Errata on Randomized Quicksort

Let \( T(n) \) be the expected number of comparisons when doing randomized quicksort on \( A[1 \cdots n] \). The pivot \( i \) can be anything \( 1 \leq i \leq n \) in the actual order, all equally likely. When you pivot on the \( i \)-th you need recursively do randomized quicksort on lists of length \( i - 1 \) and \( n - i \) (in class \( n - i + 1 \) was written but that was an error), which take \( T(i - 1) \), \( T(n - i) \) respectively. This gives the recursion

\[
T(n) = n - 1 + \frac{1}{n} \sum_{i=1}^{n} [T(i - 1) + T(n - i)]
\]

As \( i \) ranges from 1 to \( n \), \( i - 1 \) ranges from 0 to \( n - 1 \) and \( n - i \) also ranges (backwards!) from 0 to \( n - 1 \) so we can rewrite

\[
T(n) = n - 1 + \frac{2}{n} \sum_{i=0}^{n-1} T(i)
\]

with the initial values \( T(0) = T(1) = 0 \).

This recursion can be solved – though the methods are beyond the scope of this class. You should know the answer, however. It is our mantra: \( T(n) = \Theta(n \lg n) \).