Basic Algorithms, Problem Set 4

Due by 8 a.m. Wednesdy, February 24.
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> Ever since that night by the river, when Deeti had come to his help, Kalua had kept count of the days on which he was granted a glimpse of her, and the empty days in between. The tally was kept neither with any specific intension, nor as an expression of hope – for Kalua knew full well that between her and himself, none but the most tenuous connection could exist – yet the patient enumeration happened in his head whether he liked it or not; he was powerless to make it cease, for his mind, slow and plodding in some respects, had a way of seeking the safety of numbers.
> Amitav Ghosh, *Sea of Poppies*

When asked for the asymptotics answer in a form $\Theta(n^a)$ or $\Theta(\lg^b n)$ or $\Theta(n^a \lg^b n)$ for some reals a, b.

1. Consider the recursion $T(n) = 9T(n/3) + n^2$ with initial value $T(1) = 1$. Calculate the *precise* values of $T(3), T(9), T(27), T(81), T(243)$. Make a good (and correct) guess as to the general formula for $T(3^i)$ and write this as $T(n)$. (Don't worry about when n is not a power of three.) Now use the Master Theorem to give, in Thetaland, the asymptotics of $T(n)$. Check that the two answers are consistent.

2. Use the Master Theorem to give, in Thetaland, the asymptotics of these recursions:

   (a) $T(n) = 6T(n/2) + n\sqrt{n}$

   (b) $T(n) = 4T(n/2) + n^5$

   (c) $T(n) = 4T(n/2) + 7n^2 + 2n + 1$

3. Set $K = \lfloor\sqrt{N}\rfloor$. (You can ignore the floor which has negligible effect.) Let $A[1 \cdots N]$ be an (unsorted) array of numbers. Consider the following algorithm to output the $K + 1$-th largest value:

   ```
   BUILD-MAX-HEAP[A]
   FOR I=1 TO K
           EXTRACT-MAX[A]
           END FOR
   RETURN A[1]
   ```

(a) What is the time (by which we mean the number of flips of data) for the EXTRACT-MAX as a function of N and I. (Caution: The heap is getting smaller!)

(b) Express the total time for the FOR loop as a summation over I. Find the asymptotics of the sum.

(c) Analyze the total time this algorithm takes. Your answer should be $\Theta(g(n))$ for some "nice" $g(n)$.

4. Write the following sums in the form $\Theta(g(n))$ with $g(n)$ one of the standard functions. In each case give reasonable (they needn't be optimal) positive c_1, c_2 so that the sum is between $c_1 g(n)$ and $c_2 g(n)$ for n large.

   (a) $n^2 + (n+1)^2 + \ldots + (2n)^2$

   (b) $\lg^2(1) + \lg^2(2) + \ldots + \lg^2(n)$

   (c) $1^3 + \ldots + n^3$.

People complained about February; it was cold and snowy and oftentimes wet and damp and people were ready for spring. But for Cindy the light of the month had always been like a secret, and it remained a secret even now. Because in February the days were really getting longer and you could see it, if you really looked. You could see how at the end of each day the world seemed creacked open and the extra light made its way across the stark tres, and promised, it promised, that light and what a thing that was.
from *Olive Again* by Elizabeth Strout