Fundamental Limits on Synchronizing Clocks Over Networks

Nikolaos M. Freris, *Member, IEEE*, Scott R. Graham, *Senior Member, IEEE*, and P. R. Kumar, *Fellow, IEEE*

***Abstract*—We characterize what is feasible concerning clock synchronization in wireline or wireless networks. We consider a network of n nodes, equipped with affine clocks relative to a designated [1] clock that exchange packets subject to link delays. Determining all unknown parameters, i.e., skews and offsets of all the clocks as well as the delays of all the communication links, is impossible. All nodal skews, as well as all round-trip delays between every pair of nodes, can be determined correctly. Also, every transmitting node can predict precisely the time indicated by the receiver's clock at which it receives the packet. However, the vector of unknown link delays and clock offsets can only be determined up to an $(n-1)$-dimensional subspace, with each degree of freedom corresponding to the offset of one of the $(n-1)$ clocks. Invoking causality, that packets cannot be received before they are transmitted, the uncertainty set can be reduced to a polyhedron. We also investigate structured models for link delays as the sum of a transmitter-dependent delay, a receiver-dependent delay, and a known propagation delay, and identify conditions which permit a unique solution, and conditions under which the number of the residual degrees of freedom is independent of the network size. For receiver-receiver synchronization, where only receipt times are available, but no time-stamping is done by the sender, all nodal skews can still be determined, but delay differences between neighboring communication links with a common sender can only be characterized up to an affine transformation of the $(n-1)$ unknown offsets. Moreover, causality does not help reduce the uncertainty set.**

***Index Terms*—Clock offsets, clock skews, clock synchronization, delays, networked control, scheduling, sensor networks.**

I. Introduction

DISTRIBUTED clocks generally do not agree. Yet, several applications in sensor networks and networked control are grounded in accurate clock synchronization. Applications in networked control include closing control loops or coordinating events in a decentralized system, such as a traffic control or collision avoidance system. In sensor networks, clock synchronization requirements are omnipresent; in tracking, target localization, data fusion, and power-efficient duty-cycling. Scheduled operations in wireless networks, e.g., slotted protocols, also require accurate clock synchronization. This motivates the study of clock synchronization over communication networks. As we head towards the era of event-cum-time driven systems featuring the convergence of computation and communication with control, the need for well-synchronized clocks becomes increasingly important, affecting system performance, QoS, and safety.

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N. M. Freris and P. R. Kumar are with the Department of Electrical and Computer Engineering and Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: nfreris2@illinois.edu; prkumar@illinois.edu).

S. R. Graham is with the Air Force Institute of Technology, WPAFB, OH 45433-7765 USA (e-mail: scott.graham@afit.edu).

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A. Notions of Network Clock Synchronization

There are three main degrees of clock synchronization that may be required by the specifications of a particular application.

1) *Ordering of events*. In this case, the problem is to create a right chronology of the events in the entire network. For this purpose, knowledge of the exact time instants is not required, yet an ordering of events that occur at different nodes or the same node has to be determined. This is the weakest notion of synchronization but is sufficient for many applications such as financial transactions involving locks in databases, several monitoring applications, etc. This problem was first thoroughly studied in [3] where the notion of *virtual clocks* was introduced.
2) *Relative Synchronization*. Here, the goal of synchronization is to estimate the relative drift among a set of clocks in the network. This information can be then used to translate time-stamps from one clock to the units of any other clock. This has the advantage that the translation mechanism does not create undesirable dependencies by resetting clocks in hosts [2], [16] and gives rise to the notion of *relative clocks*.
3) *Absolute Synchronization*. This is the strongest notion of clock synchronization in a network. Each node has an *absolute clock* and the goal is to set all clock displays to agreement, so that a global definition of time is achieved in the entire network. Networked control applications typically require this stronger type of synchronization.

From the above definitions, it is immediate that absolute synchronization implies relative synchronization which in turn implies ordering of events in a network. However, if we define a particular node as reference, then relative synchronization can be used to achieve absolute synchronization.

In this paper, we focus on relative and absolute synchronization.

B. Problem Under Study

The specific problem that we address is the characterization of the extent to which clock synchronization is even feasible [1]. We consider clocks which run at a *constant*, but not necessarily identical speed. Each clock is characterized by its "*skew*," i.e., relative speed with respect to a reference clock, as well as an "*offset*," i.e., the time difference from the reference clock at a particular time, which, for convenience, we take to be the time 0 of the reference clock. Thus, we consider *affine* clocks.

Our purpose is to exhibit fundamental impossibility results in clock synchronization. For this purpose, we will consider the ideal scenario where neither offset nor skew drifts with time and all time-stamps are noiseless. Clearly, if clocks cannot be synchronized in such an ideal environment, then they cannot be synchronized in the presence of noise. For the same reason, we suppose that packets suffer delays dependent on the transmitter-receiver communication pair. We allow for noiseless communication where latencies in packet transfer are deterministic and time-invariant but *unknown*. In the same spirit, we will allow nodes to exchange an arbitrary number of packets containing any information that the transmitting node knows about current or past packets that the node has sent, or any information contained in past packets that it has received. This includes the time that the current packet is being sent according to the transmitter's clock, as well as the times that it received previous packets. This also includes hearsay information that another node may send to it concerning information that it received from yet other nodes. Thus, we allow for packets to contain any causally acquired information that the sender may have.

C. Characterization of Limits on Clock Synchronization

We show that while all skews can be perfectly determined, the delays of the links and the offsets of the nodes cannot be exactly determined. Specifically, the vector of all link delays and node offsets can only be characterized up to a translation of an $(n-1)$-dimensional subspace, where n is the number of nodes. In fact, we show that these $(n-1)$ indeterminable parameters can be regarded as estimates of the nodal offsets.

If we further invoke causality, i.e., that packets cannot be received before they are sent, then the uncertainty region is a polyhedron that is explicitly characterized. We provide necessary and sufficient conditions on the network topology for the polyhedron to be compact and have a nonempty interior.

We also study the problem of receiver-receiver synchronization where nodes only exchange information on the times at which they receive broadcast packets, without any sender time-stamping. We show that nodal skews can still be determined correctly, but only delay differences between neighboring communication links with a common sender, and not actual delays themselves, can be expressed affinely by $(n-1)$ unknown offsets. We prove that causality cannot be exploited and that the uncertainty set remains the entire space $\mathbb{R}^{n-1}$. Moreover, round-trip delays cannot be estimated from the known data.

We further study the case where link delays have the structure of being the sum of a transmitter-specific delay, a receiver-specific delay, and a known electromagnetic propagation delay. For such cases, we again characterize the uncertainty set.

The rest of the paper is organized as follows. In Section II we summarize related literature on clock synchronization. In Section III, we introduce the affine model for the clocks, and the assumptions on the delays. In Section IV, we formulate the problem, provide a formal description of the inter-node communication, present an impossibility result for the case of two clocks [2], and describe the structure of the residual uncertainty. In Section V, we describe the network clock synchronization problem and provide necessary and sufficient conditions on the network topology for the determination of all nodal skews. In Section VI, we prove that with no further assumption on the unknown delays, determining the offsets is impossible for *any* network topology under *any* communication scheme. We show that while skews can be reliably determined from known data, offsets involve an inherent indeterminacy. Furthermore, we outline a method for the optimal selection of the offset vector. In Section VII, we study the problem of receiver-receiver synchronization in a network, and prove the infeasibility and the fundamental fact that causality cannot be exploited in this case. In Section VIII, we study the problem when the delays have an additive decomposition in terms of transmission, reception and propagation latencies. Finally, in Section IX, we cite some concluding remarks of our work.

II. Related Literature

In distributed systems, ordering of events is crucial for many applications. Lamport [3] shows how to causally order events by defining the notion of *virtual clocks*.

In [13], the authors study the problem of synchronizing clocks in a fully connected network and show that an uncertainty of ϵ in packet delivery leads to a maximum synchronization error no less than $\epsilon(1-(1/n))$. In [12], the authors derive a polyhedral uncertainty set for link delays in a general network topology. Establishing the impossibility of determining the offset in pairwise clock synchronization was carried out in [2], [16]. A similar result is noted in [14], though without a complete rank-based proof. The main result is that while determining the relative skew between two clocks is possible, it is impossible to do so for offsets, unless delays in the two-way communication are assumed to be symmetric. The proof of this result together with an extended analysis of the uncertainty set for the problem is presented in Section IV and is used to establish the results for the network case.

The basic mechanism for synchronizing clocks is to exchange time-stamped packets, or "*pings*," between nodes. The Network Time Protocol (NTP [7]) is a widely used hierarchical protocol implemented to achieve absolute synchronization of clocks in large networks like the Internet. NTP provides accuracy in the order of milliseconds [7] by typically using GPS to achieve synchronization to external sources that are organized in levels called *stratums*. While this accuracy may be sufficient for some applications, recent applications in wireless sensor networks typically require precision in the order of microseconds (μs). Moreover, in some cases, e.g., indoors, or during solar flares, GPS may be unavailable.

In sensor networks and networked control, a variety of algorithms have been suggested for synchronization such as the Reference Broadcast Synchronization (RBS [8]) and Flooding Time Synchronization Protocol (FTSP [9]). RBS is a receiver-receiver synchronization algorithm, which uses the broadcast nature of the wireless medium. It does not make use of sender side time-stamping. Nodes broadcast packets and the nodes that receive a common transmission then record and exchange the reception times so as to estimate the receiving nodes' clock differences. The scheme attains precision within 11 μs. In FTSP

[9], the Medium Access Control (MAC) time-stamping capabilities are exploited and linear regression is used to compensate for clock drifts; the precision is of the order of 10 μs for absolute synchronization in a network with several hundreds of nodes. Elson *et al.* [6] studied fundamental properties of minimum variance estimates, and presented algorithms. Solis *et al.* [4] developed and implemented a decentralized asynchronous algorithm based on spatial smoothing, and performed comparative evaluations showing improvements. Giridhar and Kumar [5] analyzed the performance of this spatial smoothing method, both to determine asymptotic accuracy as well as convergence rates. In [15], the authors studied a stochastic differential equation model-based approach to clock synchronization.

III. Model for Clocks and Delays

A. Affine Model for Clocks

Throughout this paper, we will assume the simple model of *affine* clocks. Denoting the time of a fixed *reference clock* by t, we will assume that the display of a clock j in the network at time t, denoted by $\tau_j(t)$, satisfies

$$\tau_j(t) := a_j t + b_j. \tag{1}$$

We call the ratio of the speeds of the two clocks, a_j, as the *skew*,[1] while the difference in their displays will be referred to as the *offset* at a particular instant. Above, b_j is the offset of clock j at the time 0 of the reference clock. An affine clock can thus be represented by the pair of parameters (a_j, b_j). For the purpose of establishing fundamental impossibility results, unknown parameters such as (a_j, b_j) are considered to be *constant* time-invariant parameters, throughout the entirety of this paper. The reason for such an assumption is that if we can prove that the determination of the unknown parameters is impossible under this idealistic scenario, then impossibility will naturally carry over to the case of time-varying parameters.

We also suppose that $a_j > 0$, i.e., forward evolution of the time in all clocks. In practice, the skew takes values very close to 1.[2] On the contrary, offsets are sign-indefinite and so no constraints are imposed on the values of b_j.

For notational convenience, we fix node 1's clock to be the reference clock; hence $a_1 := 1$, $b_1 := 0$. A useful formula that provides time translation of clock j's time to clock i's time units, as can be obtained from (1), is

$$T_i^j(\tau_j) := \frac{a_i}{a_j}\tau_j + b_i - \frac{a_i}{a_j}b_j. \tag{2}$$

The reason for assuming such clock model is because this is the simplest model that captures the reality that clocks are not synchronized because of non-nominal speeds and offsets. The model assumes constant but unknown clock skew and has been validated to be accurate for some clocks [11], [17], [18]. Establishing impossibility of clock synchronization under the affine model implies impossibility for more general models that capture skew variations, even though the uncertainty sets derived in this paper do not apply in such cases. A stochastic model for clocks and delays was introduced in [15].

B. Model for Packet Delay

Delays in packet delivery constitute a fundamental limitation in synchronizing clocks over wireless networks since they can be much larger than the required synchronization precision. We will suppose that whenever a packet is sent by node i, it is received by node j after a delay of d_{ij} time units (measured in the time units of the reference clock, clock 1). The delays $\{d_{ij}\}$ are assumed to be unknown but fixed; this is again an ideal scenario used to establish general impossibility results.

By the word "*delay*" here, we mean not only the electromagnetic propagation delay, but rather the sum of all delays incurred by a packet after it is time-stamped by the transmitter and before it is time-stamped by the receiver, as discussed in Section VIII. With the exclusion of the electromagnetic propagation delay, the other delays can depend on the communication and computation platforms of the nodes involved and the load experienced at them. Due to this heterogeneity, delays cannot be expected to be symmetric or identical between links. Hence we allow $d_{ij} \neq d_{ji}$ and $d_{ij} \neq d_{ik}$.

Even though the link delays are not clock parameters, their estimation is, however, a very important bi-product of clock synchronization, since knowledge of such quantities is crucial for many applications including routing and the stability of networked control loops [2], [16].

IV. Pairwise Synchronization of Two Clocks

We shall denote the time (as measured by the i-th clock) that node i sends its k-th packet by $s_i^{(k)}$ (see Fig. 1). In a wireless network where packets are broadcast, this packet might have multiple receivers, so we avoid specifying the receiver in the above notation. We will denote by $r_{i,j}^{(k)}$ the time (as measured by the j-th clock) that node j receives the k-th packet sent by node i (see Fig. 1).[3]

In this section, we will consider the case of only two nodes trying to synchronize their clocks. Thus, suppose that clock 1 (the reference) and clock j are the only two clocks in the system. As shown in Fig. 1. (in this case $i = 1$), the two nodes are allowed to communicate repeatedly by exchanging time-stamped packets. In the k-th packet sent by node 1, the transmitting node includes its current transmission time-stamp, $s_1^{(k)}$, as measured by its clock just before the transmission. Upon receiving this packet, the receiving node j records the time (according to its local clock) just after it receives the packet, $r_{1,j}^{(k)}$. Similarly, when node j is transmitting, we assume that the time-stamps $s_j^{(k)}$, $r_{j,1}^{(k)}$ are available. Recall that all time measurements are assumed noiseless, since showing indeterminacy of the parameters even for noiseless measurements would imply such indeterminacy for noisy models, too.

We allow for every packet to contain information about all the past receipt times of all prior packets as recorded at that node, so that each node contains a full log of the transmitting/receiving

[1]Some authors [14] define the skew to be $a_j - 1$, but here we use the terminology of [2].

[2]Our analysis essentially requires only that $a_j \neq 0$; the only place where $a_j > 0$ is used is when studying causality.

[3]Note that the horizontal lines in Fig. 1 represent time displays at different clocks and are, in general, at different scales, since clocks may run at different speeds and also have an offset from one another.

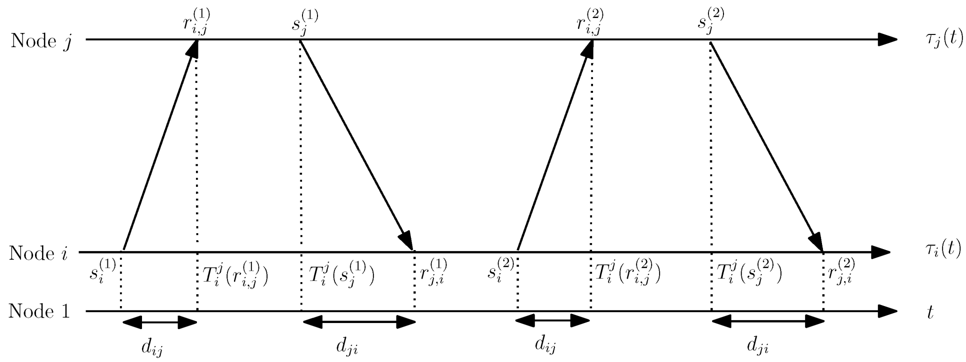

Fig. 1. Message exchanges between two nodes.

times of packets between them. Equivalently, we can suppose that there is a "genie" that has access to all transmit and receipt times for all packets, as recorded by their respective clocks.

As shown in [2], [16], it is impossible to estimate all four unknown parameters, $(a_j, b_j, d_{1j}, d_{j1})$, through any number of packet exchanges, which is stated and proven next for completeness of presentation:

1) Theorem 1. (Impossibility of pairwise synchronization [2]): Even under bilateral exchange of an infinite number of packets between the two nodes j and 1, estimation of the entire four-tuple $(a_j, b_j, d_{1j}, d_{j1})$ is impossible.

Proof: Since $a_1 = 1$, $b_1 = 0$, the translation of node j's time τ_j to the reference clock's time t is given by

$$T_1^j(\tau_j) := \frac{1}{a_j}\tau_j - \frac{1}{a_j}b_j. \tag{3}$$

For the k-th transmission of node 1 to node j, and vice-versa we have (see also Fig. 1)

$$1 \to j: \quad T_1^j\left(r_{1,j}^{(k)}\right) = s_1^{(k)} + d_{1j} \tag{4}$$

$$r_{1,j}^{(k)} = a_j T_1^j\left(r_{1,j}^{(k)}\right) + b_j \tag{5}$$

$$j \to 1: \quad r_{j,1}^{(k)} = T_1^j\left(s_j^{(k)}\right) + d_{j1} \tag{6}$$

$$s_j^{(k)} = a_j T_1^j\left(s_j^{(k)}\right) + b_j. \tag{7}$$

Substituting (4) into (5), and (6) into (7), we get

$$1 \to j: r_{1,j}^{(k)} = a_j s_1^{(k)} + a_j d_{1j} + b_j \tag{8}$$

$$j \to 1: s_j^{(k)} = a_j r_{j,1}^{(k)} - a_j d_{j1} + b_j. \tag{9}$$

In order to obtain equations that are linear in unknowns, we consider a nonlinear parametrization, $(a_j, a_j d_{1j}, a_j d_{j1}, b_j)$, of the unknowns $(a_j, b_j, d_{1j}, d_{j1})$

$$\begin{pmatrix} r_{1,j}^{(1)} \\ s_j^{(1)} \\ r_{1,j}^{(2)} \\ s_j^{(2)} \\ \vdots \end{pmatrix} = \begin{pmatrix} s_1^{(1)} & 1 & 0 & 1 \\ r_{j,1}^{(1)} & 0 & -1 & 1 \\ s_1^{(2)} & 1 & 0 & 1 \\ r_{j,1}^{(2)} & 0 & -1 & 1 \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} a_j \\ a_j d_{1j} \\ a_j d_{j1} \\ b_j \end{pmatrix}. \tag{10}$$

Denoting the vector on the LHS above by $\underline{\mathbf{y}}$, the matrix on the RHS by $\mathbf{A}$, and the vector on the RHS by $\underline{x}$, we have $\underline{\mathbf{y}} = \mathbf{A}\underline{x}$. We observe that $\underline{\mathbf{y}}$ and $\mathbf{A}$ contain known information based on the time-stamps, while $\underline{x}$ is the vector of the unknowns.

By the fact that the time-measurements are *noiseless*, we know that for the measured $(\mathbf{y}, \mathbf{A})$ there exists a solution $\underline{x}$ to $\underline{\mathbf{y}} = \mathbf{A}\underline{x}$, since the system of equations is consistent. A necessary and sufficient condition for this solution to be unique is that the matrix $\mathbf{A}$ have full-rank, i.e., rank 4. This is also the necessary and sufficient condition for the existence of a unique solution $(a_j, b_j, d_{1j}, d_{j1})$ to (10), since the parametrization $(a_j, b_j, d_{1j}, d_{j1}) \longmapsto \underline{x}$ is bijective for $a_j \neq 0$. However, the fourth column of $\mathbf{A}$ is the difference between the second and the third column, and so $\mathbf{A}$ has rank at most 3. ■

Remark 1.1: The first three columns of matrix $\mathbf{A}$ in (10) are linearly independent if and only if either not all odd or not all even entries are the same, that is to say if there are at least two distinct communication pings in one direction, and at least one ping in the other direction. In the sequel, we will assume, with no loss in generality that $s_1^{(1)} < s_1^{(2)} < \cdots$, and $s_j^{(1)} < s_j^{(2)} < \cdots$, and restrict attention to the 4×4 principal submatrix of $\mathbf{A}$, of rank 3, which yields the system

$$\begin{pmatrix} r_{1,j}^{(1)} \\ s_j^{(1)} \\ r_{1,j}^{(2)} \\ s_j^{(2)} \end{pmatrix} = \begin{pmatrix} s_1^{(1)} & 1 & 0 & 1 \\ r_{j,1}^{(1)} & 0 & -1 & 1 \\ s_1^{(2)} & 1 & 0 & 1 \\ r_{j,1}^{(2)} & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} a_j \\ a_j d_{1j} \\ a_j d_{j1} \\ b_j \end{pmatrix}. \tag{11}$$

For convenience, we denote the finite linear system in (11) as $\underline{y} = A\underline{x}$. The remaining pings contain no additional information not already contained in these four pings, for the purpose of determining the unknown parameters and estimating the delays. Note also, that from (11), it suffices that a transmitted packet contain information about its current transmit time and *just* the transmit-receipt time-stamps of two past pings, one in each direction[4] since the receipt time of the packet will also be available to the receiver.

Since the above system does not admit a unique solution, we study the set of all solutions to (11). We will call this set of all solutions as the *uncertainty set*. It is the smallest set within which the true but unknown parameter vector can be determined to lie.

[4]In fact, since $A^{(i,j)}$ is of rank 3, this system can be equivalently constructed as a 3×4 system, by using only three distinct pings (two in one direction and one in the other), however we will use, with no loss in generality, a 4×4 matrix for ease of presentation.

In the sequel, we use "^" to denote any estimate of unknown quantities, and "$*$" to refer to some special quantities that can be derived from the data.

Theorem 2. (Characterization of the Uncertainty Set in Pairwise Synchronization):

1) The skew can be precisely determined, even if there are only two one-way pings, i.e., the link is unilateral.[5]
2) The vector $(b_j, d_{1j}, d_{j1})^T$ of offset and delays can only be determined up to a translate of a one-dimensional subspace of $\mathbb{R}^3$. Each point in this one-dimensional translated subspace corresponds to a particular estimate of the unknown offset.
3) The round-trip delay $(d_{1j} + d_{j1})$ can be determined precisely.[6]
4) If we further use knowledge of causality, that packets cannot be received before they are sent, and also that $a_j > 0$, then the uncertainty set for the offset reduces to an interval, whose length is proportional to the round-trip delay.

Proof: The null space of the matrix A in (11) is spanned by the vector $(0, -1, 1, 1)^T$. Define $(a_j^*, d_{1j}^*, d_{j1}^*)$ by

$$a_j^* := \frac{r_{1,j}^{(l)} - r_{1,j}^{(k)}}{s_1^{(l)} - s_1^{(k)}} = \frac{s_j^{(l)} - s_j^{(k)}}{r_{j,1}^{(l)} - r_{j,1}^{(k)}} \tag{12}$$

$$a_j^* d_{1j}^* := r_{1,j}^{(k)} - a_j s_1^{(k)} \tag{13}$$

$$a_j^* d_{j1}^* := a_j r_{j,1}^{(k)} - s_j^{(k)} \tag{14}$$

for $k \neq l$. Note that, since the first entry of the vector spanning the null space is 0, we have that $a_j = a_j^*$, is the unique solution for the skew a_j, i.e., the skew can be determined by a ratio of differences between received time-stamps and sent time-stamps. Then the vector $(\hat{a}_j, \hat{b}_j, \hat{d}_{1j}, \hat{d}_{j1})^T$ fits the transmit and receive time-stamp data, i.e., $\underline{\hat{x}} = (\hat{a}_j, \hat{a}_j\hat{d}_{1j}, \hat{a}_j\hat{d}_{j1}, \hat{b}_j)^T$ satisfies $\underline{y} = A\underline{\hat{x}}$ (and $\underline{\mathbf{y}} = \mathbf{A}\underline{\hat{x}}$), if and only if

$$\begin{pmatrix} \hat{a}_j \\ \hat{a}_j\hat{d}_{1j} \\ \hat{a}_j\hat{d}_{j1} \\ \hat{b}_j \end{pmatrix} = \begin{pmatrix} a_j^* \\ a_j^* d_{1j}^* \\ a_j^* d_{j1}^* \\ 0 \end{pmatrix} + \hat{b}_j \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \end{pmatrix}. \tag{15}$$

Hence, $\hat{a}_j = a_j^*$, $\hat{b}_j \in \mathbb{R}$ is arbitrary, and

$$\begin{pmatrix} \hat{d}_{1j} \\ \hat{d}_{j1} \end{pmatrix} = \begin{pmatrix} d_{1j}^* \\ d_{j1}^* \end{pmatrix} + \hat{b}_j \begin{pmatrix} -\frac{1}{a_j^*} \\ \frac{1}{a_j^*} \end{pmatrix}. \tag{16}$$

This proves 1), 2), and also 3) since

$$\hat{d}_{1j} + \hat{d}_{j1} = d_{1j}^* + d_{j1}^*. \tag{17}$$

For 4), we invoke causality, that is to say that packets cannot be received before they are transmitted, which is equivalent to imposing the natural nonnegativity constraints on the delay estimates, $\hat{d}_{1j} \geq 0, \hat{d}_{j1} \geq 0$. Since $a_j^* > 0$, (16) immediately yields

$$\hat{b}_j \in \left[-a_j^* d_{j1}^*, a_j^* d_{1j}^*\right]. \tag{18}$$

The indeterminacy for the offset is $a_j^*(d_{j1}^* + d_{1j}^*)$. ■

[5]This was also shown in [14].

[6]This was also shown in [2], [12].

Remark 2.1: Note that determining the upper bound of the interval in (18), requires only transmission from node 1 to node j while determining the lower bound only needs node j to transmit to node 1.

Remark 2.2 (Use of Lamport's Global Ordering [3]): In our analysis of the consequence of causality, we have actually exploited all the available information provided by the global ordering described by Lamport in [3]. The ordering of events at a single node can be carried out trivially since all time instants are measured by the same clock with positive skew. Furthermore, causality simply implies that receipt should follow transmission, which is exactly the same as imposing the constraint $s_1^{(k)} \leq T_1^j(r_{1,j}^{(k)})$ and $s_j^{(k)} \leq T_j^1(r_{j,1}^{(k)})$ for all $k \in \mathbb{Z}^+$. By the assumption that the skew is positive, it follows from (4) and (6), that this is exactly equivalent to the nonnegativity of the link delays.

Remark 2.3: In [14], an indeterminacy of the offset was shown for a particular scheme for the case of asymmetric delays. Theorems 1 and 2 give a stronger statement than the result in [14] in two ways. They fully characterize the uncertainty set by showing that any point inside the set (18) constitutes a potential solution and any point outside it does not. In addition, our analysis does not ignore the contribution of the skew in determining the uncertainty set.

From the above characterization of the estimates, an interesting result surfaces.

Corollary 3. (Prediction of the Receiving Time by the Transmitter): The sending node can determine precisely the time, as measured by the receiver's clock, at which the receiving node will receive a sent packet.

Proof: When node 1 is transmitting, this is immediately true from (13) since $r_{1,j}^{(k)} = a_j s_1^{(k)} + a_j^* d_{1j}^*$, where $s_k^{(1)}$ is known to the transmitter, while all the other quantities are determinable by estimation. Similarly, when node j is transmitting, it follows from (14) that $r_{j,1}^{(k)} = (1/a_j^*)s_j^{(k)} + d_{j1}^*$. An alternative derivation can be made by solving (12) for $r_{1,j}^{(l)}$ (respectively $r_{j,1}^{(l)}$) since a_j^* can be determined correctly, and since the other time-stamps on the RHS of (12) are causally known to sender (we assume $l > k$). ■

When predicting such receipt times, the indeterminacy of offset and delays mutually cancel. This is an important property that can be potentially used to measure the accuracy of clock synchronization algorithms when noise is present in delays.

Since the determination of the unknowns does not allow a unique solution for the vector of offset and delays even in the simple case of pairwise communication, it is of interest to determine simple additional conditions that will allow precise characterization of offset and delays.

Theorem 4. (Sufficient Conditions for Uniqueness of Solution): All the parameters $(a_j, b_j, d_{1j}, d_{j1})$ can be uniquely determined if any one of the following conditions hold.

1) The offset b_j, or one of the delays d_{1j} or d_{j1}, is known.
2) There is a *known* affine relationship between the delays in the two directions, specifically, there exist $\alpha \neq -1, \beta$ known, such that $d_{j1} = \alpha d_{1j} + \beta$.

Proof: The first part follows directly from the fact that delays are known invertible (since $a_j \neq 0$) affine functions of

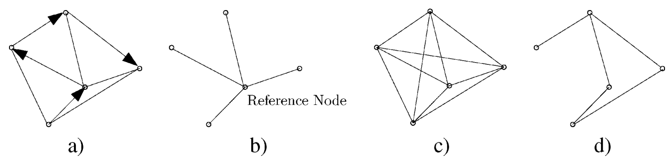

Fig. 2. General graph, star graph, complete graph, and connected graph.

the offset b_j (15), and the sum of the delays is known (17). To prove the second part of the theorem, we use the condition $d_{j1} = \alpha d_{1j} + \beta$ in (15) and get the unique solution

$$\begin{pmatrix} a_j \\ d_{1j} \\ d_{j1} \\ b_j \end{pmatrix} = \begin{pmatrix} a_j^* \\ \frac{\left(d_{1j}^*+d_{j1}^*\right)-\beta}{1+\alpha} \\ \frac{\alpha\left(d_{1j}^*+d_{j1}^*\right)+\beta}{1+\alpha} \\ a_j^* \frac{\alpha d_{1j}^*-d_{j1}^*+\beta}{1+\alpha} \end{pmatrix}. \tag{19}$$

Remark 4.1: The affine characterization of asymmetry in Theorem 4.2, includes the special case of *symmetric* delays $d_{1j} = d_{j1}$, which was established in [2], [12]. It also includes the case of *known asymmetry*, $d_{j1} - d_{1j} = \beta$, which corresponds to a known processing overhead in one of the nodes. More generally, studying the asymmetry as in [9] can lead to a full characterization of the pairwise problem. An analysis based on a decomposition of delays is given in Section VIII.

Remark 4.2 (Worst Case Error in NTP is Proportional to Round-Trip Delay): Consider two clocks which are bidirectionally connected, that is both can send packets to each other. In NTP, the offset is estimated based on the assumption that the delays in the two directions are the same, i.e., $d_{1j} = d_{j1}$. This corresponds, in the light of (19), to choosing the midpoint of the uncertainty interval (18), i.e., the point that minimizes the maximum error. Thus, the worst-case error of NTP is equal to $a_j(d_{j1} + d_{1j})/2$.

V. Network Clock Synchronization

Now we turn to the case of *networks*. We consider a network of n nodes where, again by convention, node 1 is considered to be the reference node. We will draw a directed edge from node i to node j if i can send packets to j. We will draw an undirected edge if both i and j can send packets to each other; see Fig. 2(a). We will call the resulting graph the *communication graph*. We will also occasionally refer to directed edges as unidirectional or unilateral links, and to undirected edges as bilateral or bidirectional links.

To motivate and set the stage for the general results to follow, consider the *star* communication graph shown in Fig. 2(b), where all nodes are only allowed to bilaterally exchange packets with the reference node. This is equivalent to having $(n-1)$ independent pairwise "synchronizations." From the results of the previous section, it follows immediately that determining all parameters is infeasible. There are exactly $(n-1)$ degrees of freedom in the uncertainty set of the $4(n-1)$-dimensional unknown parameter vector. These correspond precisely to the estimates of the unknown offsets of the $(n-1)$ clocks with respect to the reference clock. The delay estimates are in turn characterized as affine functions of these offset estimates.

In the general multinode case, where the graph does not necessarily correspond to a star, our goal is to similarly determine what is or is not determinable when the nodes are allowed to collaborate, i.e., when nodes other than the reference node communicate with one another. An extreme situation is when *all* nodes are allowed to exchange packets bilaterally with one another, i.e., the case of a *complete* communication graph (see Fig. 2(c)). We will show that even with such *full* collaboration the uncertainty set remains exactly the same as in the star graph, if causality is not invoked. In fact we will show that the same uncertainty set results for any directed synchronization graph that is *connected* [see Fig. 2(d)]. In the sequel, we will build up to this general case.

We first consider the problem of synchronization between two nodes neither of which is the reference node.

Corollary 5. (Pairwise Synchronization Between Two Nodes Other Than the Reference Node 0: In pairwise synchronization between two nodes i and j, neither of which is the reference node 1, the same impossibility results and structure of the uncertainty set presented in the previous section hold, with the *relative skew* a_j/a_i taking the place of the skew a_j, and the *relative offset* $b_j - (a_j/a_i)b_i$ in place of the offset b_j.

Proof: In order to define a linear system of equations, we consider the parameters $\{a_j/a_i, a_j d_{ij}, a_j d_{ji}, b_j - (a_j/a_i)b_i\}$ as unknowns in the estimation problem. This yields exactly the same system of equations as (10)

$$\begin{pmatrix} r_{i,j}^{(1)} \\ s_j^{(1)} \\ r_{i,j}^{(2)} \\ s_j^{(2)} \\ \vdots \end{pmatrix} = \begin{pmatrix} s_i^{(1)} & 1 & 0 & 1 \\ r_{j,i}^{(1)} & 0 & -1 & 1 \\ s_i^{(2)} & 1 & 0 & 1 \\ r_{j,i}^{(2)} & 0 & -1 & 1 \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} \frac{a_j}{a_i} \\ a_j d_{ij} \\ a_j d_{ji} \\ b_j - \frac{a_j}{a_i} b_i \end{pmatrix}. \tag{20}$$

Therefore the previous results continue to hold for this new parametrization. ■

Note that, as earlier, three noiseless alternating communication pings per link suffice for estimation. We can consider again a 4×4 system of rank 3, derived from (20) by considering four distinct alternating communication pings as shown in Fig. 1. For compactness of representation we denote this system, which contains all the information for the pair (i, j),

$$\underline{y}^{(i,j)} = A^{(i,j)} \underline{x}^{(i,j)}$$
$$\text{where} \quad \underline{x}^{(i,j)} := \left(\frac{a_j}{a_i}, a_j d_{ij}, a_j d_{ji}, b_j - \frac{a_j}{a_i} b_i\right)^T. \tag{21}$$

Thus, generalizing the earlier result in Theorem 2, the relative skew a_j/a_i can be determined correctly, while there is no unique solution for the unknown link delays d_{ij}, d_{ji} and relative offset $b_j - (a_j/a_i)b_i$. Moreover, due to the occurrence of the product $a_j d_{ij}$ in this parametrization, in order to express the unknown delays as affine functions of the relative offset $b_j - (a_j/a_i)b_i$, we need to have first determined the skew a_j. This is a network issue when node j does not have a direct link with the reference node 1, and we now turn to this issue.

A. Skew Estimation in Networks

In a unilateral communication link (i, j), relative skew can be determined correctly by the exchange of two noiseless pings

$$\frac{a_j}{a_i} = \frac{r_{i,j}^{(l)} - r_{i,j}^{(k)}}{s_i^{(l)} - s_i^{(k)}}. \tag{22}$$

We will consider two cases for the knowledge that nodes have:

1) *Genie*. We assume that there is a centralized view of the entire network, i.e., a genie that is aware of the transmitter time-stamp and the receiver time-stamp of every packet exchange between any pair of nodes. This implies that links do not have to be bilateral for skew estimation, since communication in one direction suffices for the estimation of relative skew through (22). The genie scenario serves as an absolute upper bound on what can be determined.
2) *Network*. This is the scenario of actual interest where each node knows only the information that it transmits or that is transmitted to it by packets. Such packets, as noted in Section IV, can contain the transmitter's time-stamp, as well as information that the transmitter has obtained from packets it has received. Note that communication along the unilateral link (i, j) implies that only the receiving node j can determine correctly the relative skew via (22), but the transmitter i is unaware of the relative skew (unless it is provided that information by some other path).

Concerning the communication graph of the network, we suppose that there is a directed graph with node set $\mathcal{N}$ and edge set $\mathcal{E}$. In the sequel, we say that there is an *undirected* path from i to j, if there is a set of nodes $(i_0 = i, i_1, \ldots, i_l = j)$ such that for every $1 \le m \le l$, either $(i_{m-1}, i_m) \in \mathcal{E}$ or $(i_m, i_{m-1}) \in \mathcal{E}$, or both. We say that there is a directed path from node i to node j if there is a set of nodes $(i_0 = i, i_1, \ldots, i_l = j)$, such that each directed edge (i_{m-1}, i_m) belongs in $\mathcal{E}$ for $1 \le m \le l$.

Theorem 6. (Necessary and Sufficient Network Topology for the Estimation of All Nodal Skews): Consider a network of n nodes.

1) In the *genie* case, all the skews $\{a_i\}_{i=2}^n$ can be determined correctly if and only if there is an undirected path from the reference node to every node in the network graph, i.e., if the graph is connected.
2) In the *network* case, every node in the network can determine its own skew (relative to the reference) if and only if there is a directed path from the reference node to that node, i.e., if and only if the graph contains a directed spanning tree rooted at the reference.
3) In the *network* case every node in the network can determine *all* nodal skews (and therefore all *relative* skews) if and only if there is a directed path from every node to all nodes, i.e., if the graph is strongly connected.

Proof:

1) Sufficiency is immediate since the skew of any node can be computed by multiplying the relative skews of the nodes along a path connecting it to the reference. Necessity follows because if the graph is disconnected, then there are $i > 1$ connected components, say $N_1, N_2, \ldots, N_i$, such that there is no packet exchange between any two components. Then multiplying the skew of all the clocks in component N_k by a positive integer θ_k, for $k = 1, 2, \ldots, i$, would create no inconsistency with any of the time-stamps.
2) For sufficiency, consider a directed path $(i_0 = 1, i_1, \ldots, i_l = j)$ from the reference node to node j. Then node i_m $(1 \le m \le l)$ can determine the ratio $a_{i_m}/a_{i_{m-1}}$ from its incoming packets along link (i_{m-1}, i_m) and communicate that ratio to all nodes i_p for $p \ge m+1$ by including that information in its outgoing packets. Node $i_l = j$ simply forms the product $(a_{i_l}/a_{i_{l-1}})(a_{i_{l-1}}/a_{i_{l-2}}) \cdots (a_{i_1}/a_{i_0}) = a_j/a_1$. To prove the necessity, let N_1 be the set containing node 1 and the set of nodes for which there is a directed path from node 1 to every node in the set, and $N_2 := \mathcal{N} \setminus N_1$. If N_2 is not empty, we can multiply the skew of all nodes in N_2 by θ, in consistency with the transmit time-stamps of packets sent by nodes in N_2 and the receipt time-stamps of packets received by nodes in N_2.
3) Sufficiency follows, since for any distinct nodes i and j, node j can determine its relative skew to node i by multiplying the relative skews along a directed path to j rooted at node i, as in the proof of sufficiency in 2). The proof of necessity is as in 1), applied to strongly connected components. ■

We note that an important special case that satisfies all the conditions of the above theorem is when all the links are bidirectional and the network is connected.

VI. Characterization of Synchronizability in Networks

Now we are ready to address what is feasible or infeasible for the problem of synchronizing clocks over networks. First, we note that in a network with n nodes and link set $\mathcal{E}$ of directed edges, there are a total of $2(n-1) + |\mathcal{E}|$ unknown parameters $\{a_i, b_i\}_{i=2}^n$ and $\{d_{ij}\}_{(i,j)\in\mathcal{E}}$. If the graph is complete, i.e., all nodes can bilaterally exchange packets with one another, then the number of unknowns in the network synchronization problem is $2(n-1) + n(n-1) = (n+2)(n-1)$.

The following theorem establishes the fundamental result that without any further assumptions, clock synchronization is impossible in any network.

Theorem 7. (Infeasibility of Clock Synchronization in Networks): Consider a network of n nodes. It is impossible to determine all $2(n-1) + |\mathcal{E}|$ unknown parameters $\{a_i, b_i, \text{and } d_{ij}$ for all i and all $j : (i, j) \in \mathcal{E}\}$ even if all pairs of nodes can exchange any number of time-stamped packets containing any information that is causally known to the transmitter.

The next theorem characterizes the uncertainty set of the unknown parameters.

Theorem 8 (Characterization of the Uncertainty Set in Network Clock Synchronization): Consider any network topology $\mathcal{G} := (\mathcal{N}, \mathcal{E})$ such that in $\mathcal{E}$ there is a directed path from any node to any other node, i.e., $\mathcal{G}$ is strongly connected.

1) All the skews $\{a_i : 2 \le i \le n\}$ can be determined correctly.
2) Every vector $\underline{\hat{d}} = (\hat{d}_{ij}, (i, j) \in \mathcal{E})$ in the uncertainty set for the delay vector $\underline{d} = (d_{ij}, (i, j) \in \mathcal{E})$ can be expressed as a known affine transformation of $(n-1)$ variables $\{\hat{b}_i : 2 \le i \le n\}$. Each $\hat{b}_i$ can be regarded as an estimate of the unknown offset b_i. Any choice of these estimates

$\{\hat{b}_i : 2 \leq i \leq n\}$ is consistent with all transmit and receipt time-stamps of all packets.

3) If causality is invoked, the uncertainty set for the estimates of the offset parameters $\{\hat{b}_i : 2 \leq i \leq n\}$ can be fully characterized as a compact polyhedron of $\mathbb{R}^{n-1}$.

4) Suppose all links in $\mathcal{E}$ are bilateral. Then the feasible polyhedron in 3) above has a non-empty interior if and only if there is no bilateral link with zero round-trip delay.

Remark 8.1: An important consequence is that the star graph and, in general, any spanning tree, results in the same uncertainty set for the parameters as the complete graph, if causality is not invoked. If causality is additionally taken into account, then additional communication links do help reduce the size of the uncertainty set.

Proof of Theorems 7,8: Theorem 8.1 is just a restatement of Theorem 6.3.

We will first prove Theorem 7 and Theorem 8.2 together, under the best case scenario of a complete graph, i.e., when all links are active and $|\mathcal{E}| = n(n-1)$. Subsequently we will show that the conclusion of Theorem 8.2 continues to hold for any strongly connected graph where there is a directed path from any node to any other node.

The complete graph contains the star graph for which we already know that the conclusions of Theorem 7 and Theorem 8.2 hold. Thus, to show Theorem 7 and Theorem 8.2 for the complete graph, it suffices to show that any estimate in the uncertainty set for the star graph will also satisfy the data from all bilateral packet exchanges, and to further express the link delays $\{d_{ij}\}$ for $i, j \neq 1$ as affine functions of the offset estimates $\{\hat{b}_i\}_{i=2}^{n}$.

In order to tackle the difficulties of a nonlinear parametrization, we introduce a *redundant parametrization* with respect to which the system of equations to be solved becomes linear. A natural selection, as in Corollary 5, is to choose the parameters $\{a_j/a_i, a_j d_{ij}, a_j d_{ji}, b_j - (a_j/a_i)b_i\}$ for $i = 2, 3, \ldots, n$, and $j = i+1, i+2, \ldots, n$, and $\{a_j, a_j d_{1j}, a_j d_{j1}, b_j\}$ for $j = 2, 3, \ldots, n$, since $a_1 = 1$, $b_1 = 0$. From the fact that there are $n(n-1)/2$ communicating pairs of nodes i and j with $i < j$, this parametrization involves $4(n(n-1)/2) = 2n(n-1)$ parameters. It is redundant, for $n > 2$, since the number of the parameters is more than the $(n+2)(n-1)$ unknowns. Hence, we have introduced an additional $(n-1)(n-2)$ redundant parameters, but this over-parametrization has the advantage that not only does the system become *linear* as before in the two-node system, but also *fully decoupled* in the communicating pairs. Using (21) and taking four alternating pings, the whole system can be put in block diagonal matrix form as follows:

$$\begin{pmatrix} \underline{y}^{(1,2)} \\ \vdots \\ \underline{y}^{(i,j)} \\ \vdots \\ \underline{y}^{(n-1,n)} \end{pmatrix} = \begin{pmatrix} A^{(1,2)} & \cdots & \mathbb{O} & \cdots & \mathbb{O} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbb{O} & \cdots & A^{(i,j)} & \cdots & \mathbb{O} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbb{O} & \cdots & \mathbb{O} & \cdots & A^{(n-1,n)} \end{pmatrix} \times \begin{pmatrix} \underline{x}^{(1,2)} \\ \vdots \\ \underline{x}^{(i,j)} \\ \vdots \\ \underline{x}^{(n-1,n)} \end{pmatrix}. \quad (23)$$

The blocks $A^{(i,j)}$ are 4×4 matrices as given by (11) and the first four rows of (20), while $\mathbb{O}$ is the 4×4 zero matrix.[7] We denote this system, by slight abuse of notation, by $\underline{y} = A\underline{x}$.

The dimension of the square matrix A is $2n(n-1) \times 2n(n-1)$. This matrix however has rank only $3n(n-1)/2$ since all the blocks in the diagonal have rank three and four columns. We cannot yet conclude nonuniqueness of solution since the parametrization itself is redundant.

By the result in (15) of Theorem 2 which, as we showed in Corollary 5, carries over for the parameters $\{a_j/a_i, a_j d_{ij}, a_j d_{ji}, b_j - (a_j/a_i)b_i\}$, we have

$$\begin{pmatrix} \widehat{\left(\frac{a_j}{a_i}\right)} \\ \widehat{(a_j d_{ij})} \\ \widehat{(a_j d_{ji})} \\ \widehat{(b_j - \frac{a_j}{a_i} b_i)} \end{pmatrix} = \begin{pmatrix} \left(\frac{a_j}{a_i}\right)^* \\ (a_j d_{ij})^* \\ (a_j d_{ji})^* \\ 0 \end{pmatrix} + \hat{\theta}_{ij} \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \end{pmatrix} \quad (24)$$

$$\text{with } \left(\frac{a_j}{a_i}\right)^* := \frac{r_{i,j}^{(l)} - r_{i,j}^{(k)}}{s_i^{(l)} - s_i^{(k)}} = \frac{s_j^{(l)} - s_j^{(k)}}{r_{j,i}^{(l)} - r_{j,i}^{(k)}} = \frac{a_j}{a_i} \quad (25)$$

$$(a_j d_{ij})^* := r_{i,j}^{(k)} - \frac{a_j}{a_i} s_i^{(k)} \quad (26)$$

$$(a_j d_{ji})^* := \frac{a_j}{a_i} r_{j,i}^{(k)} - s_j^{(k)}. \quad (27)$$

The vector in the LHS of (24) denotes estimates, parametrized by a selection of the one degree of freedom $\hat{\theta}_{ij}$. The quantities in the first vector on the RHS of (24) can be determined by two time-stamped packets k and l sent from node i to node j, and one packet sent from node j to node i, or vice-versa. In fact, if we further define

$$a_j^* := a_j \quad (28)$$

$$d_{ij}^* := \frac{1}{a_j} r_{i,j}^{(k)} - \frac{1}{a_i} s_i^{(k)} \quad (29)$$

then $(a_j d_{ij})^* = a_j^* d_{ij}^*$ and $(a_j/a_i)^* = a_j^*/a_i^*$. We will denote the system in (24) by $\hat{\underline{x}}^{(i,j)} = \underline{x}^{*(i,j)} + \hat{\theta}_{ij}\underline{n}$, where $\hat{\theta}_{ij} := \widehat{(b_j - (a_j/a_i)b_i)}$ and $\underline{n} := (0, -1, 1, 1)^T$.

Since the estimation of all the unknown parameters in the network has been decoupled in (23) into estimations for separate links, we have

$$\hat{\underline{x}} := \begin{pmatrix} \hat{\underline{x}}^{(1,2)} \\ \vdots \\ \hat{\underline{x}}^{(i,j)} \\ \vdots \\ \hat{\underline{x}}^{(n-1,n)} \end{pmatrix} = \begin{pmatrix} \underline{x}^{*(1,2)} \\ \vdots \\ \underline{x}^{*(i,j)} \\ \vdots \\ \underline{x}^{*(n-1,n)} \end{pmatrix} + \hat{\theta}_{12} \begin{pmatrix} \underline{n} \\ \vdots \\ \underline{0} \\ \vdots \\ \underline{0} \end{pmatrix} + \cdots + \hat{\theta}_{n-1,n} \begin{pmatrix} \underline{0} \\ \vdots \\ \underline{0} \\ \vdots \\ \underline{n} \end{pmatrix} \quad (30)$$

[7]The matrices $A^{(i,j)}$ are considered to be 4×4 without loss in generality. However, any one of them can be constructed as a 3×4 matrix as shown in Remark 1.1.

where $\underline{0} := (0,0,0,0)^T$. All solutions for $\underline{\hat{x}}$ consistent with the data of transmit and receipt time-stamps of all packets are of the form (30).

Now we need to determine what is the freedom in the values of the parameters $\{\hat{\theta}_{ij} : 1 \le i < j \le n\}$, so that one can choose $\{\hat{a}_i, \hat{b}_i, \hat{d}_{ij}\}$ to satisfy $\widehat{(a_j/a_i)} = \hat{a}_j/\hat{a}_i$, $\widehat{(a_j d_{ij})} = \hat{a}_j \hat{d}_{ij}$, and $(b_j - \widehat{(a_j/a_i)} b_i) = \hat{b}_j - (\hat{a}_j/\hat{a}_i)\hat{b}_i$. It is plain to check that these constraints give a set of $(n-1)(n-2)$ equations, and also no other constraints exist, hence the redundancy in the system will be eliminated.

First, from Theorem 6.3, we see that $\hat{a}_i = a_i$ for all i since the skews can be correctly determined, hence the $(n-1)(n-2)/2$ equations (for the case of a complete graph) $\widehat{(a_j/a_i)} = \hat{a}_j/\hat{a}_i$ will provide no additional constraints on $\{\hat{\theta}_{ij} : 1 \le i < j \le n\}$.

Next, $\hat{\theta}_{ij} := (b_j - \widehat{(a_j/a_i)} b_i) = \hat{b}_j - (\hat{a}_j/\hat{a}_i)\hat{b}_i = \hat{b}_j - (a_j/a_i)\hat{b}_i$, which yields $(n-1)(n-2)/2$ independent equations, so that the "free" parameters $\{\hat{\theta}_{ij}\}$ are linear functions of the offset estimates $\{\hat{b}_i : 2 \le i \le n\}$. Hence, there are only $(n-1)$ independent parameters $\{\hat{b}_i : 2 \le i \le n\}$ that determine the values of all the $\hat{\theta}_{ij}$ for $i < j$.

Next, from the equations defined by the second and third row in (24), since $\hat{a}_i = a_i$ and $\hat{\theta}_{ij} = \hat{b}_j - (a_j/a_i)\hat{b}_i$ we obtain, in both cases $i < j$ as well as $j < i$, that

$$\hat{d}_{ij} = d^*_{ij} + \frac{1}{a_i}\hat{b}_i - \frac{1}{a_j}\hat{b}_j. \tag{31}$$

Hence, we see that all delay vectors in the uncertainty set can be affinely characterized by exactly $(n-1)$ remaining degrees of freedom $\{\hat{b}_i\}_{i=2}^n$, which are the estimates for the offsets. In consequence, the estimation problem does not yield a unique solution, proving Theorem 7. Moreover, the uncertainty set of the parameter vectors in the network is an affine transformation of an $(n-1)$-dimensional subspace. This proves Theorem 8.2 for the case of a complete graph. However, a perusal shows that this proof also generalizes to the case where the link set $\mathcal{E}$ allows all skews to be determined, and the latter is guaranteed by Theorem 6.3.

Now we turn to Theorem 8.3. As in Theorem 2, we use the property that for $a_i > 0$ causality is equivalent to $d_{ij} \ge 0$, to obtain from (31) that

$$\frac{1}{a_i}\hat{b}_i - \frac{1}{a_j}\hat{b}_j \ge -d^*_{ij} \quad \text{for all } (i,j) \in \mathcal{E} \tag{32}$$

which characterizes the uncertainty set for offsets.

We now show that the uncertainty set is compact. Starting from node i, there is a directed path to the reference node 1. So applying (32) repeatedly, and using $b_1 = 0$, gives

$$\hat{b}_i \ge a_i \sum_{\text{link}(i,j) \,\in\, \text{directed path}} \left(-d^*_{ij}\right). \tag{33}$$

Similarly, starting from the reference node and considering a directed path to node i one gets an upper bound

$$\hat{b}_i \le a_i \sum_{\text{link}(i,j) \,\in\, \text{directed path}} d^*_{ij}. \tag{34}$$

To prove Theorem 8.4, we note that when all links in $\mathcal{E}$ are bilateral the inequality constraints in (32) are

$$-d^*_{ji} \le \frac{1}{a_j}\hat{b}_j - \frac{1}{a_i}\hat{b}_i \le d^*_{ij}, \quad \text{for all } (i,j) \in \mathcal{E}. \tag{35}$$

A necessary and sufficient condition for the polyhedron to be full-dimensional $(n-1)$ is that the upper bound in (35) strictly exceeds the lower bound, i.e., $d^*_{ij} + d^*_{ji} > 0$. Since we know that this is equal to the round-trip delay $d_{ij} + d_{ji}$, the result follows. ∎

Remark 8.2: There are only $(n-1)$ free parameters that one can choose if one wants the estimate to be consistent with all transmit and receipt time-stamp data. This shows that while in pairwise synchronization unknown delay asymmetry is the unique reason for indeterminacy (that is to say assuming that delays are symmetric there is no indeterminacy left), in the network case $(n > 2)$, this is not the case any more. To illustrate this effect, suppose that all communication links are bilateral, and hence the network topology is modeled by an undirected graph $(\mathcal{N}, \mathcal{E})$, and link delays are not symmetric, but yet we assume they are. Then the assumption results in $|\mathcal{E}|$ additional constraints which may exceed the number of the free parameters $(n-1)$, and one will be unable to choose symmetric delay estimates that are consistent with the data. This has also been observed in [12].

Remark 8.3: In [12], the special case where all clocks run at the same speed, all links are bilateral, and there exists a spanning tree, is examined. If we denote the set of all directed cyclic paths of the graph $(\mathcal{N}, \mathcal{E})$ by $\mathcal{C}$, then by invoking causality, it is shown in [12] that the delay vector $\underline{d}$ lies in the intersection of the positive orthant $\mathbb{R}_+^{|\mathcal{E}|}$ and the hyperplanes

$$\sum_{(i,j)\in C} d_{ij} = \sum_{(i,j)\in C} d^*_{ij} \quad \text{for all } C \in \mathcal{C}. \tag{36}$$

The latter also follows immediately from (31). It is also shown in [12] that all link delays can be expressed as known affine functions of the $(n-1)$ delays of the links of a directed spanning tree rooted at the reference node. This arises also as a plain corollary of our analysis, since the affine mapping relating the delays of the links of a directed spanning tree and the nodal offsets is invertible. The latter, in conjunction with the *sharp* characterization of the uncertainty set in Theorem 8, imply that the uncertainty set for link delays is completely characterized as the intersection of $\mathbb{R}_+^{|\mathcal{E}|}$ and the set in (36). However our analysis does not require all links to be bidirectional, nor all skews to be identical, nor does it depend on finding a spanning tree. Most importantly, we have shown that the number of degrees of freedom is *exactly* equal to $(n-1)$, thus fully characterizing the uncertainty set for both offsets and delays, whereas in [12] the set in (36) is only shown to be necessary but not sufficient. Additionally, no constraints on the offsets are provided in [12].

Remark 8.4 (Choosing an estimate in the offset feasible set): Since the offsets cannot be exactly determined, one may be interested in a min-max estimate. We pose the problem as one of finding the smallest hyper-rectangle that *contains* the polyhedron that characterizes the offset uncertainty set, with the esti-

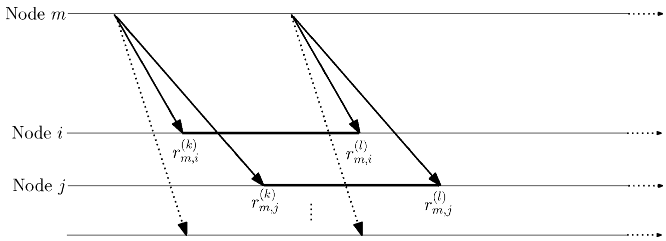

Fig. 3. Receiver-receiver synchronization.

mate then chosen as the center of the hyper-rectangle. This can be obtained by solving the Linear Program

$$\max \quad \sum_{i=1}^{n} \left(\bar{b}_i^{(i)} - \underline{b}_i^{(i)}\right)$$
$$s.t. \quad \frac{1}{a_i}\bar{b}_i^{(k)} - \frac{1}{a_j}\bar{b}_j^{(k)} \geq -d_{ij}^*, \quad k=1,\cdots,n,(i,j)\in\mathcal{E}$$
$$\frac{1}{a_i}\underline{b}_i^{(k)} - \frac{1}{a_j}\underline{b}_j^{(k)} \geq -d_{ij}^*, \quad k=1,\cdots,n,(i,j)\in\mathcal{E} \quad (37)$$

and estimating the offsets by $b_i^* = (\bar{b}_i^{(i)} + \underline{b}_i^{(i)})/2$ for $2 \leq i \leq n$, where $\bar{b}_i^{(k)}$ denotes the i-th component of the vector $\bar{b}^{(k)} \in \mathbb{R}^{n-1}$, and $\underline{b}_i^{(k)}$ denotes the i-th component of the vector $\underline{b}^{(k)} \in \mathbb{R}^{n-1}$. It would be of interest to obtain a distributed algorithm to determine such $\{b_i^*, 2 \leq i \leq n\}$.

Remark 8.5: The derivation of the uncertainty set naturally generalizes to the case that multiple nodes are assumed to be "synchronized" by letting $a_i = 1$, $b_i = 0$ for all such clocks. The case that a node has a "known" offset b_i can be treated analogously.

VII. Receiver-Receiver Clock Synchronization

In [8], a scheme called Reference Broadcast Synchronization (RBS) is proposed where nodes broadcast packets that are received by several receivers who then compare the times, according to their own clocks, at which they received common packets, in order to synchronize their clocks (see Fig. 3). In this scheme, senders never time-stamp a packet, but they include a packet identifier which receivers use to refer to packets when comparing the receipt times. We will allow a sending node to include in any packet, causal knowledge of all packet reception times by any node, that it knows. We call this scheme as *receiver-receiver synchronization*.

Since less information is available, the estimation of all unknown parameters skews, offsets, and delays remains impossible. However, it remains to check how ignorance of the transmit times will alter the uncertainty set.

Theorem 9. (Skew Estimation in Receiver-Receiver Synchronization): Consider a network topology $G = (\mathcal{N}, \mathcal{E})$ and define an undirected "comparison" graph $G' = (\mathcal{N}, \mathcal{E}')$, where $e = (i,j)$ is in $\mathcal{E}'$ if for some m both (m,i) and (m,j) are in $\mathcal{E}$. If G is strongly connected and G' is connected then via receiver-receiver communication, all skews can be determined correctly.

Proof: Consider two nodes i and j which receive common transmissions from some node, say m (see Fig. 3). Then we get

$$r_{m,i}^{(k)} = \frac{a_i}{a_m} s_m^{(k)} + a_i d_{mi} + b_i - \frac{a_i}{a_m} b_m \quad (38)$$
$$r_{m,j}^{(k)} = \frac{a_j}{a_m} s_m^{(k)} + a_j d_{mj} + b_j - \frac{a_j}{a_m} b_m \quad (39)$$

where the transmit time $s_m^{(k)}$ is also an unknown, in addition to a_i, a_j, a_m, b_i, b_j, and b_m. By considering two distinct sent times $s_m^{(k)} \neq s_m^{(l)}$ and using these equations, the relative skew can be computed as the ratio of the intervals of the corresponding receipt times in the two receiving nodes i, j as (see also Fig. 3),

$$\frac{a_j}{a_i} = \frac{r_{m,j}^{(l)} - r_{m,j}^{(k)}}{r_{m,i}^{(l)} - r_{m,i}^{(k)}}. \quad (40)$$

Since $\mathcal{E}$ provides a directed path for i to send the denominator of the RHS in (40) to j, and for j to send the numerator to i, they can both determine the ratio. The rest of the proof follows as in the sufficiency proof of Theorem 6.2. ■

It remains to examine the structure of the uncertainty set. The following theorem shows the fundamental fact that receiver-receiver communication yields a *strictly larger* uncertainty set than when transmit time-stamping is used, in that the uncertainty set is the translate of a $(2n-1)$-dimensional subspace, rather than the translate of an $(n-1)$-dimensional subspace. Moreover, causality does not help reduce the uncertainty set to a compact subset.

Theorem 10. (Characterization of Uncertainty Set in Receiver-Receiver Synchronization): Consider a network of n nodes where all links are bidirectional and the network topology satisfies the conditions of Theorem 9. Suppose receiver-receiver synchronization is used. Then,

1) The uncertainty set is the translation of a $(2n-1)$-dimensional subspace.
2) Causality does not help reduce the uncertainty set.
3) Round-trip delays cannot be estimated.

Proof: By Theorem 9 all skews $\{a_i\}_{i=2}^n$ can be determined correctly. Also, since all links are bidirectional, it follows that all nodes are transmitting packets. For a given transmitting node m, all sent times $\{s_m^{(k)}\}_{k\in\mathbb{Z}^+}$ are unknown. At each sending node, fix one of the transmit times, say $s_m^{(k)}$ arbitrarily. Then the rest (for $l \neq k$) can be uniquely determined by solving for $s_m^{(l)}$ in

$$\frac{a_i}{a_m} = \frac{r_{m,i}^{(l)} - r_{m,i}^{(k)}}{s_m^{(l)} - s_m^{(k)}}. \quad (41)$$

This gives n degrees of freedom, one for each node, in making choices for estimates of transmit times. Once these n degrees of freedom are fixed, all sent times are known and the results of Theorem 8 apply, and provide $(n-1)$ additional degrees of freedom. Therefore 1) follows. To determine the affine transformation, define

$$d_{mi}^* := \frac{1}{a_i} r_{m,i}^{(k)} - \frac{1}{a_m} s_m^{(k)}, \quad (42)$$

and exploiting (38) we get

$$d_{mi} = d_{mi}^* + \frac{1}{a_m}\hat{b}_m - \frac{1}{a_i}\hat{b}_i. \quad (43)$$

This constitutes a solution for every choice of $\{\hat{b}_i : 2 \le i \le n\}$. This is the set of all solutions, since the true solution does indeed originate from a particular choice of transmit times. In fact, for nodes, i, and j, receiving common packets, (38), (39) give

$$d_{mi} - d_{mj} = \left(\frac{1}{a_i} r_{m,i}^{(k)} - \frac{1}{a_j} r_{m,j}^{(k)}\right) - \frac{1}{a_i}\hat{b}_i + \frac{1}{a_j}\hat{b}_j. \quad (44)$$

i.e., we have eliminated all unknown sent times. These equations, for all links $(i,j) \in \mathcal{E}'$, completely characterize the uncertainty set, since for any solution they admit, a network solution can be obtained by properly determining the n degrees of freedom corresponding to sent times, through (42) and (38).

2) This is immediate from the fact that causality is equivalent to the nonnegativity of delays. However, in (44) the delays appear in differences (44), so no sign constraint can be imposed on these differences. Even though we have

$$\frac{1}{a_i}\hat{b}_i - \frac{1}{a_m}\hat{b}_m \le d_{mi}^* \quad (45)$$

d_{mi}^* can be made arbitrarily large by choosing $s_k^{(m)}$ in (42).[8]

3) This follows since the round-trip delay $d_{ij} + d_{ji}$ is equal to $d_{ij}^* + d_{ji}^*$, but the latter cannot be uniquely chosen. ■

Remark 10.1: Suppose all receiver delays are equal, i.e., $d_{mi} = d_{mj}$ for all m, i, j for which node m transmits to nodes i, j. Then from (44)

$$b_j - \frac{a_j}{a_i} b_i = r_{m,j}^{(k)} - \frac{a_j}{a_i} r_{m,i}^{(k)}. \quad (46)$$

So the relative skew $b_j - (a_j/a_i)b_i$ can be determined since the quantities on the RHS are known. Schemes such as RBS implicitly assume equality of delays. However, such equality might not provide a network-wide consistent solution, since the number of constraints to be enforced might be more than the number of free parameters.

VIII. Analysis of Structured Models of Link Delays

We now study the case where link delay has additional structure. We obtain a sufficient condition for correctness of estimates. We also provide an example where a correct estimate is attainable, and one where it is not. Moreover, we demonstrate a model where the uncertainty set of the delays is characterized by only *one* or *two* degrees of freedom, in contradistinction to the $(n-1)$ degrees of freedom in the general case. This is important since the number of the remaining degrees of freedom is then constant, independent of the network size.

A decomposition of the packet delivery delays was first introduced in [10]. We will suppose that the delay in the directed communication pair (i,j), d_{ij}, consists of the sum of three terms.

1) *A transmission delay* α_i which accounts for the processing time in the transmitter after time-stamping. This is assumed to be fixed and transmitter-dependent. This includes, in the terminology of [10], the "Send Time," the time used to construct the message at the application layer and transfer it to the MAC layer on the transmitter side, as well as the "Access Time," the delay incurred waiting for access to the transmit channel up to the point when transmission begins, and the "Transmission Time," the time it takes for the sender to transmit the message bit by bit at the physical layer.
2) *An electromagnetic propagation delay* τ_{ij} which can be estimated correctly, say by GPS, or other position information, since it only involves the distance between the nodes, and is hence assumed known.
3) *A receiving delay* β_j which accounts for the processing time in the receiver before time-stamping. This is also assumed to be fixed and receiver-dependent. This includes, in the terminology of [10], the "Reception Time," the time taken in receiving the bits and at the physical layer, and may include an overlap with Transmission Time, as well as the "Receive Time," the time it takes to reconstruct the incoming bits into a packet and pass it to the application layer where it is decoded.

To sum up, the model for the delay is

$$d_{ij} = \alpha_i + \tau_{ij} + \beta_j, \quad i,j = 1,2,\ldots,n \text{ and } i \ne j. \quad (47)$$

The important point is that by exploiting such a decomposition, the number of unknowns for the delays reduces from $n(n-1)$ to $2n$ in the case of a complete graph, since we need only estimate the variables $\{(\alpha_i, \beta_i) : 1 \le i \le n\}$. Let us denote their estimates by $\{(\hat{\alpha}_i, \hat{\beta}_i) : 1 \le i \le n\}$.

From (31), we see that the delay vector $\underline{d} \in \mathbb{R}^{|\mathcal{E}|}$ can be expressed as an affine function of the offset vector $\underline{b} \in \mathbb{R}^{n-1}$, as follows:

$$\hat{\underline{d}} = \underline{d}^* + B\hat{\underline{b}}, \quad B \in \mathbb{R}^{|\mathcal{E}|\times(n-1)} \quad (48)$$

$$\text{where} \quad B_{em} = \begin{cases} -\frac{1}{a_j^*}, & m+1 = j \\ \frac{1}{a_i^*}, & m+1 = i \\ 0, & else \end{cases} \quad (49)$$

where e is used to denote the directed link (i,j) for notational convenience in the matrix representation, and $\underline{d}^*$ is the vector containing the values d_{ij}^* as given by (13), (14), (29).

However, from the structured model assumption for the delays (47), we have

$$\hat{\underline{d}} = D\hat{\underline{\theta}} + \underline{\tau}, \quad \hat{\underline{\theta}} \in \mathbb{R}^{2n}, \quad D \in \mathbb{R}^{|\mathcal{E}|\times 2n} \quad (50)$$

$$\text{where} \quad D_{em} = \begin{cases} 1, & \text{if } m = i \text{ or } m = n+j \\ 0, & else \end{cases} \quad (51)$$

$$\hat{\underline{\theta}}_i = \begin{cases} \hat{\alpha}_i, & i = 1,2,\ldots,n \\ \hat{\beta}_{i-n}, & i = n+1,\ldots,2n \end{cases} \quad (52)$$

The vector $\underline{\tau}$ is a *known* vector with entries being the electromagnetic propagation delays $\{\tau_{ij}, (i,j) \in \mathcal{E}\}$. Notice that the matrix D is of full column rank $2n$. This is easy to see by observing that the rows corresponding to the delays $\{d_{12}, d_{13}, \ldots, d_{1n}, d_{n1}, d_{21}, d_{31}, \ldots, d_{n-1,1}, d_{23}, d_{32}\}$ form a set of $2n$ linearly independent vectors.

Substituting (50) into (48), we get

$$D\hat{\underline{\theta}} = \underline{c} + B\hat{\underline{b}} \quad (53)$$

where $\underline{c} := \underline{d}^* - \underline{\tau}$ is a known vector.

[8]Not all such bounds can be arbitrarily made large *simultaneously*, since the number of links might be larger than the total degrees of freedom.

Lemma 11. (Non-Uniqueness of Solution in the Structured Delay Model): The solution for $\underline{\hat{\theta}}$ in (53) is not unique. Hence, in general, the structured delay case is still unsolvable.

Proof: Adding $\epsilon > 0$ small enough to all α_i's and simultaneously subtracting the same value ϵ from all β_i's leaves (53) invariant. ■

More generally one may have other specific models for the delay.

Lemma 12. (Necessary and Sufficient Condition for Uniqueness of Solution in Structured Delay Models): Suppose all links are bidirectional and all round-trip delays are strictly positive. Suppose the delay vector satisfies an affine relationship $\underline{d} = A\underline{\alpha} + \underline{\beta}$ where $A, \underline{\beta}$ are known, and A has full rank. Then, there exists a unique $\underline{\hat{d}}$ that satisfies both $\underline{\hat{d}} = A\underline{\alpha} + \underline{\beta}$ for some $\underline{\alpha}$, as well as $\underline{\hat{d}} = \underline{d}^* + B\underline{\hat{b}}$, if and only if $A^T B = 0$.

Proof: Substituting $\underline{\hat{d}} = A\underline{\alpha} + \underline{\beta}$ into (48) we have

$$A\underline{\alpha} = \underline{c} + B\underline{\hat{b}}, \tag{54}$$

where $\underline{c} := \underline{d}^* - \underline{\beta}$ is a known vector.

Since A has full rank, the matrix $(A^T A)$ is square and nonsingular. Hence the pseudo-inverse of A, defined by $A^\# := (A^T A)^{-1} A^T$ exists, and satisfies $A^\# A = \mathrm{I}$, where I denotes the identity matrix. We then get

$$\underline{\alpha} = A^\# \underline{c} + (A^\# B)\underline{\hat{b}}. \tag{55}$$

Now $A^\# \underline{c}$ is known, and also B has full rank. To verify the latter, one can check that the rows corresponding to the delays $\{d_{12}, d_{13}, \ldots, d_{1n}\}$ form a diagonal $(n-1)\times(n-1)$ matrix with strictly negative values $\{-(1/a_2), -(1/a_3) \ldots, -(1/a_n)\}$. Hence, $(A^\# B)$ is also of full rank. A unique solution exists if and only if $(A^\# B)\underline{\hat{b}} = 0$. The feasible set of the vector of offset estimates, $\hat{b}$, is $\mathbb{R}^{n-1}$ or, if causality is invoked, a polyhedron. In both cases, these sets have *nonempty interior*, i.e., full *dimension* $n-1$. Thus, the condition $(A^\# B)\underline{\hat{b}} = 0$ for all "feasible" vectors $\underline{\hat{b}}$, reduces to $(A^\# B) = 0$, which is equivalent to $A^T B = 0$. ■

Example: Consider the case where there is an unknown affine relation between the transmit and receive delay, i.e., $\alpha_i = \gamma\beta_i + \delta$ for all nodes, where the parameters γ, δ are unknown and $\gamma > 0$. For each *fixed* pair (γ, δ), we obtain a known affine characterization of the asymmetry. Hence there exists a *unique* solution by Theorem 4. As one ranges[9] over various $\gamma > 0$ and δ, one obtains an uncertainty set parametrized by two degrees of freedom, namely γ, δ.

An interesting special case is when $\delta = 0$. This corresponds to the case where nodes run at a constant but unknown speed, and transmitting and receiving delays are simply inversely proportional to the speed of the processor at the node. In this case, the uncertainty set is parametrized by a *single* degree of freedom, $\gamma > 0$.

IX. Conclusion

We have characterized what is fundamentally feasible and infeasible in synchronizing clocks over wired or wireless networks. The main result is that the determination of the unknown clock offsets, and the link delays is, in general, impossible, though the skews can be determined correctly. We have also characterized the uncertainty set. The delays can be estimated up to one unknown offset for each node except the reference node, with these nodal offsets being indeterminable parameters. Invoking causality, the offset vector lies in a completely characterized polyhedron. We also have provided necessary and sufficient conditions on the network topology for this polyhedron to be compact and have a nonempty interior. If there is a known asymmetry in the delays that can be affinely characterized, a unique solution exists. Despite the uncertainty in offset and delay, a sender can predict exactly the time its packet will be received, as measured by the receiver's clock. For the problem of receiver-receiver synchronization, the nodal skews can still be determined correctly but only delay differences between neighboring communication links with a common sender can be expressed affinely with respect to the $(n-1)$ unknown offsets. Causality does not reduce the uncertainty set which remains unbounded. Last, we have studied the special case where the link delays have structure in terms of transmit and receipt delays plus known electromagnetic propagation times, and provided sufficient conditions for uniqueness of solution.

[9]Note that all values of $\gamma > 0$ are feasible, but for given γ, δ is bounded above by the fact that round-trip delays in bilateral links are known by estimation.

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Nikolaos M. Freris (M'05) received the B.S. degree in electrical and computer engineering from the National Technical University of Athens, Athens, Greece, in 2005, the M.S. degrees in electrical and computer engineering and mathematics from the University of Illinois at Urbana-Champaign, in 2007 and 2008, respectively, and the Ph.D. degree in electrical and computer engineering from the University of Illinois at Urbana-Champaign in 2010.

Since September 2010, he has been with IBM Research-Zurich, Switzerland, where he is currently a Postdoctoral Research Associate in the Mathematical and Computational Sciences Group. His research has been on wireless and sensor networks: clock synchronization, network resource allocation, random-access MAC, and scalable video streaming.

Dr. Freris was granted a Vodafone Graduate Fellowship for 2007–2008 for research excellence in the area of wireless communication networks and the Gerondelis Foundation Award for excellent graduate students of Greek descent in 2010. He is a member of the Technical Chamber of Greece.

Scott R. Graham (SM'11) received the B.S. degree in electrical engineering from Brigham Young University, Provo, UT, in 1993, the M.S. degree in electrical engineering from the Air Force Institute of Technology, Wright Patterson Air Force Base, OH, in 1999, and the Ph.D. degree in electrical and computer engineering from the University of Illinois at Urbana-Champaign in 2004.

He is currently an Adjunct Professor of Electrical and Computer Engineering at the Air Force Institute of Technology. His research interests include directional networks, networked control systems, and critical network infrastructure.

Dr. Graham is a member of Tau Beta Pi and Eta Kappa Nu.

P. R. Kumar (F'88) received the B.Tech. degree in electrical engineering (electronics) from I.I.T. Madras, Madras, India, in 1973, and the M.S. and D.Sc. degrees in systems science and mathematics from Washington University, St. Louis, in 1975 and 1977, respectively.

From 1977 to 1984, he was a Faculty Member in the Department of Mathematics, University of Maryland Baltimore County. Since 1985, he has been with the University of Illinois at Urbana-Champaign, where he is currently the Franklin W. Woeltge Professor of Electrical and Computer Engineering, Research Professor in the Coordinated Science Laboratory, Research Professor in the Information Trust Institute, and Affiliate Professor of the Department of Computer Science. He has worked on problems in game theory, adaptive control, stochastic systems, simulated annealing, neural networks, machine learning, queueing networks, manufacturing systems, scheduling, wafer fabrication plants and information theory. His current research interests are in wireless networks, sensor networks, and networked embedded control systems.

Dr. Kumar received the Donald P. Eckman Award of the American Automatic Control Council, the IEEE Field Award in Control Systems, and the Fred W. Ellersick Prize of the IEEE Communications Society. He is a member of the U.S. National Academy of Engineering. He has been awarded an honorary doctorate by ETH, Zurich, and a Guest Chair Professorship at Tsinghua University, Beijing, China. He is also the Lead Guest Chair Professor of the Group on Wireless Communications and Networking at Tsinghua University, Beijing, and is an Honorary Professor at IIT Hyderabad.