Dating Strategy

Ralph is considering asking Josephine out on a date for Saturday night. He posits the following stochastic model, based on God knows what evidence. There are three random Boolean variables.

- S — Josephine is otherwise free on Saturday. $P(S = T) = 0.6$.
- L — Josephine likes him. $P(L = T) = 0.75$.
- D — Josephine will accept his invitation to a date. The probabilities are shown below.

$P(D{=}T \mid S, L)$	$S{=}T$	$S{=}F$
$L{=}T$	0.95 (Sure!)	0.1 (J. will ditch previous engagement)
$L{=}F$	0.2 (nothing better to do)	0.0 (Are you kidding?)

Assume, finally, that S and L are absolutely independent.

A. Evaluate $P(D)$.

B. Sad to say, Josephine declined the date. Evaluate $P(L = T \mid D = F)$.

C. Faint heart never won fair lady. There is next Saturday. Assume that L remains constant. There are now two new random variables: $S2$ is the event that Josephine is free next Saturday. $D2$ is the event that she agrees to go on a date next Saturday. Assume that the distribution of $P(S2)$ is the same as $P(S)$; that $P(D2 \mid L, S2)$ is the same as $P(D \mid L, S)$; that $S2$ is absolutely independent of S, L, and D; and that $D2$ is conditionally independent of S and D given $S2$ and L.

Evaluate $P(D2 = T \mid D1 = F)$.

D. Ralph associates the following utilities to various actions and outcomes (don't ask me what the units are).

- Summoning up the courage to ask for a date has a disutility of -1, each time it has to be done.
- Being rejected has a disutility of -3, each time it happens.
- If $L = T$, then a date has a utility of 20. If $L = F$, then a date has a disutility of -6.

Draw and evaluate a decision tree for Ralph at the start, showing the following three courses of action:

- Ralph never asks for any dates.
- Ralph asks for a date this week. If it is rejected, he does not try again.
- Ralph asks for a date this week. If it is rejected, he asks again for next week.

What is Ralph's best course of action, and what is his expected utility if he adopts that?

Problem 2

A. Suppose you have a collection of N clauses in the propositional logic, where each clause has three literals with three different atoms.

You assign a truth value to each atom at random, with equal probability true or false. Let random variable K be the number of clauses satisfied by the assignment. What is $\text{Exp}(K)$ as a function of N? Justify your answer.

For example one set with $N = 5$ is

{ 1. $P \lor \neg Q \lor R$.
  2. $\neg P \lor Q \lor W$.
  3. $P \lor \neg R \lor W$.
  4. $P \lor R \lor \neg W$.
  5. $Q \lor R \lor \neg W$. }

If you randomly choose $P = F$, $Q = T$, $R = T$, $W = F$, then in that case 1, 2, 4, and 5 are satisfied but 3 is unsatisfied, so in that case, the value of K is 4.

Hint: This is an *easy* problem; your answer should not be more than three or four sentences long. Determining the probability distribution of K is difficult, and depends on the particular set of clauses. For example, $\text{P}(K = N)$ is equal to 0 only if the set of clauses is unsatisfiable, which, as we've seen, is a hard problem (co-NP-complete). But $\text{Exp}(K)$ is the same, regardless of what the clauses are (even if the collection is just N repetitions of the same clause, say.)

B. Find the probability distribution for K for the above specific example of five clauses. (There is no clever way to do this; you just have to enumerate all 16 different valuations.)