Fundamental Algorithms, Problem Set 3
Due Tuesday, February 18 9 a.m. via GRADESCOPE
GRADESCOPE instructions on website

Well, you see, Haresh Chacha, its like this. First you have ten, that’s just ten, that is, ten to the first power. Then you have a hundred, which is ten times ten, which makes it ten to the second power. Then you have a thousand which is ten to the third power. Then you have ten thousand, which is ten to the fourth power - but this is where the problem begins, don’t you see? We don’t have a special word for that, and we really should. . . .

But you know, said Haresh, I think there is a special word for ten thousand. The Chinese tanners of Calcutta once told me that they used the number ten-thousand as a standard unit of counting. What they call it I can’t remember . . .

Bhaskar was electrified. But Haresh Chacha you must find that number for me, he said. You must find out what they call it. I have to know, he said, his eyes burning with mystical fire and his small frog-like features taking on an astonishing radiance.

– from A Suitable Boy by Vikran Seth

1. Write each of the following functions as $\Theta(g(n))$ where $g(n)$ is one of the standard forms: $2n^4 - 11n + 98$; $6n + 43n \log n$; $63n^2 + 14n \log^5 n$; $3 + \frac{2}{n}$

2. Illustrate the operation of RADIX-SORT on the list: COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX following the Figure in the Radix-Sort section. (Use alphabetical order and sort one letter at a time.)

3. Given $A[1 \cdots N]$ with $0 \leq A[I] < N^N$ for all $I$.
   (a) How long will COUNTING-SORT take?
   (b) How long will RADIX-SORT take using base $N$?

4. Just for Fun: What is the next term in the sequence

   $4, 14, 23, 34, 42, 50, \ldots$

   1Just for Fun problems are not graded!
5. Write the time \( T(N) \) (don’t worry about the output!) for the following algorithms in the form \( T(N) = \Theta(g(N)) \) for a standard \( g(N) \). For time, consider the total number of times \( X++, I=2*I, J++, J=2*J \) respectively are applied. (Note: * means multiplication, ++ means increment one.) The hardest is the last one, there is an outer FOR I loop, write the time it takes inside the loop as a function of \( I \) and (perhaps) \( N \). Then try (!) to add over \( I = 1 \) to \( N \).

(a) \( X=0 \)
   FOR I=1 TO N
   do FOR J=1 TO N
      X ++
   (b) \( I=1 \)
      WHILE I < N
      do I = 2*I
   (c) FOR I=1 TO N
      do J=1
         WHILE J*J < I
         do J ++
   (d) FOR I = 1 to N
      J=I
      WHILE J < N
      do J=2*J

6. Prof. Squander decides to do Bucket Sort on \( n \) items with \( n^2 \) buckets while his student Ima Hogg decides to do Bucket Sort on \( n \) items with \( n^{1/2} \) buckets. Assume that the items are indeed uniformly distributed. Assume that Ima’s algorithm for sorting inside a bucket takes time \( O(m^2) \) when the bucket has \( m \) items.

(a) Argue that Prof. Squander has made a poor choice of the number of buckets by analyzing the time of Bucket Sort in his case.
(b) Argue that Ima has made a poor choice of the number of buckets by analyzing the time of Bucket Sort in her case.

A good traveler has no fixed plans and is not intent on arriving. A good artist lets his intuition lead him wherever it wants. A good scientist has freed himself of concepts and keeps his mind open to what is.
Laozi, Philosopher and Writer, 6th century B.C.