Lecture 7 clarifications #33

A Anupam Gupta STAFF
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Hi all: a question and a clarification from yesterday:

1. Rama asked if the assumption |X_i - X_0| \leq iC would be enough for concentration as in Azuma's inequality (along with the sequence (X_i) being a martingale).

   Madhusudhan pointed out the following example where X_0, ... X_{n-1} = 0. But X_n = +/- n wp 1/2. It satisfies the properties, but is far from being concentrated (it only takes value n or -n, neither of which are close to its mean 0). Really, the fact that it's a bunch of small per-step changes makes it concentrated, and hence "somewhat predictable".
2. One could ask: can one get concentration of the form

$$\Pr\left[f - Ef| \ > \lambda\right] \ \leq \exp(-\lambda^2/\mu) \qquad (*)$$

   instead of the form

$$\Pr\left[f - Ef| \ > \lambda\right] \ \leq \exp(-\lambda^2/n) \qquad (**)$$

   for 1-Lipschitz non-negative functions? (These are often called "scale-free" bounds, since they don't depend on the number of variables n but only on the expectation mu = Ef.) Sadly, not without more assumptions. E.g.: Let X_i's be Bernoulli(1/2) and

$$f \ = \ \max\left(0, \sum X_i \ - \frac{n}{2} + 100\sqrt{n}\right)$$

   Since the mean of the sum is n/2, the shifting by (n/2 - 100sqrt(n)) makes the mean 100sqrt(n). Moreover, the variance is sqrt(n) with Gaussian tails, so truncating to the positive part (max with zero) does not change the mean by much. So the mean of f is now O(sqrt(n)). And the variance is still about Omega(sqrt(n)). Which means we cannot hope to get the deviation of the type (*), it still remains of the type (**).

   If we do want (*), we need to make more assumptions. E.g., if the function is submodular or convex or satisfies other conditions like in Freedman's inequality, then one can get (*).