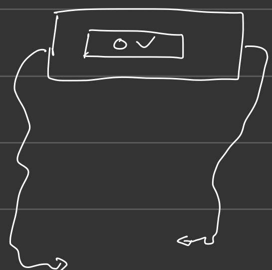
# INTRO TO QUANTUM COMPUTING

what can we do with Quantum computers?

## Classical Computers:

| Logical bits | Physical bits |
|---|---|
| $\boxed{0}$ | Low Voltage |
| $\boxed{1}$ | High Voltage |

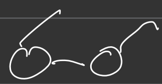



Voltmeter

Make the physical implementation smaller and smaller...
to the Quantum scale!

| Logical bits | | Physical bits |
|---|---|---|
| $\boxed{0}$ | $\gamma$<br>photon | horizontally polarized |
| $\boxed{1}$ | | vertically polarized |

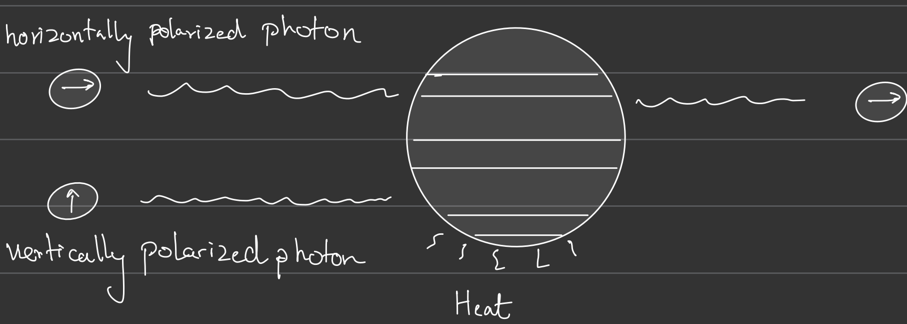
# Measurement?



Polarizing Sunglasses

(same idea as 3D movie)

## Cartoon:



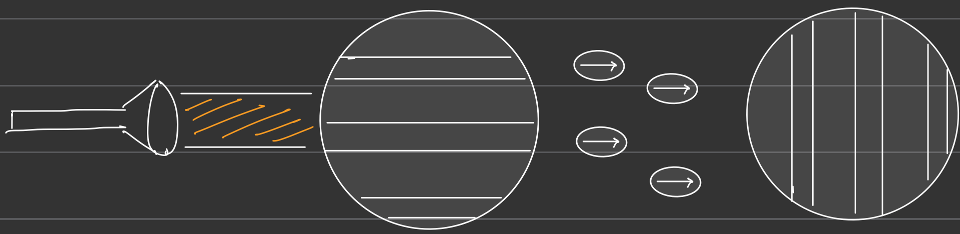

Horizontally Polarized Filter

- Same idea for vertically polarized filters

## Fun Experiments with Polarized Filters:

①

(2)



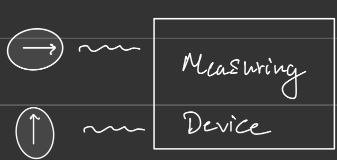
No light

Exercise: Explain the following "party trick"

(mathematics to explain it is in this lecture)

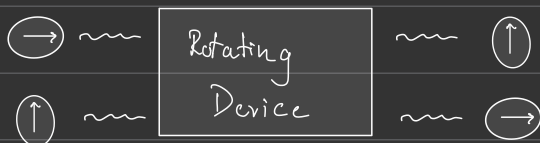
??

## Mathematical Abstraction:




Read out: 0

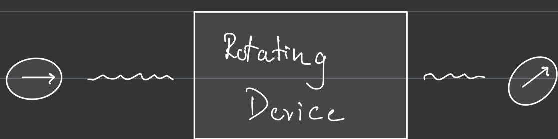
Read out: 1




(Obs. this is the NOT Gate)

- Cannot really do interesting computation with 1 bit.

Punchline: Can do interesting computation with 1 "qubit"!




"superposition" of 0 and 1

<u>Logical Qubit</u>:

$$x \cdot \underbrace{|0\rangle}_{\text{logical zero}} + y \underbrace{|1\rangle}_{\text{logical one}}$$

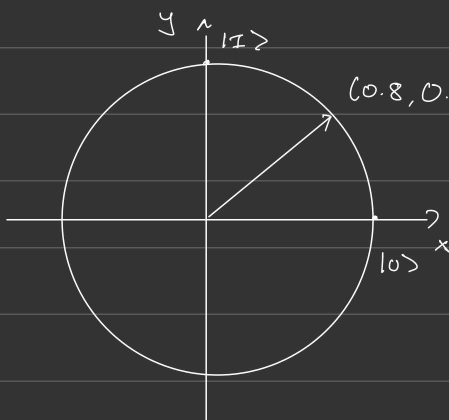
for real numbers $x, y$. s.t. $x^2 + y^2 = 1$

- Some amount of logical zero and some amount of logical one.

Ex. Equal superposition of $|0\rangle$ and $|1\rangle$

$$\underbrace{\sqrt{\tfrac{1}{2}}}_{\text{"amplitude"}}\,|0\rangle + \underbrace{\sqrt{\tfrac{1}{2}}}\,|1\rangle$$

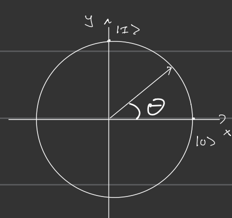
Geometric Interpretation: $x\,|0\rangle + y\,|1\rangle$ s.t. $x^2+y^2 = 1$




$(0.8, 0.6) \iff \underbrace{0.8\,|0\rangle + 0.6\,|1\rangle}$

accurate if $\boxed{\begin{array}{l} |0\rangle = (1, 0) \\ |1\rangle = (0, 1) \end{array}}$

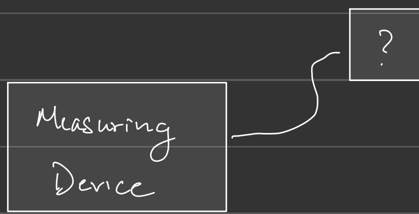
$|0\rangle \rightsquigarrow \boxed{\text{Rotate by } \theta} \rightsquigarrow \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$




(recall $\sin^2\theta + \cos^2\theta = 1$)

## What happens when you measure an arbitrary qubit?

$\cos\theta\,|0\rangle + \sin\theta\,|1\rangle \rightsquigarrow$




### Law of Quantum Mechanics:

$$\left.\begin{aligned} \Pr[\text{ readout is } |0\rangle \,] &= \cos^2\theta \\ \Pr[\text{ readout is } |1\rangle \,] &= \sin^2\theta \end{aligned}\right\}$$ the natural thing you expect to happen

**Recap:** We have the following mathematical formalization of storing and manipulating a single qubit:

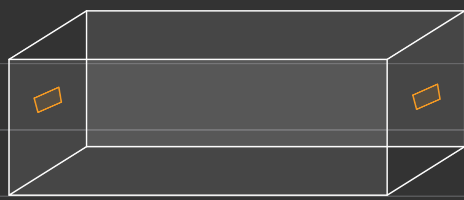
1. Description: $\cos\theta\,|0\rangle + \sin\theta\,|1\rangle$
2. Measurement: $\Pr[\,|0\rangle\,] = \cos^2\theta$, $\Pr[\,|1\rangle\,] = \sin^2\theta$

Computation: $|0\rangle \xmapsto[\text{by } \theta]{\text{rotate}} \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$

# Elitzur – Vaidman "Bomb"

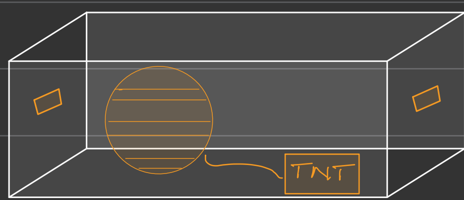
(Physics though experiment / Quantum Algorithm)

**Case 1:**



Empty box

**Case 2:**




Bomb

Horizontal filter with heat triggered fuse

**Rules:**

1. If filter measures $|0\rangle$, photon passes through
2. If filter measures $|1\rangle$, heat sets off bomb.

**Goal:** Figure out whether the box has a bomb without it exploding.

- If you know nothing about quantum, and you've only seen horizontal or vertical photons (classical bits) this problem is impossible.

Explanation: You only have two choices: input $|0\rangle$ or $|1\rangle$.

- Consider $|0\rangle$: you get no information at all
  - If the box is empty, the photon passes through
  - If the box has a bomb, the same thing happens
- Consider $|1\rangle$: Bad idea
  - If the box is empty you see a $|1\rangle$ come out
  - If the box has a bomb, you certainly see an explosion!

    (recall $|1\rangle$ gets absorbed by the filter w.p. 1)

Warm Up:

Key Idea: Taking advantage of super-positions!

Pass $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ through the box

Case 1: Nothing happens

Case 2: w.p. 0.5 the photon gets absorbed and the bomb explodes.

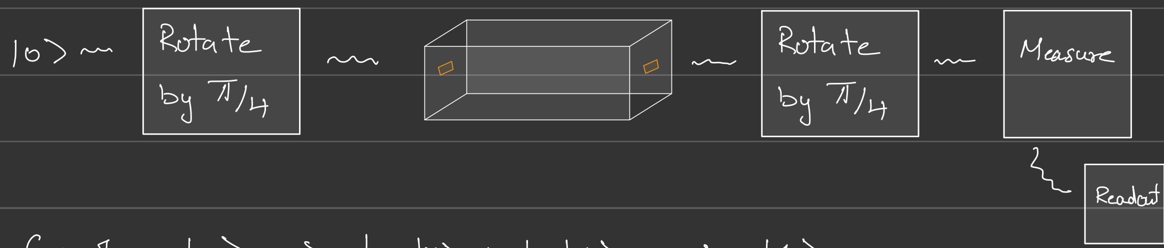
w.p. 0.5 the photon passes through and is $\underbrace{\text{horizontally polarized}}$ i.e. $|0\rangle$

In this case, we have some non-trivial signal.

Algorithm:

1. Start with qubit $= |0\rangle$
2. Rotate $45°$ or $\pi/4$ radians
3. Pass through box
4. Rotate $\pi/4$ radians
5. readout := Measure (qubit)

If readout $= |0\rangle$
conclude "Bomb"
Else "Inconclusive"




<u>Case 1:</u> $|0\rangle \rightarrow \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \longrightarrow |1\rangle$

Readout = $|1\rangle$ w.p. 1 and the algo. outputs "inconclusive"

<u>Case2:</u>

$|0\rangle \rightarrow \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

w.p. 0.5 $\rightarrow$ $|1\rangle$ is meavred by filter and "BOOM"

w.p. 0.5 $\rightarrow$ $|0\rangle$ is measured $\longrightarrow \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

- w.p. 0.5 $\rightarrow$ $|1\rangle$ is measured and Readout is "Inconclusive"
- w.p. 0.5 $\rightarrow$ $|0\rangle$ is measured and readout is "Bomb"

<u>Recap:</u> Empty box: 100% chance "Inconclusive"

Bomb: 50% chance of explosion

25% chance "Inconclusive"

$\underbrace{\text{25\% chance "Bomb"}}$

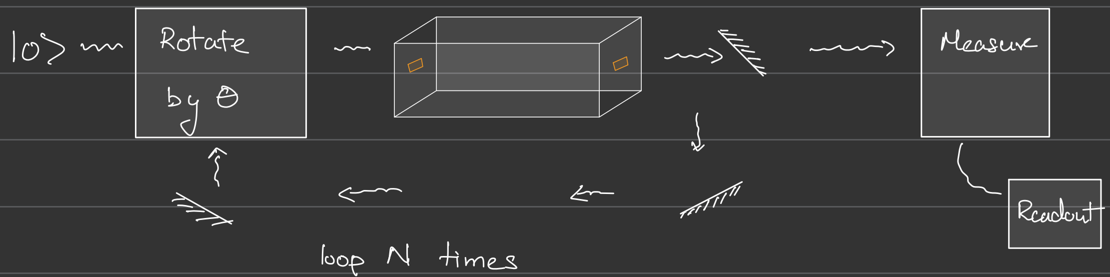
Detected bomb without exploding!

(not achievable with classical bits)

# Elitzer - Vaidman Algorithm :

① Let N be "Safety level" specified by user
② Start with qubit = $|0\rangle$
③ Let $\theta = \pi/2N$
④ For $t = 1, 2 \ldots N$
- Rotate by $\theta$
- Pass through mystery box

⑤ readout := Measure (qubit)
⑥ If readout = $|0\rangle$ print "Bomb"
Else "Empty"




Case 1: Bounce around N times

$$|0\rangle \longrightarrow |1\rangle$$

Readout := $|1\rangle$ w.p. 1 and algorithm outputs "Empty."

Case 2:

- photon is "essentially horizontal", very high chance that filter measures it to be $|0\rangle$ (will calc precisely later)

- photon at angle $\theta$ goes into the box again and again the chance the measurement is $|0\rangle$ again

- Assuming the likely thing happens in each iteration the photon comes out in state $|0\rangle$ at the end

- Readout $= |0\rangle$ w.p. $\underline{1}$ and we output "Bomb" without ever exploding!

Summary:

1. Empty: 100% chance of detecting empty
2. Bomb: 100% chance of detecting bomb assuming we don't explode

What is the chance of explosion?

- Two competing effects: as you make N large
  the angle gets closer to 0 (decrease pr. of exploding)
  But the photon goes through the box many more times
  (increase pr. of exploding)
- Not obvious if increasing N helps.

$$\Pr[\text{any explosion}] \leq \Pr\{\text{explode at } t=1\} + \Pr\{\text{explode at } t=2\} \cdots + \Pr[\text{explode at } t=N]$$

Union bound

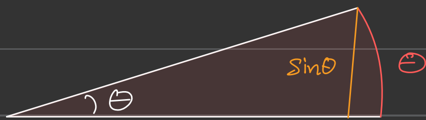
$$\Pr[\text{explode at } t=1] = \Pr\{\text{explode at } t=2\} \cdots = \Pr[\ t=N]$$

( the photon is always entering in state $\cos\theta|0\rangle + \sin\theta|1\rangle$ )

$$= N \Pr\{\text{explode at } t=1\}$$


$\Pr[\text{measing } |1\rangle]$
$= (\sin\theta)^2$

$$= N(\sin\theta)^2$$




$\therefore \sin\theta \leq \theta$

$$\leq N\theta^2 = N(\pi/2N)^2 = \pi^2/4N \leq 2.5/N$$

Take N as big as you want. $Pr[\text{exploding}] \leq 2.5/N$

- whenever you don't explode, the algorithm is correct w.p. 1.

What can you do with more than one qubit?

1. Grover Search
2. Shor's Algorithm