Some problems that should be easy for people good at math but might be hard for the current generation of AIs

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My guess is that a strong high school math student should be able to answer any of these in a few minutes without need for pen and paper or computer. If that doesn't seem right, let me know. On the other hand, I conjecture that these are hard for the currrent (2022) generation of AI problems that solve math problems from natural language descriptions. If anyone has any information about this, or is in a position to run experiments, *please* let me know. If you would like to suggest more, I'll be happy to consider them. Please, no standard brain-teasers that can be Googled.

Problems 3 and 4 ask for proof, because one can guess the answer "no" just from the fact that the question is being asked (if the answer were "yes", the question would not be interesting). For the others, only an answer is required, not a proof. For some of these, it would take some work to formulate a rigorous proof. That's part of the point.

Problem 1

Arrange in increasing order. (a) $11^{11^{-11}}$. (b) $11^{(-11)^{11}}$. (c) $(-11)^{11^{11}}$. (d) $(-10)^{10^{10}}$

Problem 2

Arrange in increasing order: (a) $10^{10^{10}} + 13^{13^{13}}$; (b) $7^{7^{7^{7^{7}}}} + 102$; (c) $105!$.

Problem 3

Prove that there do not exist primes p and q such that $q = p + 109$.

Problem 4

Prove that there does not exist a prime p such that $p+32, p+64, p+128$ and $p+256$ are all prime. Hint: Reduce mod 5.

Problem 5

Let C be the circle centered at the origin of radius 1. Let v be the point $\langle 0, 1\rangle$. Let S be the square inscribed in C, one of whose vertices is v, and let H be the regular hexagon inscribed in C one of whose vertices is v. Label vertex v as 1, and label the points on the circle that are vertices of either S or H as 2, 3, 4 ... in increasing order, going clockwise around C. What are the coordinates of vertex 7?

Problem 6

Let P be a cube of side length 3. Let U and V be opposite faces. Let a and e be vertices in U and V respectively that are connected by an edge. Starting with a and moving counterclockwise around U as viewed by someone in the center of the cube, label the other vertices of U as b, c, d. Starting with e and moving counterclockwise around V as viewed by someone in the center of the cube, label the other vertices of V as f, g, h. Let m be the point on edge eh that is distance 1 from e. What is the distance from b to m?

Problem 7

Let S be the surface of a sphere. Let A be a convex polyhedron all of whose vertices lie on S. Let $N \geq 4$ be an integer. Describe how to construct a convex polyhedron B with N vertices, all of whose vertices lie on S, such that B is disjoint from A.

Problem 8

Let N be an positive integer. How many roots does the equation $x^2 \sin(\pi x/2) = 36$ have for x between 0 and $4N$?

Problem 9

How many solutions does the equation $\sin(x^{1/100} + \cos(x)) = 0$ have for positive x?

Problem 10

How many non-negative solutions x does the equation $\sin(\lceil x \rceil^2) = 0$ have? ($\lceil x \rceil$ is x rounded up to the next integer.)

Problem 11

How many solutions does the equation $\sin(\cos(x)) = 1$ have for real x?

Problem 12

You have a deck of cards with 6 spades, 7 hearts, 8 diamonds, and 9 spades. You deal a hand of five cards at random. What is the probability that all the cards are of the same suit? You may express your answer as an arithmetic expression using the function $C(n,k) = n!/(k!(n-k)!)$, the number of combinations of k elements from a set of n.

Problem 13

You have a deck of 60 cards labelled 1..60. You deal a hand of 20 cards at random. What is the probability that the cards in your hand can be ordered as an exact arithmetic series? You may express your answer as an arithmetic expression using the function $C(n,k) = n!/(k!(n-k)!)$, the number of combinations of k elements from a set of n.

Problem 14

You have a deck of 60 cards labelled 1..60. You deal a hand of 20 cards at random. To your astonishment, you find that the cards in your hand can be ordered as an exact arithmetic series. What is the probability that the successive difference is 1?

Problem 15

A and B are two disjoint, interlocked circles in three-dimensional space. P is the plane containing A. Which of these might be true: (a) P is disjoint from B. (b) P intersects B at one point; (c) P intersects B at two points; (d) P intersects B at finitely many, more than two, points; (e) B lies in P.

Problem 16

Does there exist an ellipse with major axis of length 10 and circumference 50?

Problem 17

Does there exist an ellipse with minor axis of length 10 and circumference 50?

Problem 18

Let A and B be two ellipses in the plane. A has major axis 10 and minor axis 5. B has major axis 2 and minor axis 1. The center of B is inside A. Which of the following are possible: (a) A is entirely inside B. (b) B is entirely inside A. (c) A and B intersect in a single point. (d) A and B intersect in two points. (e) A and B intersect in 3 points. (f) A and B intersect in 4 points. (g) A and B intersect in a finite number of points greater than 4. (h) A and B coincide along an arc of finite length. Hint: 5 points determine a conic.