Basic Algorithms, Problem Set 7

Due by 8 a.m. Wednesdy, March 17.
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What you need is that your brain is open. – Paul Erdős

1. Determine an LCS of 10010101 and 010110110 using the algorithm studied.

2. Write all the parenthesizations of $ABCDE$. Associate them in a natural way with (setting $n = 5$) the terms $P(i)P(n-i)$, $i = 1, 2, 3, 4$ given in the recursion for $P(n)$.

3. Let $x_1, \ldots, x_m$ be a sequence of distinct real numbers. For $1 \leq i \leq m$ let $INC[i]$ denote the length of the longest increasing subsequence ending with x_i. Let $DEC[i]$ denote the length of the longest decreasing subsequence ending with x_i. `Caution:` The subsequence must *use* x_i. For example, $20, 30, 4, 50, 10$. Now $INC[5] = 2$ because of $4, 10$ – we do *not* count $20, 30, 50$.

   (a) Find an efficient method for finding the values $INC[i]$, $1 \leq i \leq m$. (You should find $INC[i]$ based on the previously found $INC[j]$, $1 \leq j < i$. Your algorithm should take time $O(i)$ for each particular i and thus $O(m^2)$ overall.)

   (b) Let LIS denote the length of the longest increasing subsequence of $x_1, \ldots, x_m$. Show how to find LIS from the values $INC[i]$. Your algorithm, *starting with* the $INC[i]$, should take time $O(m)$. Similarly, let DIS denote the length of the longest decreasing subsequence of $x_1, \ldots, x_m$. Show how to find DIS from the values $DEC[i]$.

   (c) Suppose $i < j$. *Prove* that it is impossible to have $INC[i] = INC[j]$ *and* $DEC[i] = DEC[j]$. (`Hint:` Show that *if* $x_i < x_j$ then $INC[j] \geq INC[i] + 1$.)

   (d) Deduce (*assume* (3c)) the following celebrated result (called the Monotone Subsequence Theorem) of Paul Erdős and George Szekeres: Let $m = ab + 1$. Then any sequence $x_1, \ldots, x_m$ of distinct real numbers either $LIS > a$ or $DIS > b$. (Idea: Assume not and look at the pairs $(INC[i], DEC[i])$. Try it with $m = 5, a = 2, b = 2$ on sequence $20, 30, 4, 50, 10$) Paul Erdős was a great twentieth century mathematician, whose work remains highly influential in many areas.

4. Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $5, 10, 3, 12, 5, 50, 6$.

5. Some exercises on logarithms:

   (a) Write $\lg(4^n/\sqrt{n})$ in simplest form. What is its asymptotic value.

   (b) Which is bigger, 5^{313340} or 7^{271251}? Give reason. (You can use a calculator but you can't use any numbers bigger than 10^9.)

   (c) Simplify $n^2 \lg(n^2)$ and $\lg^2(n^3)$.

   (d) Solve (for x) the equation $e^{-x^2/2} = \frac{1}{n}$.

   (e) Write $\log_n 2^n$ and $\log_n n^2$ in simple form.

   (f) What is the relationship between $\lg n$ and $\log_3 n$?

   (g) Assume $i < n$. How many times need i be doubled before it reaches (or exceeds) n?

   (h) Write $\lg[n^n e^{-n}\sqrt{2\pi n}]$ precisely as a sum in simplest form. What is it asymptotic to as $n \to \infty$? What is interesting about the bracketed expression?

I feel sunk in that long corridor between old values, actions, modes of thought, and those that I seek, that I'm working towards.
letter from Barack Obama at age 21)