

# Basic Algorithms, Assignment 4, Solutions

Due, Thursday, Oct 4

## MIDTERM October 9 In Class!

Wednesday, October 3: NO OFFICE HOURS

**SPECIAL PREMIDTERM OFFICE HOURS** Monday, October 9,  
7-8:30p.m.

1. When  $H$  is a heap with length fifty million and  $\text{HEAPIFY-UP}(A, 300)$  is called what is the maximum number of exchanges that can take place. What is the minimiml number of exchanges that can take place.  
Solution. Minimal is zero, when  $A(300)$  is already bigger than its parent. Then maximum is if you exchange up to the root with 150, 75, 37, 18, 9, 4, 2, 1 for 8 exchanges.
2. When  $H$  is a heap with length fifty million and  $\text{HEAPIFY-DOWN}(A, 300)$  is called what is the maximum number of exchanges that can take place. What is the minimiml number of exchanges that can take place.  
Solution. Minimal is zero, when  $A(300)$  is already bigger than its children. Then maximum is if you exchange down to the leaf with 600, 1200, 2400, .... You would do this  $\lfloor \lg(50000000/300) \rfloor = 17$  times.
3. Consider a heap  $H$  with length 1023.<sup>1</sup> Assume the elements of the array are distinct. Let  $x$  be the third largest element in the array. What are the possible positions for  $x$ .  
Solution: It can be anywhere on the second or third levels but nowhere else so it can be in positions 2, 3, 4, 5, 6, 7.  
Let  $y = H[700]$ . Can  $y$  be the largest element in the array?  
Solution: Yes, the largest element can be anywhere in the last row.  
Can  $y$  be the smallest element in the array?  
Solution: No, thats always at the top in position one.  
What is the smallest  $i$  so that it is possible that  $y$  is the  $i$ -th smallest element of the array.  
Its ancestors must be smaller than it but thats it. Node  $x$  has  $\lfloor \lg x \rfloor$  ancestors (or you can just enumerate them for 700: 350, 175, ... so that  $i = 9$ .

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<sup>1</sup>Did you recognize 1023 as a special number? Its one less than  $1024 = 2^{10}$ . The binary tree with that many nodes just fills out a row!

4. Page 69, Problem 8. (Do part (b) only for  $k = 3$ .)

Solution: (With two jars) Drop the first jar from  $x, 2x, 3x, \dots$  until it crashes. When it crashes between  $ix$  and  $(i + 1)x$  do the second jar one by one between them. So the first jar is used at most  $N/x$  times and the second one at most  $x$  times for a total of  $\frac{N}{x} + x$  times. Some calculus gives that it is best to take  $x = \sqrt{N}$  and then the total drops is  $2\sqrt{N}$  which is certainly  $o(N)$ .

With three jars again drop the first jar from  $x, 2x, 3x, \dots$  until it crashes. This takes  $\frac{N}{x}$  drops. Now we have  $x$  possibilities and we are in the two jar case so we can do the rest with  $2\sqrt{x}$  drops. The total is  $\frac{N}{x} + 2\sqrt{x}$ . Some calculus gives that it is best to take  $x = N^{1/3}$  and then the total drops is  $3N^{1/3}$  which is  $o(\sqrt{N})$  so the third jar helped. This pattern continues.

A person who never made a mistake never tried anything new.

– Albert Einstein