Fundamental Algorithms, Assignment 11

Due April 27/8 in Recitation.

My work has always tried to unite the true with the beautiful and when I had to choose one or the other I usually chose the beautiful. – Hermann Weyl

1. Consider `Dumb Prim` for MST. The high level idea is the same but to find the minimal weight of an edge $\{v,w\}$, $v \in S$, $w \notin S$, one looks at all the weights and finds the minimum in the usual way. (There is no updating in `Dumb Prim`.) Assume that all pairs $\{v,w\}$ have a weight. Let n be the number of vertices.
   (a) When $|S| = i$ what is the time to add a vertex to S as a function of n and i.
   (b) What is the total time for `Dumb Prim` as a sum over i.
   (c) Evaluate the above sum as $\Theta(g(n))$ for some nice function $g(n)$. (Caution: The time is *not* an increasing function of i. For example, when $i = n-1$ the time is quite quick.)
   (d) Compare the time for `Dumb Prim` with `Prim` as discussed in class
2. Consider Prim's Algorithm for MST on the complete graph with vertex set $\{1,\ldots,n\}$. Assume that edge $\{i,j\}$ has weight $(j-i)^2$. Let the root vertex $r = 1$. Show the pattern as Prim's Algorithm is applied. In particular, Let $n = 100$ and consider the situation when the tree created has 73 elements and π and *key* have been updated.
   (a) What are these 73 elements.
   (b) What are $\pi[84]$ and $key[84]$.
3. Find $d = \gcd(89,55)$ and x,y with $89x + 55y = 1$. [Remark: This is part of a pattern with two consecutive numbers from the Fibonacci sequence $0,1,1,2,3,5,8,13,21,34,55,89,\ldots$]
4. Find $\frac{211}{507}$ in Z_{1000}.
5. Solve the system
   $x \equiv 34 \bmod 101$
   $x \equiv 59 \bmod 103$.
6. Using the Island-Hopping Method to find 2^{1000} modulo 1001 using a Calculator but NOT using multiple precision arithmetic. (You should never have an intermediate value more than a million.)

7. (extra from last week!) Suppose that during Kruskal's Algorithm (for MST) and some point we have $SIZE[v] = 37$. What is the interpretation of that in the case when $\pi[v] = v$? What is the interpretation of that in the case when $\pi[v] = u \neq v$? Let w be a vertex. How many different values can $\pi[w]$ have during the course of Kruskal's algorithm? How many different values (as a function of V, the number of vertices) can $SIZE[w]$ have during the course of Kruskal's algorithm? (That is, the maximal number possible.)

The universe is not only queerer than we suppose but queerer than we *can* suppose.
– J.B.S. Haldane