Fundamental Algorithms, Assignment 7
Due March 9/10 in Recitation

I am slow to learn and slow to forget that which I have learned. My mind is like a piece of steel; very hard to scratch anything on it and almost impossible after you get it there to rub it out.
– Abraham Lincoln

1. (Continuation of Problem from Last Assignment)) Consider a Binary Search Tree T with vertices a, b, c, d, e, f, g, h and $ROOT[T] = a$ and with the following values (N means NIL)

vertex	a	b	c	d	e	f	g	h
parent	N	e	e	a	d	g	c	a
left	h	N	N	e	c	N	f	N
right	d	N	g	N	b	N	N	N

   (a) Which is the successor of c. Illustrate how the program `SUCCESSOR` will find it.

   (b) Which is the minimal element? Illustrate how the program `MIN` will find it.

   (c) Illustrate the program `DELETE[e]`

2. Draw binary search trees of height 2,3,4,5,6 on the set of keys {1,4,5,10,16,17,21}.

3. What is the difference between the binary-search property and the heap property? (*) Can the heap property be used to print out the keys of an n-node tree in sorted order in $O(n)$ time? Explain how or why not.

4. You are given an array $A[1 \cdots n]$, whose values come from a universe Ω. (In application, the values would be the keys of records.) You want to test if there are any duplicates, if there are any $1 \leq i < j \leq n$ such that $A[i] = A[j]$. You are given a hash function $h : \Omega \rightarrow \{1, \ldots, n\}$ and a table $T[1 \cdots n]$ of linked lists, initially all empty. Using the hash function, give an algorithm that returns `BAD` if there is a duplicate and `GOOD` if there is no duplicate. Discuss the time of the algorithm under the assumption that calculating the hast function takes unit time.

5. What would the BST tree look like if you start with the root a_1 with $key[a_1] = 1$ (and nothing else) and then you apply

$$INSERT[a_2], \ldots, INSERT[a_n]$$

in that order where $key[a_i] = i$ for each $2 \leq i \leq n$? Suppose the same assumptions of starting with a_1 and the key values but the INSERT commands were done in *reverse* order

$$INSERT[a_n], \ldots, INSERT[a_2]$$

In the novel I never wrote, I wanted the hero to be a computer programmer because it was the most poetic and romantic occupation I could think of [...] I conceived of him, whose professional life was spent in the sanctum of the night (when, I was told, the computers, too valuable to be unemployed by industry during the day, are free, as it were, to frolic) devising idioms whereby problems might be fed to the machines and emerge, under binary percussion, as the music of truth - I conceived of him as being too fine, transluscent, and scrupulous to live in our coarse age. He was to be, if the metaphor is biological, an evolutionary abortion, a mammalian mutation crushed underfoot by dinosours, and, if the metaphor is mathematical, a hypothetical ultimate, one digit beyond the last real number. The title of the book was to be $N + 1$.
from *The Music School* by John Updike