# Lecture 9 post-lecture comments #41

A Anupam Gupta STAFF
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Hi all:

We'll go over the coupling lemma (saying that the mixing time is at most the coupling time), and the proof that the Glauber dynamics has small coupling time, next Thursday, before we do whatever we're going to do next time.

Some comments (and answers to questions) about today's lecture:

1. Romain asked if DNF counting could possibly be done **exactly**. We don't expect so (unless P=NP or worse): the problem is something called #P-hard, so counting this is at least as hard as counting the number of CNF solutions, which is at least as hard as deciding if the formula is satisfiable. Indeed, consider some CNF formula $\psi$. Then its negation $\bar{\psi}$ can be written as a DNF formula of the same size. (Hint: use DeMorgan's laws.) Now if we could count its satisfying assignments exactly, we could subtract that from 2^n and count those satisfying $\psi$ exactly too. So, unless P=NP, we cannot.

2. He also pointed out that the stationary distribution of the Markov chain on colorings was uniform because it was **symmetric**: that P_{\chi,\chi'} = P_{\chi',\chi}. This is indeed true, and here is a proof. (Feel free to suggest other proofs in the comments below, if you like.) Since $\sum_{\chi'} P_{\chi,\chi'} = 1$, because it's a Markov chain, we can use symmetry to now claim that $\sum_{\chi'} P_{\chi',\chi} = 1$. And hence one can verify that setting $\pi^*(\chi') = 1/|\Omega|$ for every coloring $\chi'$ satisfies $\pi^* \times P = \pi^*$. Now since there is a unique stationary distribution (because the MC is ergodic), the uniform distribution is the stationary distribution.

   This is much nicer than the proof in the notes, I will try to update those soon.

3. Rama asked if the definition of coupling required that we have the **"sticky" property**, that $X_{t+1} = Y_{t+1}$ if $X_t = Y_t$. I realized that the notes I was referring to were not consistent, and then I realized that it did not matter for the purposes of what we are doing. We can indeed assume this sticky property if we want, WLOG.

   Indeed, given any coupling, we can change it to be a "sticky" coupling: just follow the old coupling until the two states are equal, and thereafter evolve them identically --- this is another valid coupling. Moreover, if we want small coupling time, this sticky coupling has no greater coupling time than the original coupling.