coefficient $C(60,20)$.

Finally, the probability that a randomly drawn sequence of 20 cards forms an arithmetic sequence with a common difference of 3 is the ratio of the number of such sequences to the total number of possible sequences, which is $3/C(60,20)$.

The exact probability that a randomly drawn sequence of 20 cards from a deck of 60 forms an arithmetic sequence with a common difference of 3 is

$$\frac{1}{1397281501935165}$$

Comment: Correct answer to a different question.

Human author asked: But's what's the probability, *conditional on the event* that the 20 cards do form an arithmetic sequence?

GPT-4+WA continues

Ah, I see. If we already know that the 20 cards form an arithmetic sequence, then we need to find the probability that the common difference of this sequence is 3.

First, let's find out how many arithmetic sequences of length 20 exist within a sequence of length 60.

An arithmetic sequence is defined by its first term and its common difference. The first term of the sequence can be any number from 1 to 60. The common difference can be any number from 1 to $60 - 19 = 41$. (to ensure that the last term does not exceed 60). Therefore, the total number of such sequences is $60 \cdot 41 = 2460$.

As we found out earlier, there are 3 arithmetic sequences with a common difference of 3 and length 20 within a sequence of length 60.

Therefore, the conditional probability that the common difference is 3, given that the 20 cards form an arithmetic sequence, is $3/2460$.

The exact conditional probability that the common difference of an arithmetic sequence of 20 cards drawn from a deck of 60 is 3, given that the cards form an arithmetic sequence, is 1/820.

Comment: Completely confused calculation of the number of arithmetic sequences of length 20.

GPT4+CI first Answer: $38/C(60,20) = 9.07 \cdot 10^{-15}$.

Comment. As with GPT4+WA, the question has been misinterpreted and GPT tried to compute the unconditional probability of drawing an arithmetic sequence with common difference 3. However, this time, it miscounted the number of such sequences, so this answer is wrong, even for that question.

Human author asked: But's what's the probability, *conditional on the event* that the 20 cards do form an arithmetic sequence?

GPT4+CI second answer: 1/3

GPT4+CI Explanation: The possible common differences are 1, 2, or 3.

Given that the cards form an arithmetic sequence, each of these common differences is equally likely. So the conditional probability that the common difference is 3, given that the cards form an arithmetic sequence, is 1/3.

Comment: Still wrong

28. You draw 5 cards from a deck of cards labelled 1 ... 200. To your surprise the cards can be arranged as a geometric series. What is the probability that the smallest number is less than or equal to 7? (Note that the ratio need not be an integer.)

Answer: 9/16

Explanation: The possible increasing ratios are 2, 3, and 3/2.

If the ratio is 2, the sequence has the form $a, 2a, 4a, 8a, 16a$. a can be any integer from 1 to 12. In 7 of these, $a \leq 7$.

If the ratio is 3, the sequence has the form $a, 3a, 9a, 27a, 81a$. a is either 1 or 2. In both of these, $a \leq 7$.

If the ratio is 3/2, the sequence has the form $16a, 24a, 36a, 54a, 81a$, and a can be either 1 or 2. In both of them the smallest number is greater than 7.

So the probability is 9/16.

GPT4+WA Answer: 2/316956255

GPT4+WA Explanation: *The only indication of how GPT arrived at this result in its request to Wolfram Alpha:*

```
REQUEST TO WOLFRAM \\
\begin{verbatim}
{ "input": "geometricSeriesProbability[n_, k_, r_] := Module[{total =
Binomial[n, k], favourable = 0}, For[i = 1, i <= r, i++, For[j = 1, j
<= n, j++, If[i*j^(k - 1) <= n, favourable++;];];];
Return[favourable/total];]; geometricSeriesProbability[200, 5, 7]" }
```

```
RESPONSE FROM WOLFRAM \\
"2/316956255"
0
```

Comment: We have no idea what GPT thought it was calculating.

GPT4+CI Answer: 3/845216840.

CI Code executed: i

```
# Calculate the maximum possible value of r when a = 7
r_max = (200 / a_max) ** (1 / 4)
```

```

# Count the possible geometric sequences for each a from 1 to 7
count = 0
for a in range(1, a_max + 1):
    for r in range(2, int((200 / a) ** (1 / 4)) + 1):
        if all(a * r ** i <= total_cards for i in range(5)):
            count += 1

# Calculate the total possible geometric sequences
total_sequences = sp.binomial(total_cards, 5)

# Calculate the probability
probability = sp.Rational(count, total_sequences)
probability

```

Comment: Again, the answer is way off the mark, and it is not at all clear what, if anything, this code actually corresponds to.

29. In this question and the following three, assume that the satellite is moving in a closed orbit around the Earth and that the only influence on the satellite's motion is the Earth's gravity. Assume that the Earth is a perfect sphere. Ignore the revolution of the Earth around the sun, but do not ignore the rotation of the Earth around its axis.

A satellite in a circular geosynchronous orbit remains directly above the point on the earth's equator 0° N, 50° E. What is the straight line distance from the satellite to Houston?

Answer: 46,934 km.

Matlab:

```

% Method 1: Compute the positions of Houston and the satellite
% as 3 dimensional vectors
r = 6378.1 % km. Earth's radius
q = 35786 % height of geosynchronous orbit over earth's surface
rs = r+q % distance from center of earth to satellite
deg2rad = pi/180; % degrees to radians
hlat = 29.75*deg2rad % Houston latitude and longitude
hlong = -95.38*deg2rad
uh = [cos(hlat)*cos(hlong), cos(hlat)*sin(hlong),sin(hlat)] % unit vector Houston
h = r*uh % Houston location as vector
slong = 50*deg2rad
us = [cos(slong),sin(slong),0]
s = rs*us % Satellite location as vector
answer1 = norm(s-h)

```

```

% Method 2 Apply law of cosines to triangle Houston - center of earth- satellite
cosTheta = dot(us,uh) % cosine of angle at center of earth
sqrt(r^2 + rs^2-2*cosTheta*r*rs)

```

GPT4+WA Answer: $3.58 \cdot 10^7$ meters

GPT4+WA Explanation:

```

REQUEST TO WOLFRAM \\
{ "input": "R = 6.371009*10^6; r = 4.2164*10^7; lat1 = 0; lon1 = 50;
lat2 = 29 + 46/60 + 50/3600; lon2 = -(95 + 23/60 + 11/3600); dlat =

```

```

lat2 - lat1; dlon = lon2 - lon1; a = Sin[dlat/2]^2 +
Cos[lat1]*Cos[lat2]*Sin[dlon/2]^2; c = 2*Atan2[Sqrt[a], Sqrt[1 - a]];
d = R*c; h = Sqrt[r^2 + d^2 - 2*r*d*Cos[c]]; h" }
RESPONSE FROM WOLFRAM
"Sqrt[1.777802896 ^15 + 1.62359022712324 ^14
Atan2[Sqrt[Sin[10721/720]^2 + Cos[10721/360] Sin[523391/7200]^2],
Sqrt[1 - Sin[10721/720]^2 - Cos[10721/360] Sin[523391/7200]^2]]^2 -
1.074508893904 ^15 Atan2[Sqrt[Sin[10721/720]^2 + Cos[10721/360]
Sin[523391/7200]^2], Sqrt[1 - Sin[10721/720]^2 - Cos[10721/360]
Sin[523391/7200]^2]] Cos[2 Atan2[Sqrt[Sin[10721/720]^2 +
Cos[10721/360] Sin[523391/7200]^2], Sqrt[1 - Sin[10721/720]^2 -
Cos[10721/360] Sin[523391/7200]^2]]]]]"
0

```

Here is the calculation for the straight line distance from the satellite to Houston:

First, we need to convert the coordinates of Houston and the point on the Earth's equator directly below the satellite into radians. The coordinates for Houston are approximately 29.78° N, 95.39° W, and the coordinates for the point on the equator are 0° N, 50° E.

The conversion from degrees to radians is done by multiplying the degree measurement by $180/\pi$.

Let's denote:

R as the Earth's radius, which is approximately $6.371009 \cdot 10^6$ meters,

r as the radius of the geosynchronous orbit, which is approximately $4.2164 \cdot 10^7$ meters,

lat1 and lon1 as the latitude and longitude of the point on the equator, which are 0° and t_0° respectively.

lat2 and lon2 as the latitude and longitude of Houston, which are approximately $29.78i^\circ$ and -95.39° respectively.

We can calculate the change in latitude (Δlat) and longitude (Δlon) as follows:

$$\Delta\text{lat}=\text{lat2}-\text{lat1}$$

$$\Delta\text{lon}=\text{lon2}-\text{lon1}$$

Next, we calculate a, c, and d

$$a = \sin^2(\Delta\text{lat}/2) + \cos(\text{lat1}) \cos(\text{lat2}) \sin^2(2\Delta\text{lon}/2)$$

$$c = 2\text{atan2}(a, 1 - a)$$

$$d = Rc.$$

Finally, we can calculate the straight line distance (h) from the satellite to Houston:

$$h = \sqrt{r^2 + d^2 - 2rd \cos(c)}$$

The result of this calculation is approximately 3.581073.58107 meters.

Comment: We can make no sense of the angles being passed as arguments to the trigonometric functions in the request to Wolfram Alpha, either if these are interpreted as radians or as degrees.

Broadly speaking, GPT is approaching the problem as follows: Let H be Houston, S be the satellite, and P the earth point underneath the satellite. GPT is using the given information and trigonometric formulas to compute the distance HP and the angle HPS; the given information to compute PS, and then using the law of cosines to compute the length HS. We don't recognize the formulas being used, but they may be right. However, there seems to

be a problem with the angles being passed as argument. Also, GPT seems to be taking the length PS to be the radius of the geosynchronous orbit, whereas it should be the height of the geosynchronous orbit above the earth's surface.

GPT4+CI Answer: 44,771.83 km

Comment: Close but no cigar. GPT computes the distance along the earth's surface from Houston to the point under the satellite, computes the height of the satellite above that point, and then assumes that the line from Houston to the satellite is a right triangle with those two sides.

30. A satellite in a circular geosynchronous orbit passes directly above the North and South Poles. When it crosses the North Pole, its velocity is in the plane of the 0° circle of longitude. At what longitude does it pass directly above a point in the Tropic of Cancer? (An answer will be marked correct if it is within 3 degrees of the correct answer.)

Answer: 66.56° W.

Explanation: The period of a geosynchronous orbit is equal to a sidereal day. In that time, the satellite executes a 360° revolution and the earth executes a 360° rotation, so their angular velocities are equal. So the longitude is equal to the difference in latitude between the North Pole and the Tropic of Cancer. Since the Tropic of Cancer is at 23.44 degrees, the difference is 66.56 degrees.

GPT4+WA Answer: Premise of the question is impossible.

GPT4+WA Explanation: It seems like there might be a misunderstanding in the premises of the problem.

A geosynchronous orbit is one where the satellite completes one full orbit in 24 hours, the same time it takes for the Earth to rotate once on its axis. It's a type of high Earth orbit, and when it's circular and in the Earth's equatorial plane, it's specifically a geostationary orbit, meaning the satellite appears stationary relative to an observer on the Earth.

If a satellite passes directly over the North and South poles, it's not in a geosynchronous or geostationary orbit. This is a polar orbit, where the satellite travels over the poles on each revolution around the Earth. These are fundamentally different orbits.

Geostationary satellites always stay above the same point on the Earth's equator, not over the poles or the Tropic of Cancer (or any other latitude other than 0). For polar orbit satellites, they can cross any point of longitude but not at geosynchronous speeds.

So the problem as described is a mix of two different types of orbits which is contradictory. The velocity of the satellite can't be in the plane of the 0 circle of longitude (the Prime Meridian) while simultaneously passing over the North and South Poles and the Tropic of Cancer. It might be best to clarify the question or check the initial premises.

Comment: Wrong.

GPT4+CI Answer: 23.5 degrees longitude.

Comment: Wrong. GPT fails to subtract from 90 degrees. It also finishes with a completely confused comment: "The satellite passes directly above a point in the Tropic of Cancer when it is at 23.5 degrees longitude. However, this is a highly simplified model. In reality, a satellite in a polar orbit wouldn't remain in the plane of a particular longitude line due to the Earth's rotation. This solution assumes that the satellite's orbit somehow adjusts to remain in this plane, which isn't physically realistic." Finally, note that GPT fails to specify whether this is 23.5 degrees East or West.

31. A satellite orbits the earth in a circular orbit. It completes an orbit every 14 hours 40 minutes. How high is it above the Earth's surface?

Answer: 24,042 km

Matlab:

```
% By Kepler's 3rd law, R^3 is proportional to T^2, so R is proportional to T^(2/3)
r = 6378.1 % km. Earth's radius
q = r+35786 % km. distance of geosynchronous orbit from center of earth
e = 23+(56/60) % hours. Period of geosynchronous orbit.
z = q*((14+(40/60))/e)^(2/3) % Distance from satellite to center of earth
answer = z-r
```

GPT4+WA Answer: $2.400475 \cdot 10^7$ m

Comment: Correct.

GPT4+CI Answer: 24,047.5 km.

Comment: Correct.

32. A satellite orbits the earth in a circular orbit. It passes directly over the North and South poles and completes an orbit every 14 hours 40 minutes. On one orbit going southward it was directly above the earth location 40° N, 10° W at 1:00 PM EST. At what time will it next cross the plane that contains the circle of latitude 40° N, and what will be its longitude? (An answer will be marked correct if the time is within 5 minutes of the correct answer, and the longitude is within 3 degrees.)

Answer: Time: 2:19 PM. Longitude: 29.77 degrees W (= $20^\circ \circ 40'$ W)

Matlab:

```
% Let c be the center of the earth.
% Let P be the plane containing the circle of 40 degrees latitude.
% Let s be the position of the satellite when it crosses P
% Let a be the point where the line from c to the North Pole crosses P.
% Let r be the radius of the earth and let h be the height of the satellite
```

```
% cas is a right triangle. |ca| = r*sin(40 degrees). |cs| = r+h
% So the angle acs = arcsin(|ca|/|cs|).
% The angle traversed since 1 PM is 40-(90-acs)
% The rest of the calculation is as in problem 29
```

```
r = 6378.1 % km. Earth's radius
h = 24042 % height of satellite over Earth's surface. Problem 30
w = 14+(40/60) % period of satellite
d = 23+(56/60) % sidereal day
deg2rad = pi/180
ca = r*sin(40*deg2rad)
cs = r+24042 % Problem 30
acs = asin(ca/cs)
at = 40-(acs/deg2rad) % angle traversed in degrees
lat = 10 + (w/d)*at % latitude
t = 1 + w*(at/360) % time
```

GPT4+WA Answer: 11:00 PM EST, at 65° W longitude.

:GPT4+WA Explanation

DA

A satellite orbits the earth in a circular orbit. It passes directly over the North and South poles and completes an orbit every 14 hours 40 minutes. On one orbit going southward it was directly above the earth location 40° N, 10° W at 1:00 PM EST. At what time will it next cross the plane that contains the circle of latitude 40° N, and what will be its longitude?



This problem can be solved by understanding the motion of the satellite and the rotation of the Earth.

The satellite completes an orbit every 14 hours and 40 minutes, or 880 minutes. This means it moves at a speed of $360^\circ/880$ minutes = 0.4091° per minute in its orbit.

The Earth rotates once every 24 hours, or 1440 minutes. This means it rotates at a speed of $360^\circ/1440$ minutes = 0.25° per minute.

The relative speed of the satellite to the Earth's rotation is therefore $0.4091^\circ - 0.25^\circ = 0.1591^\circ$ per minute.

The satellite was last directly above 40° N, 10° W at 1:00 PM EST. We want to know when it will next be directly above 40° N, at any longitude.

Since the satellite passes directly over the North and South poles, it will cross the 40° N latitude line twice during each orbit - once going northward and once going southward. We're interested in the next crossing, which will be the northward crossing.

Half an orbit later, the satellite will be at the same longitude but at 40° S. This takes 440 minutes. However, during this time, the Earth has rotated. The rotation of the Earth during 440 minutes is $440 * 0.25^\circ = 110^\circ$.

So, when the satellite is at 40° S, it will be over the longitude 10° W - $110^\circ = -120^\circ$ W (or 120° W).

The satellite then needs to travel from 40° S to 40° N. This is a quarter of its orbit, so it takes 220 minutes. During this time, the Earth rotates by $220 * 0.25^\circ = 55^\circ$.

So, when the satellite is next at 40° N, it will be at the longitude 120° W - $55^\circ = 65^\circ$ W.

The total time for this to occur is 440 minutes (to get to 40° S) + 220 minutes (to get back to 40° N) = 660 minutes.

So, the satellite will next cross the 40° N latitude line at 1:00 PM EST + 660 minutes = 11:00 PM EST, and it will be at 65° W longitude.

Comment: GPT4 doesn't get the distinction between being over the point at 40 degrees latitude and in the plane of the 40 degree latitude circle. The calculation isn't right, even for the alternate interpretation.

We ran a further test (not included in our tally, since it was devised after the fact): "A satellite is 10000 km above the earth's surface in the plane that contains the 40 degree circle of latitude. What is the latitude of the point directly below the satellite?" This completely confused the system; GPT called WA ten times, and finally gave up.

GPT4+CI Answer: 3:40 EST on the next day, 150° W.

Comment: Like GPT4+WA, GPT4+CI fails to understand the difference between the two geometric concepts, and does not find the right answer even for the problem with the alternate interpretation. Asked the above follow-up question, it gave the answer " 40° ". going through some circular trigonometric calculations to find it.