Distributed Systems

Lecture 2

- Ordering Events
- Safety
- Liveness

https://cs.nyu.edu/~apanda/classes/sp25/

Announcements

- Lab 1 now posted on website
  Due Feb 14
- Only about half-the-class is on Campuswire
  → We have had at least 2 technical questions
  → Expectation: Everyone joins.
  ↳ We use it for announcements, etc.

Last Week

- Asynchronous model

- I/O Automata

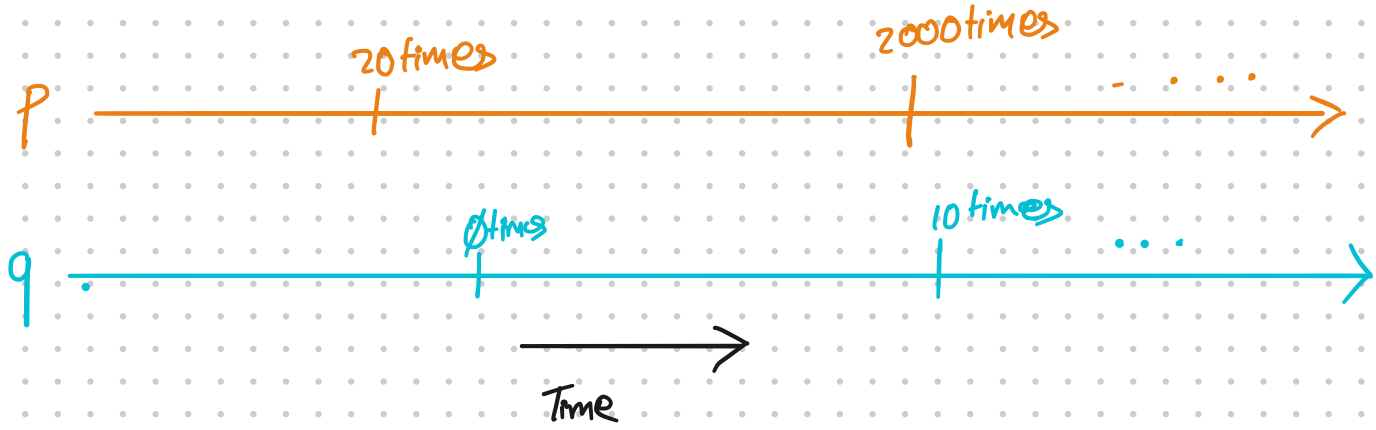
Clarifying Fairness

E occurs infinitely often

$\forall n \in \mathbb{Z}^+ \; \exists$ time t after which $|E| \geq n$

From last class } If process p sends m to q infinitely often

then q receives m infinitely often



Do not need all the messages to get through.

Fairness $\not\Rightarrow$ Reliable.

But really the bit of fairness that will be important to us for most of the semester is the simpler form (weak fairness)

- P sets a timer
- Message sent to P over a reliable channel
- ...

$\Rightarrow \exists$ Time t e is delivered

- Timer goes off
- P receives & processes message
- ...

Some Math Preliminaries

- Partial & Total Orders
  - ↳ Feature in today's discussion
  - ⇒ Play a starring role next week when we discuss LINEARIZABILITY.

- $\leq$: Relation defining a reflexive/weak/non-strict order

  Defined over some set X — $\mathbb{Z}$, Events, ...

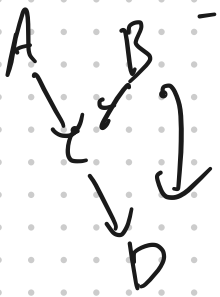
  $a, b, c \in X$

  1. $a \leq a$ [Reflexive]
  2. $a \leq b \wedge b \leq a \Rightarrow a = b$ [Antisymmetry]
  3. $a \leq b \wedge b \leq c \Rightarrow a \leq c$ [Transitive]

- Total order

  $\forall a, b \in X \quad a \leq b$ or $b \leq a$

A — A → C ← C — Natural numbers $\{ \leq$



- $X \equiv \mathbb{Z} \times \mathbb{Z}$ $(0,0)$, $(1,0)$, $(1,2)$ ...
- Lexicographic order

$$(a_0, b_0) \leq (a_1, b_1) \equiv$$
$$a_0 < a_1 \text{ OR } (a_0 = a_1 \text{ AND } b_0 \leq b_1)$$

- Partial Order
  - Not all elements in X can be ordered

$$X \equiv \mathbb{Z} \times \mathbb{Z}$$
$$(a_0, b_0) \leq (a_1, b_1) \equiv$$
$$a_0 \leq a_1 \text{ AND } b_0 \leq b_1$$

- $(0,0) \leq (0,0)$? ✓
- $(1,2) \leq (2,3)$? ✓
- $(1,2) \leq (2,1)$? ✗
- $(2,1) \leq (1,2)$? ↑

$< \leq \mathbb{Z}^+$

- Last bit of math I promise.
  - Can view Relations as sets

$R, \leq : R \times R \quad (r_0, r_1) \in \leq \iff r_0 \leq r_1$

$(1,2) \in \leq$

$(2,1) \notin \leq$

$((1,2), (2,3)) \in \leq$

$(1,2),(2,1)) \notin \leq$ $((2,1),(1,2)) \notin \leq$

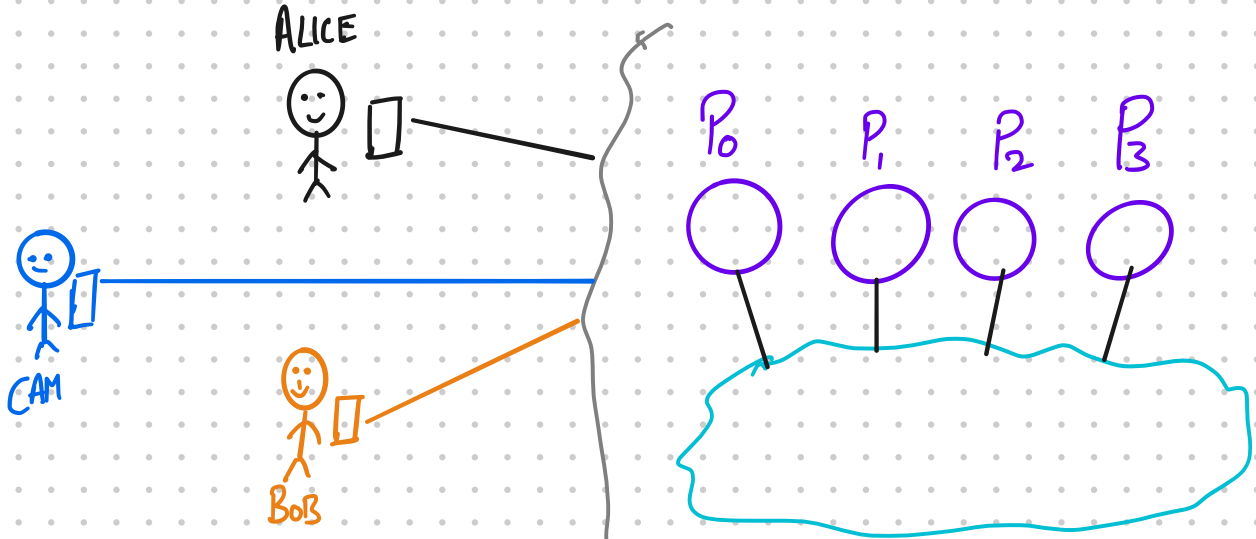
- Paper will talk about finding $\leq_A$ s.t.

$$\leq_A \supseteq \leq_B$$

Meaning $x \leq_B y \Rightarrow x \leq_A y$

But might have $c \leq_A d$ where $(c,d) \notin \leq_B$

OK! Back to Correctness OR Time OR...

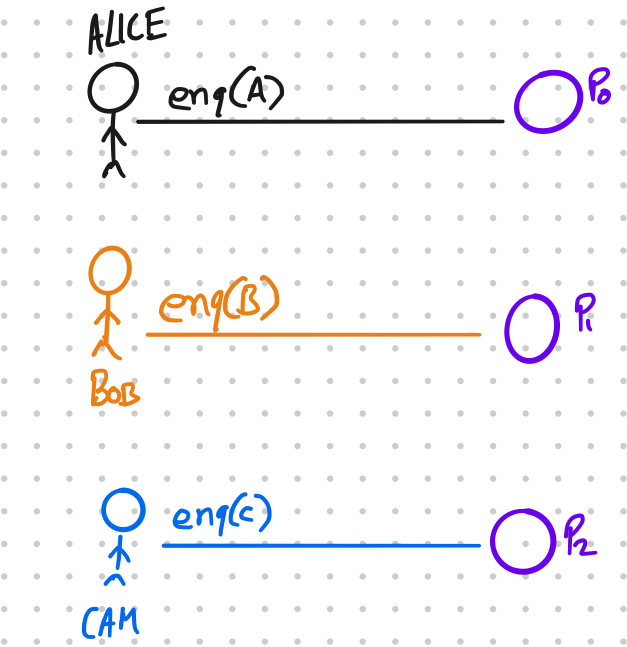


① Messages/orders/... ordered first come first serve

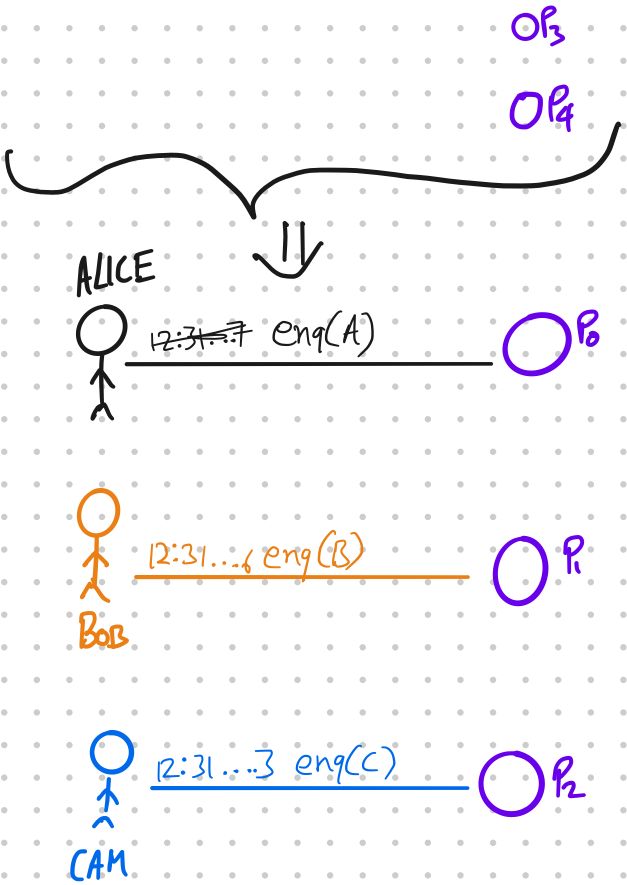
② FIFO Queue

③ ...

NEED TO ORDER OPERATIONS/EVENTS/THINGS/...



ABC
CAB
...
} What is the correct queue?



PROBLEMS

① Relativity: No notion of a universal time

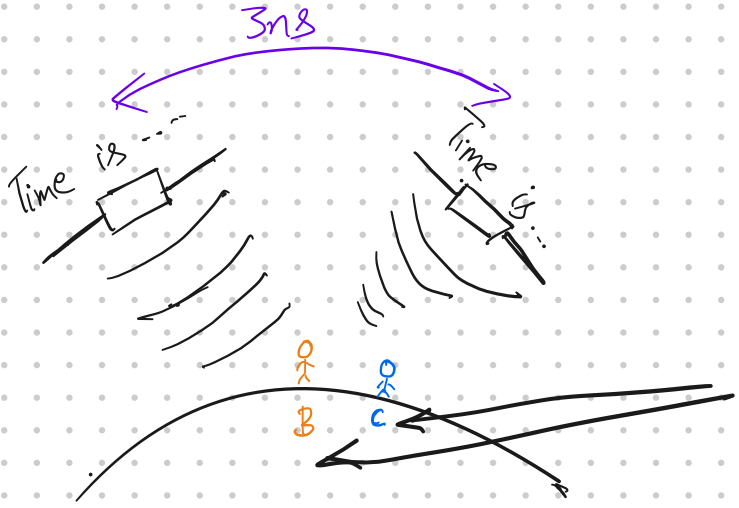
Alice's clock runs at a diff rate than Bob's or Charlie's, etc.

FUN BLOG: EXPERIMENTAL CONFIRMATION:

② Limits on time synchronization

- Impossible in asynchronous model

- Perfect sync impossible even with assumptions about message delays (See optional paper for this week)



~ μs to ms

Implication: To use clocks to sync

- Assume min interval b/w events that need to be synced

AND/OR – How to order events that occur too close to each other.

What we will assume: TIME-FREE MODEL

- Will not read clocks/time in our protocols, not attach time to messages, etc.
- Will use timers (for timeout) but

Reflects reality

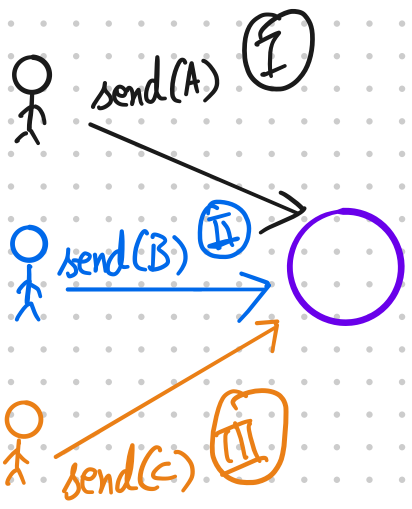
Timer goes off non-det. some time after timeout expires

Set timer for 5 sec

in most cases!

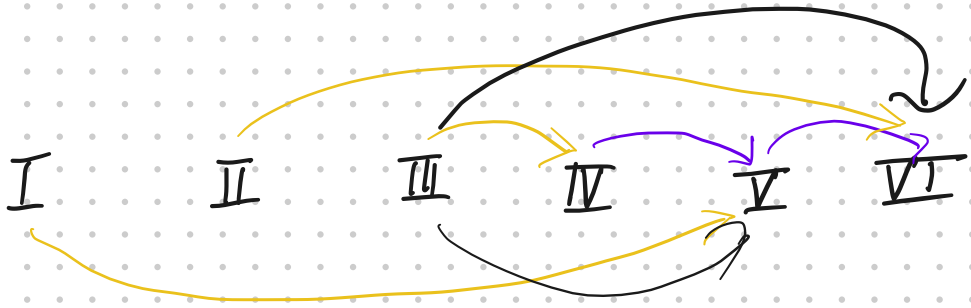
⇒ Protocol sees timer event in $[5s, \infty)$

What can we order without clocks?

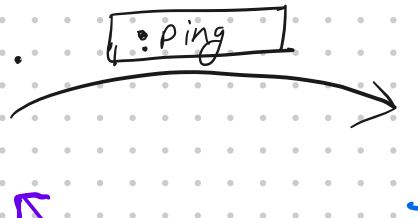


recv(): C IV
recv(): A V
recv(): B VI

$\lesssim$ Lamport's Happens Before




```
on init:
    send(B, :ping)
on recv :ping from C:
    halt
```




```
on recv :ping from A
    send(C, :ping)
```

B

:ping

C

```
on recv :ping from B
    send(A, :ping)
```

A: ... receive

A: send → B: receive

B: send → C: receive

C: send → A: receive

OBSERVE: ① Happens-Before is a partial order

- Orders events that are CAUSALLY CONNECTED

② Can equivalently consider how to order states

A.send
|||
A after init

B.receive + B.send
||
B after receiving :ping from A

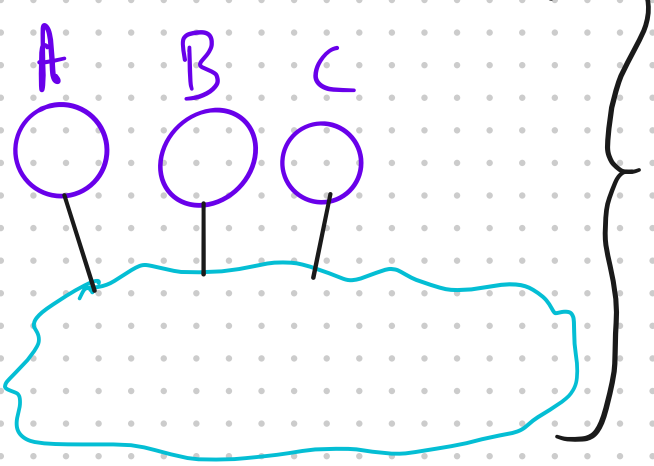
C.receive + C.send
|||
C after

A.receive
|||
A after

CORRECTNESS

Specify how state evolves during execution

STATE:

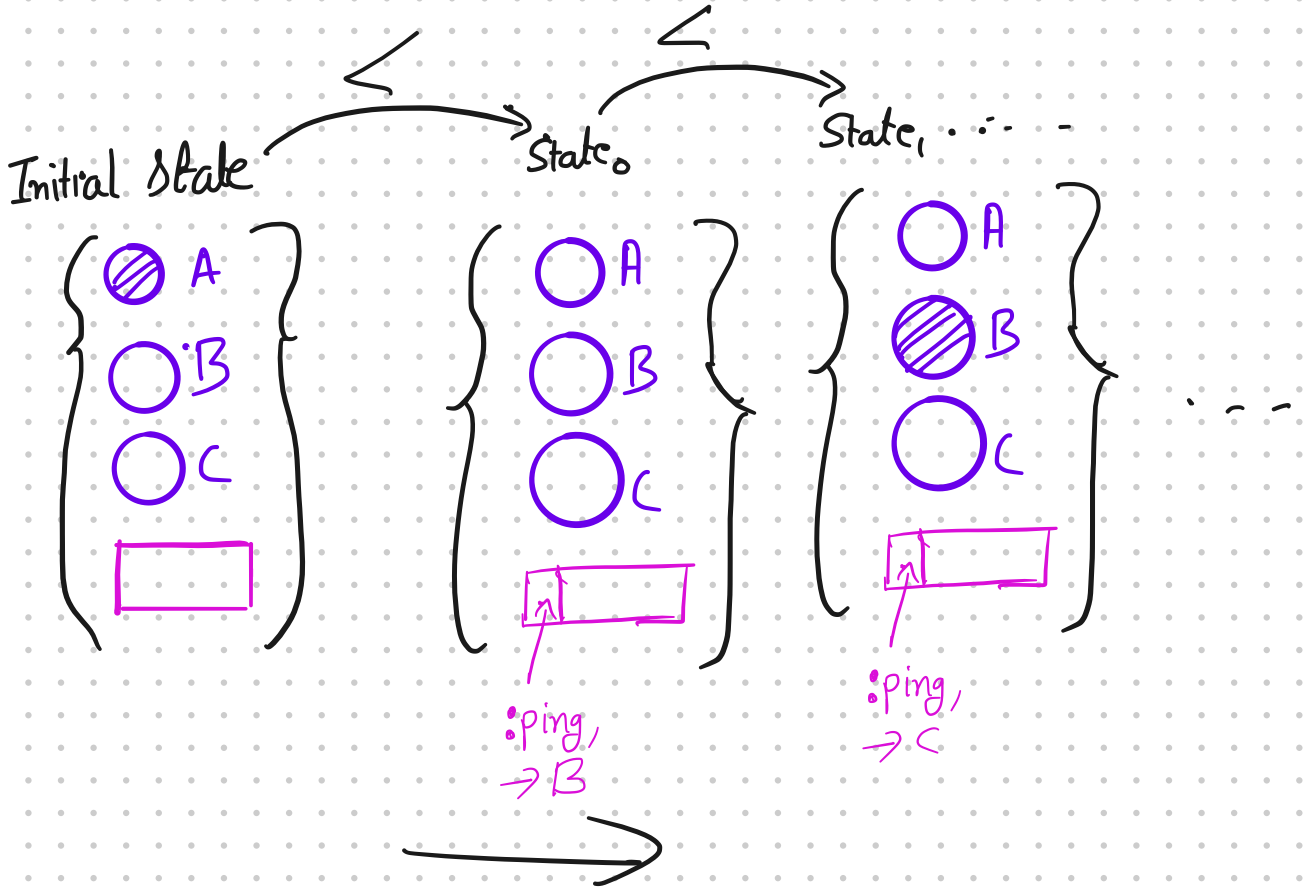


A, B, C's memory

+

Messages that have been sent but not received or dropped

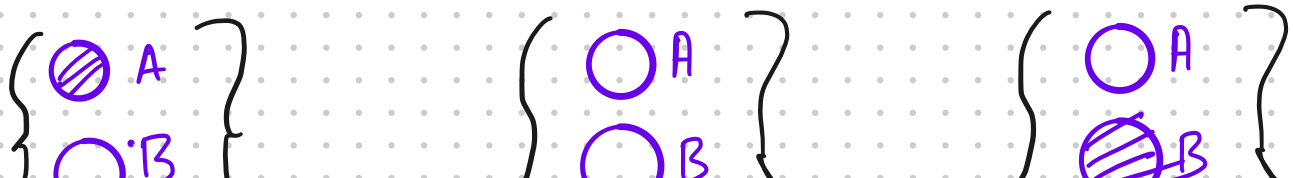
Pending messages

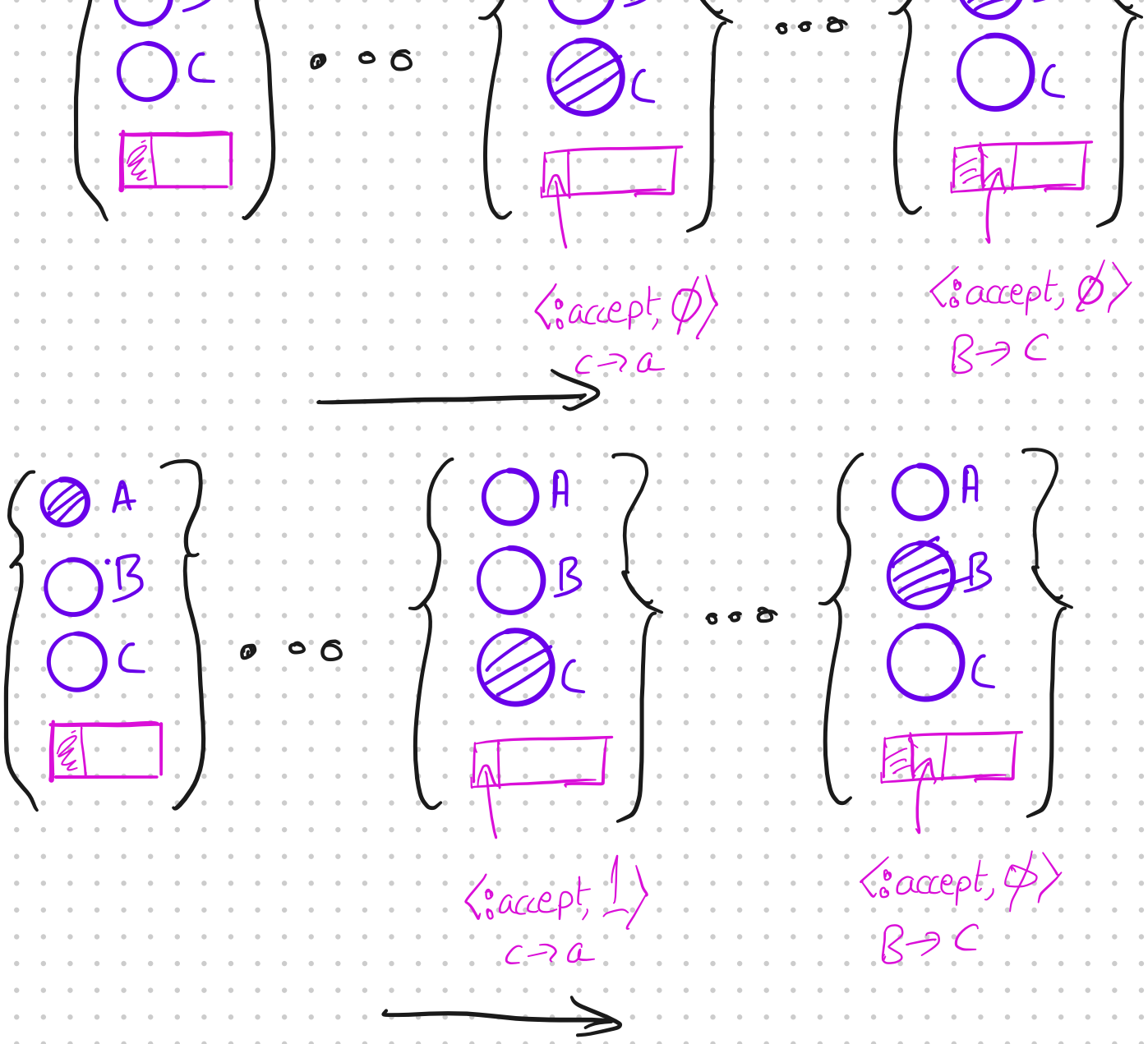


P

Safety: Some bad state evolution never occurs OR some predicate never holds.

- If a process p sends ⟨:accept, ∅⟩ then after that no process sends ⟨:accept, 1⟩





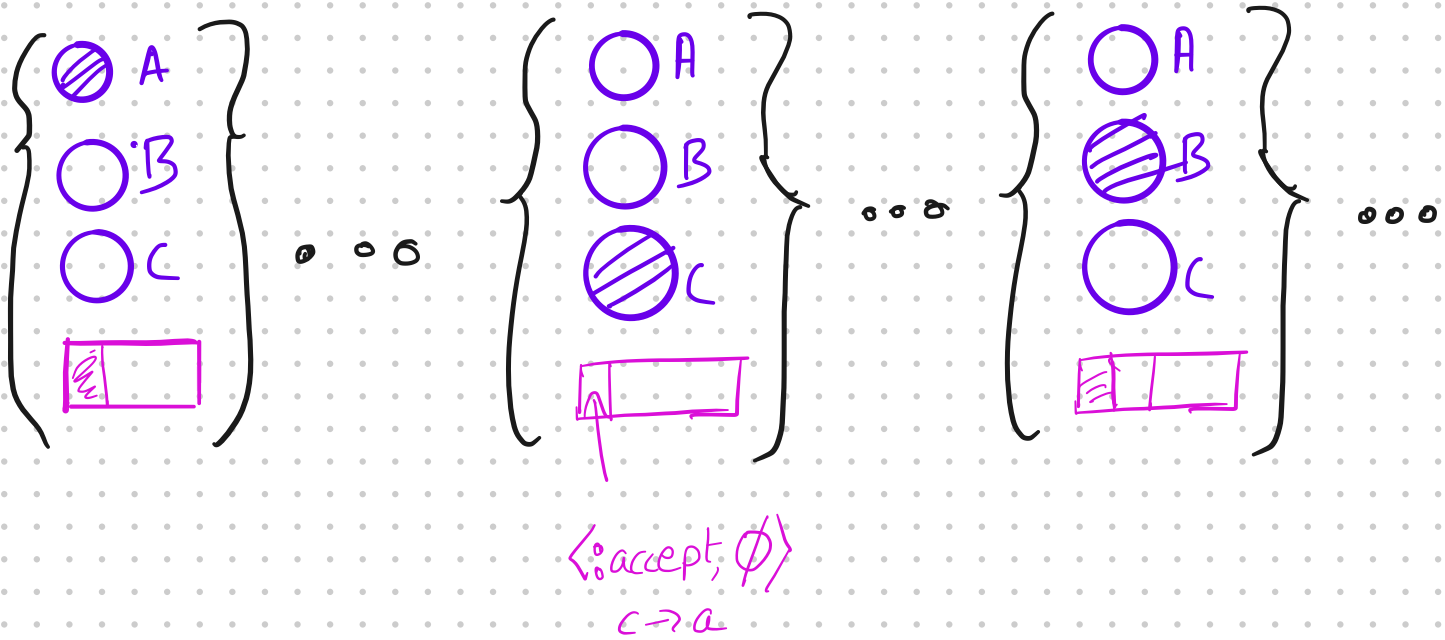
OBSERVATIONS:

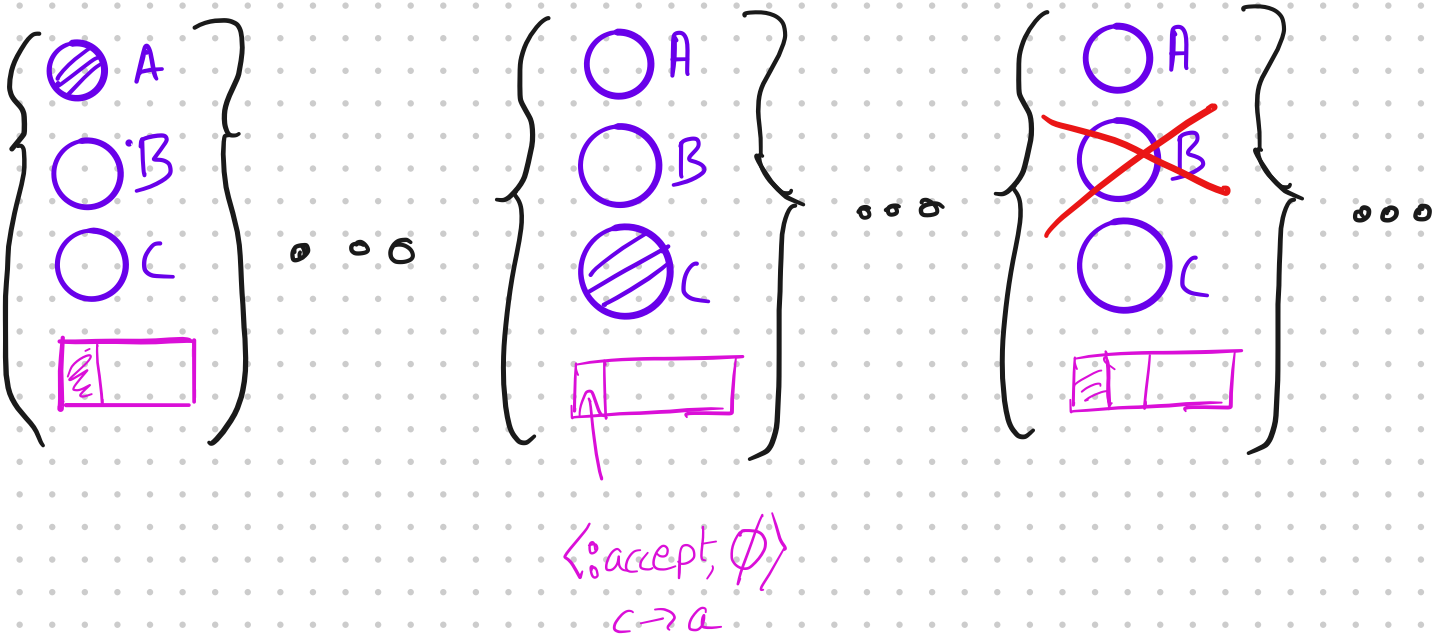
- Given a sequence of states (TRACE) can determine if safety condition has been VIOLATED by evaluating predicate.
- Once a safety condition is violated, no subsequent change is going to fix it/make it hold.

liveness : Some good thing eventually happens OR some

predicate eventually holds

- If a process p sends ⟨:accept, ∅⟩ then
  EVENTUALLY all processes send ⟨:accept, ∅⟩





OBSERVATION: Just evaluating predicate is not enough

Instead given a trace T ($s_0, s_1, s_2, \ldots, s_n$) need to ask

if $\exists$ a sequence σ of states that

(a) Is valid as an extension to T

↳ (OR VALID STARTING FROM s_N)

E.g.,- If in fail-stop model, no failed process acts.

- ...

(b) Predicate holds for $T\sigma \models P$

Paper: Uniform and Absolute liveness

↳ Additional properties that hold for σ

- Uniform: $\exists \sigma$ THAT APPLIES TO ALL EXECUTIONS

- ABSOLUTE: IF $\sigma \models P$ then $\forall T$, $T\sigma \models P$

$T_3\sigma \not\models P$

$T_1\sigma \not\models P$

$T_2\sigma \not\models P$

$T_i \models P$

$T_i \not\models P$

$T_1 \rightarrow \sigma_1$ $\quad \sigma'$

$T_2 \rightarrow \sigma_2$ $\quad T_1\sigma' \not\models P$

$T_2\sigma' \not\models P$

$T_2 T_1 \sigma' \not\models P$

Absolute

$\exists T \models P$ $\quad$ $P_i \models P$

$T_1 P_i \not\models P$

T_1 P_1

T_2 P_i

T_3 P

T_4 P