Fundamental Algorithms, Assignment 9
Due April 6 8:00 a.m. via Gradescope

The cautious seldom err. – Confucius

1. Consider the undirected graph with vertices 1, 2, 3, 4, 5 and adjacency lists (arrows omitted) 1 : 25, 2 : 1534, 3 : 24, 4 : 253, 5 : 412. Show the $d$ and $\pi$ values that result from running BFS, using 3 as a source. Nice picture, please!

2. Show the $d$ and $\pi$ values that result from running BFS on the undirected graph of Figure A, using vertex $u$ as the source.

3. We are given a set $V$ of boxers. Between any two pairs of boxers there may or may not be a rivalry. Assume the rivalries form a graph $G$ which is given by an adjacency list representation, that is, $\text{Adj}[v]$ is a list of the rivals of $v$. Let $n$ be the number of boxers (or nodes) and $r$ the number of rivalries (or edges). Give a $O(n + r)$ time algorithm that determines whether it is possible to designate some of boxers as $\text{GOOD}$ and the others as $\text{BAD}$ such that each rivalry is between a $\text{GOOD}$ boxer and a $\text{BAD}$ boxer. If it is possible to perform such a designation your algorithm should produce it.

Here is the approach: Create a new field $\text{TYPE}[v]$ with the values $\text{GOOD}$ and $\text{BAD}$. Assume that the boxers are in a list $L$ so that you can program: For all $v \in L$. The idea will be to apply $\text{BFS}[v]$ – when you hit a new vertex its value will be determined. A cautionary note: $\text{BFS}[v]$ might not hit all the vertices so, just like we had $\text{DFS}$ and $\text{DFS-VISIT}$ you should have an overall $\text{BFS-MASTER}$ (that will run through the list $L$) and, when appropriate, call $\text{BFS}[v]$.

Note: The cognescenti will recognize that we are determining if a graph is bipartite!

4. Show how DFS works on Figure B. All lists are alphabetical except we put R before Q so it is the first letter. Show the discovery and finishing time for each vertex.

5. Show the ordering of the vertices produced by $\text{TOP-SORT}$ when it is run on Figure C, with all lists alphabetical.

---

1Figures are on the website – not Gradescope – for Assignment 9
6. Let $S(n)$ satisfy initial condition $S(1) = 4$ and recursion $S(n) = S(n/7) + 11$ Assume $n$ is a power of 7. Give a precise formula for $S(n)$.

7. **Not to be Submitted!** If one person is purple on December 10, 2019 and the number of purple people doubles every five days, at what day does the number of purple people reach $7 \cdot 10^9$?

What is night for all beings is the time of waking for the disciplined soul. Bhavagad Gita, II.69