Random Graphs G22.3033-007
Assignment 2. Due Monday, February 6,2006

1. Suppose $n \geq 2$ and let $A_1, \ldots, A_m \subseteq \Omega$ all have size n. Suppose $m < 4^{n-1}$. Show that there is a coloring of Ω by 4 colors so that no edge is monochromatic.

2. Suppose $n \geq 4$ and let $A_1, \ldots, A_m \subseteq \Omega$ all have size n. Suppose $m < \frac{4^{n-1}}{3^n}$. Prove that there is a coloring of Ω by 4 colors so that in every edge all 4 colors are represented.

3. The expected number of isolated trees [just take this as a fact] on k vertices in $G(n,p)$ is given by $f(n,k,p) := \binom{n}{k} k^{k-2} p^{k-1} (1-p)^B$ with $B = k(n-k) + \binom{k}{2} - k + 1$. Set $p = \frac{1}{n}$. Let c be a positive constant. Find the asymptotics of $f(n,k,p)$ when $k \sim cn^{2/3}$. (*) Express the limit as $n \to \infty$ of the sum of $f(n,k,p)$ for $n^{2/3} \leq k < 2n^{2/3}$ as a definite integral and use a computer package to evaluate the integral numerically.

4. Consider Boolean expressions on atoms $x_1, \ldots, x_n$. By a k-clause C we mean an expression of the form $y_{i_1} \vee \ldots \vee y_{i_k}$ where each y_{i_j} is either x_{i_j} or $\overline{x}_{i_j}$. Prove a theorem of the following form [you fill in the $m = m(k)$] by the probabilistic method: For any m k-clauses $C_1, \ldots, C_m$ there is a truth assignment such that $C_1 \wedge \ldots \wedge C_m$ is satisfied.

Working with Paul Erdős was like taking a walk in the hills. Every time when I thought that we had achieved our goal and deserved a rest, Paul pointed to the top of another hill and off we would go.
– Fan Chung