Basic Algorithms, Problem Set 1
NOT TO BE SUBMITTED

The world can be divided into those who love New York City and
those who don’t. Those who love New York tend to be unusually
lively people. They have to be. Characteristically, they are
ambitious, curious, intellectually vigorous, culturally alive. Such
people give New York City institutions great dynamism and some
eccentricity.
James Hester 1924-2015, NYU President

1. Let $A$ is a max-heap with heapsize fifty million, being used as a priori-
   tity queue. Suppose $\textsc{Heap-Increase-Key}(A, 300, \text{key})$ is called. What is
   the maximum number of exchanges that can take place. What is the
   minimal number of exchanges that can take place.

2. When $A$ is a array with length fifty million and $\textsc{Max-Heapify}(A, 300)$
   is called. What is the maximum number of exchanges that can take
   place. What is the minimalist number of exchanges that can take
   place.

3. Consider a min-heap $H$ with length 1023.  
   Assume the elements
   of the array are distinct. Let $x$ be the third smallest element in the
   array. What are the possible positions for $x$. Let $y = H[700]$. Can $y$
   be the largest element in the array? Can $y$ be the smallest element in
   the array? Give all $i$ for which it is possible that $y$ is the $i$-th smallest
   element of the array.

4. Using the figures in the text as a model, illustrate the operation of
   $\textsc{Build-Max-Heap}$ on the array $A = (5, 3, 17, 10, 84, 19, 6, 22, 9)$

5. Shower Exercise² Recite the powers of 2 from 1 to 65536. If your
   mother tongue is other than English alternate between English and
   your mother tongue.

6. The operation $\textsc{Heap-Delete}(A, t)$ deletes the item in node $t$ from heap
   $A$. Give an implementation of $\textsc{Heap-Delete}$ that runs in $O(\lg n)$ time
   for an $n$-element max-heap.

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¹Did you recognize 1023 as a special number? Its one less than $1024 = 2^{10}$. The binary
tree with that many nodes just fills out a row!

²These are occasional somewhat fanciful exercises to give you a gut feeling for the
material.
7. Let $A$ be an array of length 127 in which the values are distinct and in increasing order. In the procedure BUILD-MAX-HEAP($A$) precisely how many times will two elements of the array be exchanged? Now suppose the values are distinct and in decreasing order. Again, in the procedure BUILD-MAX-HEAP($A$) precisely how many times will two elements of the array be exchanged?

Computer science is no more about computers than astronomy is about telescopes.
Edsger Dijkstra