Basic Algorithms, Problem Set 5

Due by 8 a.m. Wednesdy, March 3.
Send to Jingshuai: jj2903@nyu.edu.

> There's something you must remember ... you're braver than you believe, and stronger than you seem, and smarter than you think.
> Christopher Robin to Winnie-the-Pooh

1. Consider hashing with chaining using as hash function the sum of the numerical values of the letters (A=1,B=2,...,Z=26) mod 7. For example, h(JOE)= 10+15+5mod7 = 2. Starting with an empty table apply the following operations. Show the state of the hash table after each one. (In the case of Search tell what places were examined and in what order.)
   Insert COBB
   Insert RUTH
   Insert ROSE
   Search BUZ
   Insert DOC
   Delete COBB

2. Sunway TaihuLight [1] is tested on Karatsuba's algorithm and it takes 8 minutes to multiply two numbers, each having one billion digits. How long would it take to multiply two numbers, each having eight billion digits. (Assume the addition in Karatsuba's algorithm takes negligible time.)

3. **Just For Fun!**[2] Here is a puzzle from *Mathematical Puzzles*, a great new book by Peter Winkler:
   You have 25 horses and can race them in groups of five, but having no stopwatch you can only observe the order of finish. Find the *top three* horses (with their order) with seven races.

[1] supercomputer
[2] While Just for Fun problems are not graded, you are welcome to submit what you came up with!

4. This problem gives some approaches toward finding the median of n elements. Assume n odd for convenience.

   (a) Assume a routine `YUE[A]` which takes input $A[1 \cdots n]$ and returns that i so that $A[i]$ is in position precisely $\frac{n}{2} + \sqrt{n}$ [3] when the data is ordered. Assume further [4] that `YUE` takes time $O(n)$. Using `YUE` create an algorithm `MEDIAN[A]` that returns that i such that $A[i]$ is the precise median of the data and such that `MEDIAN[A]` takes time $O(n)$. (Or, if you wish, you can make the output the value of the median.) Give the analysis of the time.

   (b) (*) [5] Now suppose instead of `YUE[A]` you had `CARLOS[A]` which takes input $A[1 \cdots n]$ and returns some i so that $A[i]$ is in *some* position between $\frac{n}{2} - \sqrt{n}$ and $\frac{n}{2} + \sqrt{n}$ when the data is ordered and takes time $O(n)$. However, `CARLOS[A]` doesn't tell you precisely where $A[i]$ lies in the ordering. Using `CARLOS` create an algorithm `MEDIAN[A]` that returns that i such that $A[i]$ is the precise median of the data and such that `MEDIAN[A]` takes time $O(n)$. (Or, if you wish, you can make the output the value of the median.) Give the analysis of the time.

Usually when we hear or read something new, we just compare it to our own ideas. If it is the same, we accept it and say that it is correct. If it is not, we say it is incorrect. In either case, we learn nothing.
Thich Nhat Hanh

[3] Technically, the floor of that
[4] While YUE is fictitious, something similar does exist
[5] (*) represents a more challenging problem but still part of the assignment