Random Graphs G22.3033-007
Assignment 7. Due Monday, March 27,2006

`Note`: Benny Sudakov will give the Math Colloquium talk on Probabilistic Combinatorics on Monday, March 27, at 3:45 p.m. in wwh 1302.

1. Set $\Omega = [n] \times [n]$. Define a random set $C \subset \Omega$ by

   $$\Pr[(x, y) \in C] = p = \frac{c}{n}$$

   the events $(x, y) \in C$ mutually independent. A horozontal bond is a pair $(x, y), (x+1, y) \in C$ and a vertical bond is a pair $(x, y), (x, y+1) \in C$. Find the expected number of bonds. Use Janson's Inequality to bound the probability there are no bonds in both directions and find the limiting probability as $n \to \infty$.

2. Let $G \sim G(n, p)$ with $p = \frac{c}{n}$.

   (a) Condition on $12, 13, 23 \in G$. Use Janson's Inequality to find an asymptotic formula for the probability there are no *other* triangles in G.

   (b) Find an asymptotic formula for the probability that G contains precisely one triangle.

3. Find a positive constant c so that there exists a two-coloring of K_n such that every subset of size $\frac{n}{2}$ has discrepancy less than $(c+o(1))n^{3/2}$. (The discrepency is the absolute value of the difference between the number of edges of the two colors.) Try to make c as small as you can. (Of course, you'll examine a random coloring!)

4. Let $G \sim G(n, p)$ with $p = cn^{-2/3}$ and let v, w be two distinct fixed vertices of G. Use Janson's Inequality to find the limiting probability that v, w are *not* joined by a path of length three.

Every Sunday my wife and I took a romantic little walk to Grandchester, a lovely, lovely little town near Cambridge, and we ate lunch at a pub there. We would stroll along the road reciting pi to each other; she would do twenty places, then I would do twenty and so forth.
– John Conway