Propositional Calculus

Atomic sentence: p, q, r . . .

Boolean operators:

¬p — not p.
p∧q — p and q.
p∨q — p or q.
p⇒q — if p then q
p⇔q — p if and only if q.

Sentence: Either an atomic sentence or a Boolean operator applied to sentences.
Examples:

p.
p ∨ q
¬p ⇔ (q ∨ p).

A *literal* is either an atomic sentence or the negation of an atomic sentence.
Examples: p, q, ¬p, ¬q.

A sentence is in *conjunctive normal form* (CNF) if it is the disjunction of literals. A set of sentences is in CNF if each sentence is in CNF.
Example: The following set of sentences is in CNF.

p.
¬p ∨ q ∨ r.
q ∨ ¬r.

Converting a sentence to CNF:

1. Replace every occurrence of $\alpha \Leftrightarrow \beta$ by $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   When this is complete, the sentence will have no occurrence of ⇔.
2. Replace every occurrence of $\alpha \Rightarrow \beta$ by $\neg\alpha \vee \beta$. When this is complete, the only Boolean operators will be ∨, ¬, and ∧.
3. Replace every occurrence of $\neg(\alpha \vee \beta)$ by $\neg\alpha \wedge \neg\beta$; every occurrence of $\neg(\alpha \wedge \beta)$ by $\neg\alpha \vee \neg\beta$; and every occurrence of $\neg\neg\alpha$ by α. Repeat as long as applicable. When this is done, all negations will be next to an atomic sentence.
4. Replace every occurrence of $(\alpha \wedge \beta) \vee \gamma$ by $(\alpha \vee \gamma) \wedge (\beta \vee \gamma)$, and every occurrence of $\alpha \vee (\beta \wedge \gamma)$ by $(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$. Repeat as long as applicable. When this is done, all conjunctions will be at top level.
5. Break up the top-level conjunctions into separate sentences. That is, replace $\alpha \wedge \beta$ by the two sentences α and β. When this is done, the set will be in CNF.

Example:

Start: $(p \Rightarrow q) \Leftrightarrow r$.
After step 1: $((p \Rightarrow q) \Rightarrow r) \wedge (r \Rightarrow (p \Rightarrow q))$.
After step 2: $(\neg(\neg p \vee q) \vee r) \wedge (\neg r \vee (\neg p \vee q))$.
Step 3(a): $((\neg\neg p \wedge \neg q) \vee r) \wedge (\neg r \vee (\neg p \vee q))$.
After step 3: $((p \wedge \neg q) \vee r) \wedge (\neg r \vee (\neg p \vee q))$.
After step 4: $((p \vee r) \wedge (\neg q \vee r)) \wedge (\neg r \vee (\neg p \vee q))$.
After step 5: { $p \vee r$.
$\neg q \vee r$.
$\neg r \vee \neg p \vee q$. }