

## Fundamental Algorithms, Assignment 13

Not to be Submitted!

What is the fastest, cheapest and most efficient way to get from point A to point B? That consumes him, and all parts of his life.  
– Sean Stanton on Travis Kalanick, Uber CEO

1. Suppose that we are doing Dijkstra's Algorithm on vertex set  $V = \{1, \dots, 500\}$  with source vertex  $s = 1$  and at some time we have  $S = \{1, \dots, 100\}$ . What is the interpretation of  $\pi[v], d[v]$  for  $v \in S$ ? What is the interpretation of  $\pi[v], d[v]$  for  $v \notin S$ ? Which  $v$  will have  $\pi[v] = NIL$  at this time. For those  $v$  what will be  $d[v]$ ?
2. Suppose, as with Dijkstra's Algorithm, the input is a directed graph,  $G$ , a source vertex  $s$ , and a weight function  $w$ . But now further assume that the weight function only takes on the values one and two. Modify Dijkstra's algorithm – replacing the MIN-HEAP with a more suitable data structure – so that the total time is  $O(E + V)$ .
3. Let  $G$  be a DAG on vertices  $1, \dots, n$  and suppose we are *given* that the ordering  $1 \dots n$  is a Topological Sort. Let  $COUNT[i, j]$  denote the number of paths from  $i$  to  $j$ . Let  $s$ , a “source vertex” be given. Give an efficient algorithm (appropriately modifying the methods of the chapter) to find  $COUNT[s, j]$  for all  $j$ .

A clever man commits no minor blunders. – Goethe