

“Arbitrary Numerical” Science Problems for GPT4 + Wolfram Alpha and GPT4 + Code Interpreter: Problems

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This document includes 32 problems in high school and college-level physics and mathematics with numerical or vector answers.

Unless otherwise noted, all questions should be marked correct if the relative error is less than 1%. If the answer is a vector, then the question will be marked correct if $|\vec{u} - \vec{v}|/|\vec{v}| < 0r., 1$ where \vec{v} is the correct value and \vec{v} is the proposed answer.

1. Object A is a spherical shell of gold, 1 meter in radius, 10 cm thick. Object B is a spherical shell of copper, 10 cm in radius, 5 cm thick. They are placed so that there is an 8 cm gap between them. What is the net gravitational force they exert on each other?
2. Let b, c, d be points at the center of Buenos Aires, Cincinnati, and Delhi, respectively. Consider the plane P that contains b, c, and d (it cuts through the earth). What is the area in square kilometers of the triangle b,c,d lying in P?
3. X is a regular tetrahedron of zinc with a side length of 2 moving at 100 m/sec. Y is a molecule of NaCl that is moving with speed v . X and Y have equal kinetic energy. What is $1 - (v/c)$, where c is the speed of light?
4. Viewed from Vega, what is the angle between Sirius and the Sun?
5. You have an empty cylindrical open container whose inside has a diameter of 8 centimeters and a height of 20 centimeters. and a pitcher with 200 ccs of water. You first empty the pitcher into the cylinder, then put a solid rock cube, 4 cm on a side, into the container so that it is sitting flush against the bottom of the container. What is the height of the water in the container?
6. There are six small spherical masses placed at various positions in space. Measured in a coordinate system with a 1 meter unit length:
A has mass 2 kg and is at $\langle 2, 0, 0 \rangle$;
B has mass 3 kg and is at $\langle 0, 2, 0 \rangle$;
C has mass 1 kg and is at $\langle 0, 0, 2 \rangle$;
D has mass 4 kg and is at $\langle 1, 1, 1 \rangle$;
E has mass 5 kg and is at $\langle 1, 0, 1 \rangle$;
F has mass 2 kg and is at $\langle 0, 1, 1 \rangle$.
What is the instantaneous acceleration of F under the gravitational attraction of A,B,C,D,E?
Express your answer as a vector.

7. How many total eclipses of the moon were there between Jules Verne's death and Neil Armstrong's moon landing? An exact integer value is required.
8. A quantity of chlorine gas is in a right prism whose base is a triangle with sides 5 cm, 7 cm, and 4 cm and whose altitude is 8 cm. The temperature is the freezing point of mercury, and the pressure is 2 atmospheres. What is the mass of the chlorine?
9. A train has two whistles, one at middle C and one at the F above middle C. The train is driving past a station without stopping or changing speed. On approaching the station, it blows the low whistle; when it passes the station, it switches to the high whistle. However, to the people standing at the station, it sounds like the whistle dropped by a whole tone. How fast is the train moving?
10. A physical process generates photons whose energies follow a random distribution of the following form: For positive energy e , the probability density at e is proportional to the value of e in a Gaussian distribution with mean 2 Ev and standard deviation 0.01 Ev. The probability of a negative value is zero. What is the expected value of the wavelength of a photon produced by this process? (Give the mathematical answer, assuming that the above description is exact, and assuming the standard relation between energy and wavelength in a photo. The answer is not physically plausible.)
11. The wavelengths of the photons in a beam of light are uniformly distributed across the range of visible (to humans) light. What is the mean energy of a photon in electron volts?
12. A pendulum is hanging on a 2 meter cord attached to the ceiling 3 meters above the floor. It is brought to a position 25 degrees from the vertical and released. It swings past the bottom and the cord is cut when it is 10 degrees from the vertical on the far side. Then the bob flies through the air and hits the ground. What is the distance from the point where the bob is released to the point where it hit the ground?
13. An spherical asteroid 500 km in diameter travels in an essentially perfect circular orbit of radius 2.4 astronomical units. On January 4, 2023, the earth was at perihelion, and as it happens, the asteroid was in perfect opposition. What was the solid angle of the asteroid in the sky as seen by an earth observer?
14. Two twin stars, one of 5 solar masses, the other of 10 solar masses, orbit each other in circular orbits in a period of 4 earth years. How far apart are they, in astronomical units?
15. Draw a circle, on the earth's surface, going through Cairo, Peking, and Moscow. Let S be the area of the part of the earth's surface inside the circle and let P be the area of the circle in the plane of the circle. What is S/P ?
16. Assume that the probability of having any particular isotope of a chemical follows their frequency on earth. What is the probability that a randomly constructed molecule of glucose will have 4 atoms of ^{12}C , 2 atoms of ^{13}C , 11 atoms of ^1H , 1 atom of ^2H , 3 atoms of ^{18}O , 3 atoms of ^{16}O .
17. Two ^{31}P phosphorus nuclei, with no electrons, are isolated in space, with coordinates $\langle 0,0,0 \rangle$ and $\langle 10,10,10 \rangle$ in a coordinate system whose unit length is 1 Angstrom. What is the instantaneous acceleration (a vector with unit length of Angstrom/sec²) of the nucleus at the origin due to the electrostatic force?
18. An irregular (house-shaped) pentagon has vertices numbered 1 through 5 in order. The pentagon has right angles at vertices 1, 3, and 4, and 135-degree angles at 2 and 5. Side 2-3 and 4-5 have length 1 and side 3-4 has length 2. The pentagon is placed on a planar coordinate system so that the numbering of the vertices is in clockwise order, vertex 3 is at the origin, and vertex 5 is on the positive y-axis. What are the coordinates of vertex 1?

19. Consider a cube with unit length sides, where the vertices of one face are numbered A..D in counterclockwise order, as viewed from the center of the cube; the vertices of the opposite face are named E to H; and there are edges AE, BF, CG, and DH. Rotate the cube so that vertex A is at the origin, vertex G is on the positive z axis, and vertex B is in the x-z plane, with positive x coordinate. What are the coordinates of vertex E?

20. Joe and Jim each have a bank account which they started on January 1, 2000. Joe started his account with \$1000; Jim started his with \$900. Joe's account pays 5% every December 31, and he adds an additional \$500 every January 1. Jim's account pays 10% every December 31. When will Jim's account have more money in it than Joe's?

21. Two points \mathbf{p} and \mathbf{q} are independently randomly chosen following a uniform distribution in a three-dimensional sphere of radius 10^7 . With a relative error of less than 1%, give an estimate with an error no greater than 1% that the Euclidean distance from \mathbf{p} to \mathbf{q} is less than 1?

22. A point \mathbf{p} is chosen at random within the 100-dimensional box $\mathbf{B} = [0, 100]^{100}$ following a uniform distribution. What is the probability that the Euclidean distance from \mathbf{p} to the boundary of \mathbf{B} is less than 1?

23. A point \mathbf{p} is chosen at random within the 100-dimensional box $\mathbf{B} = [0, 1]^{100}$ following a uniform distribution. What is the mean value of the Euclidean distance from \mathbf{p} to the boundary of \mathbf{B} ?

24. A pound of graphite is inside a closed cylindrical container with a radius of 50 cm and a height of 200 cm, filled with air at room temperature (22° C) and atmospheric pressure. The graphite reacts with the oxygen, producing equal masses of carbon monoxide and carbon dioxide until one of the reactant chemicals is exhausted. How many moles of each chemical is in the container at the end?

25. Suppose that you have a gram of pure radium 233. What is the mass of the helium that will generate via fission in a week?

26. A small school has 100 students and three extramural activities: the basketball team, the chess club, and the drama society. There are 60 students in the band, 48 in the chess club, and 55 in the drama club. There are 18 students who do both band and chess, 28 who do both band and drama, 31 who do both chess and drama, and 8 students who are in all three groups. What is the probability that a student is not in the band given that they are either in the drama club or the chess club but not both?

27. You draw 20 cards from a deck of cards labelled 1..60. To your surprise, the cards can be arranged as an arithmetic sequence. What is the probability that the successive difference is 3? Give an exact answer as a rational number.

28. You draw 5 cards from a deck of cards labelled 1 ... 200. To your surprise the cards can be arranged as a geometric series. What is the probability that the smallest number is less than or equal to 7? (Note that the ratio need not be an integer.)

29. In this question and the following three, assume that the satellite is moving in a closed orbit around the Earth and that the only influence on the satellite's motion is the Earth's gravity. Assume that the Earth is a perfect sphere. Ignore the revolution of the Earth around the sun, but do not ignore the rotation of the Earth around its axis.

A satellite in a circular geosynchronous orbit remains directly above the point on the earth's equator 0° N , 50° E . What is the straight line distance from the satellite to Houston?

30. A satellite in a circular geosynchronous orbit passes directly above the North and South Poles. When it crosses the North Pole, its velocity is in the plane of the 0° circle of longitude. At what longitude does it pass directly above a point in the Tropic of Cancer? (An answer will be marked correct if it is within 3 degrees of the correct answer.)

31. A satellite orbits the earth in a circular orbit. It completes an orbit every 14 hours 40 minutes. How high is it above the Earth's surface?

32. A satellite orbits the earth in a circular orbit. It passes directly over the North and South poles and completes an orbit every 14 hours 40 minutes. On one orbit going southward it was directly above the earth location 40° N, 10° W at 1:00 PM EST. At what time will it next cross the plane that contains the circle of latitude 40° N, and what will be its longitude? (An answer will be marked correct if the time is within 5 minutes of the correct answer, and the longitude is within 3 degrees.)